ICMI STUDY 25
Study Conference

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Study Themes

ICMI STUDY 25

Theme A | Theoretical Perspectives on Studying Mathematics Teacher Collaboration
Theme Co-Chairs: João Pedro da Ponte, Portugal
Takeshi Miyakawa, Japan

Theme B | Contexts, Forms and Outcomes of Mathematics Teacher Collaboration
Theme Co-Chairs: Cristina Esteley, Argentina
Rongjin Huang, USA

Theme C | Roles, Identities and Interactions of Various Participants in Mathematics Teacher Collaboration
Theme Co-Chairs: Shelley Dole, Australia
Ronnie Karsenty, Israel

Theme D | Tools and Resources Used/Designed for Teacher Collaboration and Resulting from Teacher Collaboration
Theme Co-Chairs: Ornella Robutti, Italy
Luc Trouche, France
CONTENT-SPECIFIC THEORY ELEMENTS FOR EXPLAINING AND ENHANCING TEACHERS’ PROFESSIONAL GROWTH IN COLLABORATIVE GROUPS

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In order to explain and enhance teachers’ professional growth in collaborative groups, sociocultural theoretical perspectives and the construct of community of inquiry have been established. This plenary paper builds upon these achievements and suggests eight kinds of theory elements that can refine these perspectives and allow theorizing in more content-specific ways, where content refers to both, classroom content and PD content. The paper suggests an adapted model for professional growth as the framework for content-specific theorizing.

As the insightful ICME survey (Robutti et al., 2016; Jaworski et al., 2017) has shown, collaborative groups form a widespread and promising environment for teachers’ professional development (PD), and many different variants have already been explored in research and PD practice. However, the ICME survey also revealed that only 85 out of the 316 papers on teacher collaborative learning that were analyzed explicitly referred to theoretical perspectives on collaboration. This finding shows the need for more comprehensive and systematic theoretical foundations for explaining and enhancing PD in collaborative groups (PDCG). This plenary paper argues that in order to elaborate a theoretical foundation for teachers’ professional growth in collaborative groups, further theory elements should be integrated at the classroom and the PD levels. In particular, it advocates including a theory of professional growth (Clarke & Hollingsworth, 2002) and content-specific theory elements capturing the PD learning content. In this way, the paper focuses on content-specific theorizing, i.e. of creating new theoretical contributions and the relation to the background theoretical perspectives the research builds upon (Mason & Waywood, 1996; Prediger, 2019a). To illustrate my meta-theoretical messages, I discuss a vignette and results from two PD research projects (Prediger, Fischer, Selter, & Schöber, 2019; Prediger, Kuhl, Büscher, & Buró, in press) where secondary mathematics and special needs teachers develop their practices for fostering slow learners and for differentiating in inclusive classrooms.

After a brief introduction of PDCG and the construct of practices in Section 1, an introductory vignette in Section 2 illustrates why we need more theorizing. Section 3 disentangles the kinds of theory elements required for a theoretical foundation of PDCG by exploiting the analogy of classroom and PD levels. On this meta-theoretical base, Section 4 specifies the relevant theory elements for the introduced vignette and the PD it stems from, which provides the concrete material for the meta-theoretical reflections in Section 5.
1. Existing background theories for professional development in collaborative groups: Communities of practice and inquiry

According to the ICME survey (Robutti et al., 2016), 80% of 85 papers analyzed that explicitly referred to teacher collaboration in their theory sections addressed the theoretical construct of community, often in the sense of community of practice (Wenger, 1998; Lave & Wenger, 1991) or community of inquiry (Jaworski, 2006). Whereas some articles only referred to community in order to name the PD setting for practical reasons, most articles also referred to underlying sociocultural theories of learning. Within these sociocultural theories, learning is conceptualized as being situated in communities of social practice, where novices are successively drawn into practices, first from a legitimate peripheral position then to the center of the experienced practitioners. When these theoretical perspectives are related to teachers’ learning, teaching is thereby conceptualized as a set of social practices, and professional growth is described by increasing engagement and alignment.

Jaworski (2006) has enriched this theoretical perspective by emphasizing that PD itself is shaped by a set of social practices in which alignment should not be assimilation, but critical alignment. This offers the opportunity for collective professional growth within a community of practice rather than the stability of only enculturating novices. She is widely cited for the enriched construct of communities of inquiry in which “participants . . . align with aspects of practice while critically questioning roles and purposes as a part of their participation for ongoing regeneration of the practice” (p. 190). In both perspectives, practices form the key theoretical construct. Practices in mathematics classrooms can be defined as “ways of acting that have emerged . . . [that make] it possible to characterize mathematics as a complex human activity and in that it brings meaning to the fore by eschewing a focus on socially accepted ways of behaving” (Cobb, Stephan, McClain, & Gravemeijer, 2001, p. 120). To apply the analogy, teaching practices are ways of acting that have socially emerged to manage typical situational demands at the classroom level, while practices of inquiry refer to those at the PD level (see below).

Putnam and Borko (2000) had already advertised lifting sociocultural, situated theoretical perspectives from the classroom level to the PD level. In order to effectively exploit these perspectives, they underlined the need for further considerations on (1) where to situate teachers’ learning experiences, (2) the nature of discourse communities for teacher learning, and (3) the importance of tools in teachers’ professional learning experiences. In this plenary paper, I extend their call for further considerations into a call for further theorizing.

2. Why we need more theorizing: An introductory vignette from a community of inquiry

Vignette. The first phase our PD research project on inclusion involved observing communities of inquiry of mathematics teachers and special needs teachers who were attempting to collectively develop their teaching practices for a new pedagogical demand, namely, integrating students with special needs who were previously taught in segregated special needs schools. The vignette is from the first meeting in which the researcher facilitator started collaboration of 18 months with the teacher group. At the moment of this first meeting, the teachers had already spent 9 months on finding ways for differentiating their teaching material in order to adapt to students’ diverse mathematical profiles. After nine months of intense collective work, they were proud to have substantially changed their
teaching practices in order to adapt to all students’ abilities, mainly in task-based individualized settings. When the researcher facilitator first met them, Paul, the mathematics teacher, reported about Suleika, one of their students with learning difficulties, and showed two of her products on multi-digit subtraction (in Figure 1):

Paul: Suleika can calculate the subtraction well, only the carries pose problems for her. But we can handle this successfully by differentiated tasks: I only give her subtractions without carries.

Although the community agreed on the success of their changes, Maria, the special needs teacher, took three months for convincing her colleagues to participate in a PD program, as she saw an additional problem to be tackled:

Maria: I tried to teach them subtraction with carries several times, but they always forget it.

**Analysis.** Paul’s utterance was a characteristic expression of this community’s adaptive differentiating practices: Students get individualized tasks that they can succeed in. Within the frame of the teachers’ collective evaluation, the differentiation practice proved to be successful, as Suleika was able to complete her tasks. In spite of the intense engagement of these teachers’ community of inquiry, they could not develop more productive practices for enhancing Suleika’s learning. Although Suleika’s second product reveals serious struggle with place value understanding (see Figure 1), this was not treated by the teachers. Maria’s additional concerns that students “always forget” had also not yet entered the teachers’ collectively shared space of discourse.

**What the analysis leaves out.** This brief analysis reveals the importance of shared orientations of what the teachers consider relevant in their community of practice. However, the language of analysis provided by the theoretical constructs from Section 1 is not yet well enough elaborated to identify the critical points in more detail. A rough analysis outside the given theoretical framework reveals: For them, differentiation meant adapting to students’ abilities rather than really strengthening their learning, so their shared criterion for evaluation was reduced to task completion, not to learning growth. The reduction to this criterion also reduced the need for critical alignment with their own teaching practices. At the same time, the teacher community did not distinguish between procedural and conceptual knowledge, as they did not problematize whether teaching Suleika the algorithm with carries might miss the conceptual base. A theoretical foundation that can really inform facilitators’ work in supporting the development of this community of inquiry will need to include these aspects. It will also need to find a language for explaining why Maria was not able to introduce her “students forgetting” criterion into the shared discourse.

Section 3 will introduce the meta-perspective on generic and content-specific theory elements for the first steps towards a content-specific sociocultural theory of professional growth. Section 4 then discusses the concrete elements that can explain the vignette and guide action in supporting a community’s professional growth.
3. What kind of theory elements are required for a theoretical foundation for PDCG?

This chapter starts with structural meta-theoretical clarifications: What is a theory, and why do I only speak about theory elements? What kind of theory elements are required for a theoretical foundation of PDCG that provides a framework for explaining and enhancing teachers’ professional growth? Theory elements vary in their logical structure, in size and in their function (Prediger, 2019a).

Niss (2007) characterizes a theory by its logical structure as an “organized network of concepts (including ideas, notions, distinctions, terms, etc.) and claims about . . . objects, processes, situations, and phenomena” (p. 1308). The claims can be basic hypotheses, statements logically derived from the fundamental claims, or empirically grounded propositions about connections and mechanisms.

Theories vary in size: some encompass a well-elaborated theoretical perspective with a complex network of constructs and propositions (such as sociocultural theory) and are sometimes reduced to single constructs or claims (such as the construct communities of practice). Rather than networking complete theoretical perspectives (Bikner-Ahsbahs & Prediger, 2014), this paper focuses on the local integration of several constructs and claims; this networking strategy is more suitable for fields that are not yet mature enough for big theories (as Jaworski, 2006, stated for the field of PDCG).

In order to decide which theory elements have to be integrated for a theoretical foundation for PDCG, a distinction according to their function in the design and research process us useful (Prediger, 2019a) and use the structural analogies between the classroom level and the PD level (see Figure 2). On the classroom level, theory elements (i.e., constructs, basic assumptions, and empirically grounded connections) for four main functions have been established (Mason & Waywood, 1996; Prediger, 2019a):

- C1 for specifying and structuring the mathematical learning content,
- C2 for explaining mechanisms of mathematics learning,
- C3 for explaining the nature and background of mathematics teaching, and
- C4 for designing and enacting learning environments.

![Figure 2: Lifting theory elements from the classroom to the PD tetrahedron](image)

As Figure 2 visualizes, these functions refer to different parts of the didactical tetrahedron on the classroom level. Exploiting the structural analogy between the classroom didactical tetrahedron and...
the PD tetrahedron makes it possible to lift the elements (Prediger, Roesken-Winter, & Leuders, 2019), inferring the need for analogous theory elements on the PD level:

- PD1 for specifying and structuring the PD content;
- PD2 for explaining mechanisms of teachers’ professional growth;
- PD3 for explaining the nature and background of facilitating PDs, if a facilitator exists; and
- PD4 for designing and enacting PD environments.

4. Generic and content-specific theory elements for explaining and enhancing PDCG

Coming back to the vignette from Section 2, the researcher facilitator intended to receive a profound understanding of the collaborative group’s practices and challenges before changing the role from observer/analyzer to a facilitator of the PDCG and before supporting the group to develop their critical inquiry stance and their teaching practices.

These dual practical goals, the facilitator’s understanding and then intervening, have a counterpart on the theorizing side: The repeated (and much more systematic) analysis of these kinds of vignettes can enhance the researchers’ theoretical understanding. Systematically connecting the theoretical elements can generate a theoretical underpinning for typical facilitation practices and designs for PDCG. That is how we aim to find a theoretical foundation for enhancing teachers’ professional growth. For this theorizing purpose, it proved to be highly relevant to unpack the theory elements on the classroom and PD levels, not only by means of generic theory elements, which apply to all classroom and PD content, but also content-specific theory elements, in this case, where the classroom content is understanding multi-digit subtractions and the PD content is differentiating in inclusive mathematics classrooms.

Although it is not possible to demonstrate the theorizing process with all its details here, this section intends to show the power of working with articulated theory elements unpacked down to the level of the mathematical content. I will successively introduce theory elements for all the different functions and use them for the analysis. The analysis starts at the classroom level to build the ground for analyzing the group’s teaching practices and the group’s processes of professional growth later at the PD level. It ends with a look at how the facilitator reacted and how this experience informed the PD design for future collaborative groups.

**Introducing theory elements for C1.** On the classroom level, the theory elements C1 for specifying the classroom content in view are printed in Table 1. The table distinguishes conceptual understanding and procedural skills (Hiebert & Carpenter, 1992), but also the actual learning content and its foundations from previous years. The examples in Table 1 relate to the classroom content for multi-digit subtraction and its conceptual underpinnings (discussed in Hiebert

<table>
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<tr>
<th>Actual content (Grade 5)</th>
<th>Conceptual understanding of actual topic, in this case, regrouping units while subtraction</th>
<th>New procedures, in this case, multi-digit subtraction procedure with carry</th>
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<tr>
<td>Foundations from previous years (Grade 2)</td>
<td>Basic conceptual needs, in this case, place-value understanding (meaning of digits)</td>
<td>Basic skills, in this case, basic subtraction facts up to 10</td>
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Table 1: Theory elements of C1 for specifying the classroom content, with content trajectory for structuring it
& Wearne, 1996). The arrow indicates the typical trajectory according to which the content can be structured (see C2): basic conceptual needs to underpin basic skills, and these are necessary for building further conceptual understanding, which then underpins the new procedures.

Analyzing the vignette with respect to C1. In Suleika’s case (see Figure 1), she mastered the basic skills of subtraction facts up to 10 and used them for multi-digit subtraction without carry. Subtraction with carry, however, is based on the conceptual understanding of decomposing numbers into digits. Suleika could not build on this due to the basic conceptual need of fundamental place-value understanding (which becomes visible in her decomposition of 443; see Figure 1).

Introducing theory elements for C2 and C4. The basic mechanism of learning C2 that helps to explain Suleika’s challenge in remembering the procedure of subtraction is that sustainable learning always requires connecting to previous knowledge, best accomplished with settled understandings (Hiebert & Carpenter, 1992, p. 67). Hence, students who have no access to specific basic conceptual understandings cannot continue learning along the content trajectory, in other words, the actual conceptual understanding or the procedural skills building upon these basic needs are not accessible. This proposition about the structure of the generic content trajectory has been empirically proven in long-term assessment studies (e.g., by Moser Opitz, 2007) and resonates with early results on understanding multi-digit subtraction (Hiebert & Wearne, 1996). Consequently, our design research group has designed a learning environment that enables teachers to formatively assess and foster students’ basic conceptual needs (Prediger, Fischer, Selter, & Schöber, 2019). The theory elements C4 underlying the learning environment include three design principles (for their empirical and theoretical justification, see Moser Opitz et al., 2017): DP1, focusing on conceptual understanding; DP2, ensuring adaptivity using diagnostic tasks; and DP3, promoting discourse. This learning environment was shown to be effective for giving students safe access to the basic conceptual needs, with significantly higher learning gains than the control group (Prediger et al., 2019). However, the learning gains varied largely and relied heavily on the teachers, so further PD research is required for optimizing support for all collaborative groups.

Analyzing the vignette with respect to C2 and C4. In the vignette, Suleika’s learning pathway was not aligned to the trajectory C2 in Table 1, and the fact that the teachers tried to teach her later stages of the content trajectory without taking into account the earlier stages might explain why she always forgot the content. However, the teachers’ practices for differentiating did not rely on the categories from C1 (Table 1), nor on the propositions C2 and design principles C4. Instead, they were based on the practices the community of inquiry collectively developed, independent from the design research team. Rather than blaming the teachers for not using this approach, theory elements C3 are required which allow the researcher facilitator to explain their practices and their forms of inquiry, acknowledging the enormous efforts they conducted in their community.

Introducing theory elements for C3. In line with Wenger (1998) and Jaworski (2006), teachers’ professional practices have been defined as socially established ways of mastering recurrent situational demands in mathematics classrooms (such as differentiating and fostering low achievers). To explain these teaching practices in more depth, this paper refers to Bromme’s (1992) situated construct of teacher professional expertise and adapts it to a situated sociocultural perspective. According to Bromme, practices are visibly characterized by shared pedagogical tools (e.g., tasks, teacher
moves), but are explainable only by the underlying orientations (socially shared beliefs) and the activated categories for perceiving, thinking, and evaluating. Bromme’s general framework gains its explanatory power for content-related purposes when filled in content-specific ways (Prediger, 2019b): To explain teachers’ practices, we identify the socially shared content-specific pedagogical tools, orientations, and filtering categories that underlie their utterances and visible behavior for mastering the self-posed situational demands.

Analyzing the vignette with respect to C3. With respect to our vignette, the practices to be analyzed by the theory elements C3 are Paul’s and Maria’s (and their colleagues’) differentiating practices for their inclusive classroom and their fostering practices for her student Suleika. The teacher community was driven by the shared inclusive orientation (a good inclusive classroom is adaptive to students’ abilities) (orientations are indicated by <…>), hence design principle DP2 here serves as the shared guiding orientation. In order to realize it, they use the pedagogical tools of differentiated tasks and activity settings of individualized learning. The major category by which they perceive the initiated processes and by which they evaluate and optimize their differentiating practice is ||task completion|| (categories are indicated by ||...||). Applying this category, the teachers evaluate the short-term success by assessing whether a student was able to complete the task with the given support and simplifications. And, indeed, simplifying the mathematical demands involved in subtractions without carries proved to be efficient for fulfilling their evaluation category ||task completion||. However, this category is guided by a <short-term> orientation, whereas <long-term> would refer to ||learning growth||.

Deriving theory elements for PD1. Brief analysis revealed first content-specific categories and orientations for the PD content “differentiating and fostering students in inclusive classrooms” that the observed collaborative group held. Careful analysis of many other vignettes revealed a specification of further content-specific categories and orientations that could bring the teachers’ collective inquiry forward. The researchers who observed several groups finally structured them in Figure 3, which includes, as a nested core, the categories for specifying and structuring the classroom content. Figure 3 also later turned out to be a useful tool for communication with teachers (although it was not existent during the vignette itself). The C1 categories can help teachers to make decisions about learning goals and assess students’ learning pathways along the content trajectory. However, the PD1 <short-term or long-term orientation> determines whether teachers focus on ||learning growth|| or on ||task completion|| as their major category of thinking, perceiving, and evaluating their practices.

Introducing theory elements for PD2. To explain the mechanisms of teachers’ professional growth, I follow the ideas of Jaworski’s (2006) communities of inquiry and exploit them content specifically by an adapted version of Clarke and Hollingsworth’s (2002) model of professional growth.

![Figure 3: Specifying the PD content: Teachers’ categories and orientations](image)
Jaworski (2006) emphasizes the condition for community growth that communities do not only align with current practices but engage in inquiry for critical alignment of their practices (and of new input). For this, she describes the shared categories of evaluation as crucial. This idea can be combined with Clarke and Hollingsworth’s (2002) model for professional growth, which has often been cited for designing and enacting PD environments (PD4) and for explaining teachers’ professional growth (PD2), although mostly without explicit focus on collaborative groups. Clarke and Hollingsworth (2002) provided a model that relates the different domains of a change environment without bringing them into a naive chain of implementation steps, from the external domain (with external sources of information, stimulus, or classroom resources) via changing the personal domain (teachers’ knowledge or attitudes, here conceptualized from a sociocultural perspective as the collective domain of teachers’ shared orientations and categories) to the domain of practice (here conceptualized as the domain of inquiry for new practices) and then to the domain of consequence (with salient outcomes such as students’ learning gains). Rather than assuming such a simple unidirectional chain of effects, they emphasize the mutual interplay of different domains with different pathways of enactment and reflection. This model is suitable for our purposes as it builds upon Putnam and Borko’s (2000) call for authentic situated learning opportunities, which is here realized by the emphasis on the domain of inquiry. As the model was originally articulated in generic terms and mainly for individual teachers, it is here adapted to the sociocultural perspective and content-specific explorations, as indicated for the vignette in Figure 4. Particularly, this adaptation also allows analysis of the connection between individual and collective concerns, which is crucial for the conditions of collaborative growth.

**Analyzing the vignette with respect to PD2 and PD1.** Until the facilitator came in and provided external sources, the vignette can be analyzed in the three lower domains: the collective domain, the domain of inquiry, and the domain of consequences. Based on the status quo in the collective domain (as presented in the analysis by C3), the community of inquiry was working hard on new differentiation practices (domain of inquiry). However, their current focus on the evaluation category ||task completion|| had substantial impact on the way they perceived their success in the domain of consequence. At the same time, outside the shared collective domain, Maria, the special needs teacher, puts a second evaluation category on the table, ||forgetting||. The category of ||forgetting|| is a remarkable
one, since, while it refers to the category identified as crucial in PD1, learning growth, it does not relate students’ learning to teachers’ practices, explaining unsuccessful learning growth solely within the students. In this way, it has not yet initiated a focused reflection in the domain of inquiry, but a call for external support. Indeed, Paul and other colleagues articulated that they were not interested in forgetting, as this cannot guide their teaching and assumes too much responsibility by the students (Jackson et al., 2017). Here, incompatibility of teachers’ orientations explain why the community of inquiry could not adopt Maria’s concerns. The researcher facilitator first met the community of inquiry in this moment. With the theory elements of the specified PD content in mind (see Figure 3), she noticed that the teachers were not concerned about Suleika’s place value understanding, although her written product in Figure 1 provides a strong evidence of its peculiarity. Thus, the teachers did not activate the category basic conceptual needs for assessing Suleika’s work. The researcher facilitator assessed that the community of inquiry was neither driven by the orientation understanding before procedures nor by long-term rather than short-term, which would have led them to focus on her basic conceptual needs rather than trying to teach her multi-digit subtraction without any place value understanding. A first rough approximation of this analysis allowed the researcher facilitator to explain why the teachers’ enormous efforts had not yet led to satisfying long-term results and how to enter the discourse with the collaborative group in order to turn Maria’s concern into a collective and productive concern.

Consequences for PD3 and PD4 in the continued vignette. In order to draw consequences for PD3 and PD4, no further theory elements are required, as all relevant aspects can be articulated by the model in Figure 4. The facilitator researcher’s practice for enhancing professional growth started with listening to the teachers and analyzing their collective efforts in all three domains by means of the theory elements PD1 and PD2 (Figures 3 and 4). Following the principle of building upon teachers’ collective resources, she realized that understanding before procedures was not the ideal starting point to discuss in this community, as being too far from their actual collective concern. Instead, she chose short-term versus long-term orientation in order to relate Paul’s and Maria’s points and build upon Paul’s intention to consider aspects that they can influence in order to establish a new shared orientation. The following utterances are synthesized from a longer conversation:

Facilitator: Paul says you can handle Suleika’s difficulties successfully by giving her only subtractions without carries. However, Maria does not seem to be satisfied with the learning outcome. What is the problem with Suleika always forgetting the procedure, Maria?

Facilitator: You also seem to be interested in the long-term learning. Can we go back some steps and check what Suleika can master on her learning pathway towards multi-digit subtraction? I see how she decomposed the numbers (443 = 400 – 400 – 300); do you think this could have any impact on her ability to remember the procedure of managing carries?

After 30 minutes of discussion, Paul, Maria, and their colleagues collectively decided that they needed to go back in the content trajectory to stabilize Suleika’s learning pathway in a more sustainable way. It took them much longer to realize that they needed to provide learning opportunities for the basic conceptual needs, and that this might be also the more productive practice of differentiation. Thus, the first external offer provided by the facilitator could strengthen Maria’s implicit orientation long-term instead of short term, which also opened the teachers to other evaluation categories for their teaching success. However, to enact teaching practices towards the new evaluation category
learning growth in basic conceptual needs, they required further external offers, namely pedagogical tools for formatively assessing students’ basic conceptual needs and teaching material for fostering them.

The externally offered curriculum materials for formative assessment and remediating sessions not only provided them with the required pedagogical tools, which they could now integrate into their practices, but also with access to the detailed pedagogical content knowledge on basic conceptual needs for other mathematical topics, such as place value understanding on the number line and meanings of multiplication and division (see Prediger et al., 2019). Once the teachers had incorporated these categories and orientations in their collective domain, their inquiries resulted in bigger changes of their practices and a closer approximation to the newly set goals: fostering all students’ basic conceptual needs in order to assure adaptivity to the individual learning pathways. Based on these experiences, it took another year before the orientation understanding before procedures really started to guide their work. Interestingly, it entered their collective domain via the domain of inquiry, when experimenting with the curriculum materials for all students and experiencing “lovely aha-moments, when students say ‘now, I really got it!’” In this way, the teachers’ pathways of long-term collective professional growth reflected an interesting interplay between the four domains.

Tentative content-specific theorizing on PD2 and PD4. As these considerations illustrate, the adapted model for PDCG can serve as a theoretical framework for explaining teachers’ professional growth (PD2) and for taking actions to enhance it (PD4). However, the general model only provides the framework for necessary content-specific theorizing. The analysis of this vignette and many further cases (e.g., Prediger et al., in press) resulted in the first tentative theorizing about teacher communities’ learning pathways towards striving for differentiating practices (PD2) and the roles of external resources such as classroom material in supporting the process (PD4): When communities of inquiry work on innovative practices (in this case, on their differentiating practices), their evaluation categories in the domain of consequence might be the most crucial to develop.

Lessons learned. The various case studies from the PD projects in views are currently theorized. One central theoretical contribution is the empirically grounded hypothesis is that the teacher communities’ learning of the PD content “inclusive mathematics teaching” might be characterized as progression of evaluation categories in four stages:

1. ||Work intensity|| “All students work eagerly.” (no matter on what)
2. ||Task completion|| “All students complete the tasks.”
3. ||Procedural learning growth|| “Students develop procedural skills with adaptive demands.”
4. ||Conceptual learning growth|| “Students develop conceptual understanding, if necessary on basic conceptual needs.”

Hence, initiating shifts in the evaluation categories might be the most crucial external input required to allow teacher communities to continue their independent inquiries. Additionally, providing curriculum materials for formative assessment and fostering sessions may not only provide a pedagogical tool for strengthening teaching practices, but it may also influence the collective domain, namely the shared pedagogical content focus on further basic conceptual needs in different mathematical content. These two hypotheses will require further systematic investigation before they can count as empirically grounded theory elements.
With respect to Putnam’s and Borko’s (2000) three areas of consideration, I conclude the following hypotheses for PDCG:

1. It is worth situating teachers’ learning experiences in communities of inquiry with emphasis on the domain of inquiry.
2. Classroom materials can support the teachers’ formative assessment and fostering practices and at the same time serve as a tool to extend the communities’ shared PCK.
3. The nature of the discourse in the PDCG is heavily influenced by shared orientations and categories, specifically by evaluation categories. Shifting these categories to the domain of consequence seems to be a crucial starting point for the external domain.

5. Meta-theoretical reflections on the necessary topic-specific theory elements

This paper intends to contribute to developing theoretical foundations for explaining and enhancing teachers’ professional growth in collaborative groups. Building upon the general sociocultural perspective on teachers’ practices (Wenger, 1998) and the construct of communities of inquiry (Jaworski, 2006), it used the exemplification in one vignette in the community of inquiry to show:

- Theory elements of PD content (PD1), teacher learning (PD2), facilitating (PD3), and PD settings (PD4) are all useful and necessary for explaining and enhancing professional growth.
- The structures of the big theoretical frameworks (communities of inquiry, models of professional growth) are helpful in understanding the complexities and intertwinment of different domains. However, they mainly provide a generic search space. Informing the concrete analysis and especially the concrete PD design and facilitation, they must be elaborated in content-specific ways for different PD content (Prediger et al., 2019). Whereas content on the classroom level refers to specific mathematics topics, content on the PD level refers to the specific teaching practices that the communities of inquiry have chosen to work on, in our case, differentiating and fostering the learning of students with difficulties.
- Although PD practices are always content specific, research papers and particularly theorizing processes tend to abstract from these contents; however, we should talk more about content-specific theorizing.
- The generic theory elements gain their explanative power when being filled in content-specific ways, and this also requires the nesting of corresponding theory elements (C1–C4) on the classroom level into the PD level. The more this nested structure is unpacked, the more we learn in content-specific ways about the PD content (ibid.).

Whereas the above arguments are not specific to the form of PD in view here, PDCG, the main result of the current study might be very characteristic for the collaborative setting: The shared evaluation categories seem to be the crucial point, more than the shared knowledge or orientations as a whole. As long as individual evaluation categories have not really entered the collective domain (as Maria’s ‖forgetting‖), they cannot unfold their influence, and this can also hinder the professional growth of the individual within the collaborative group. This interplay of individual and collective learning in particular will require substantial further empirically grounded theorizing.

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