RESEARCHING MATHEMATICS EDUCATION AND LANGUAGE DIVERSITY: THE THEORIES WE USE AND DEVELOP, AND WHAT THEY ENABLE US TO SEE, SAY OR DO

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INTRODUCTION TO THE RESEARCH FORUM: THEORIES AND THEORETICAL CHALLENGES IN MATHEMATICS EDUCATION RESEARCH ON LANGUAGE DIVERSITY

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This Research Forum focuses on and discusses a range of theories and approaches to theory used in research on mathematics education and language diversity. This topic is particularly appropriate for a PME conference held in South Africa, a country with 11 official languages, and a leader on research in this field for several decades. A Research Forum on this topic, ‘Researching mathematics education in multilingual contexts: Theory, methodology and the teaching of mathematics’, was last held in 2004 during PME28 in Bergen, Norway, under the coordination of Richard Barwell and Philip Clarkson, in which Judit Moschkovich and Mamokgethi Phakeng (Setati) also participated in the roles of contributor and discussant. The coordinators and whole group of contributors at that time were concerned with the significance of theory in research into multilingualism within mathematics education and its interdisciplinary nature. In the introduction and discussion of theories of language mostly close to applied linguistics but also linguistic anthropology and other fields, Barwell and Clarkson (2004, pp. 227-228) specifically asked, “What theories are relevant to work in mathematics education? How might these theories be applied in mathematics education? What are the challenges which arise from working with theories from other disciplines?” One of the challenges exemplified was the visibility of mathematics in linguistic analyses of mathematics classroom interaction, and the related more general challenge of working with both language and mathematics while keeping them both in view in mathematics education and mathematics education research.

Investigation on this topic has received much attention and grown since then, with manuscripts, edited volumes and new terrains and initiatives of study oriented to interrogating and addressing the mesh of affordances, limitations and objectives of
different theories to move research and practice forward. In the debates on which language(s) to use and why, how and when to use them in, for example, multilingual mathematics classrooms (Bose & Phakeng, 2017), research has interpreted and adopted broadly sociocultural and discursive perspectives. Nonetheless, recent reviews of research on mathematics education, language and language diversity (Barwell et al., 2016; Planas, Morgan & Schütte, 2018; Planas & Schütte, 2018; Radford & Barwell, 2016; and others) show the complexity of perspectives and theoretical grounds contributing to the domain. We thus continue to struggle with the major endeavour and need of articulating and developing theory. In this endeavour, a careful discussion of the tapestry of theories and epistemologies can help to uncover traditions and frameworks, both outside and inside mathematics education, with a possible goal to more clearly delimit the understanding, scope and range of the area of study and of some of its substantial transformations and distinctive changes.

In this Research Forum, we pose to ourselves, to the community of our common area of study and to the broader PME audience and readers the following guiding questions:

- What theories have been used and developed in research on mathematics education and language diversity and why?
- What have the theories made visible/invisible and what have they enabled us to say and/or to do as researchers and/or practitioners?

Four decades since the inception of the PME conferences, the world is becoming more multilingual and increasing awareness of the importance of the languages of all groups and communities. It is timely to examine what theories have been used or whose development has emerged to explore the complexities of language diversity in mathematics classrooms. PME43 is an opportune time for us as researchers to reflect on and interrogate the theories that inform our mathematics education research in settings of language diversity. Systemic functional linguistics, anthropological linguistics and sociolinguistics, sociocultural and discourse theories, social and cultural semiotics, structural and applied linguistics and more increasingly often combinations of them, to name a few, show that a single theory cannot fully explain the progress of this area of study. Considering this, it is crucial to create opportunities for conversation among researchers using existing theories, and for the generation of theoretical revision, refinement and articulation. While the contributors to this Forum have developed well-recognized expertise using specific theories, their reflections serve to frame and open a discussion of the ways in which theories may influence one another in order to deepen and extend insights. As the named theories largely stem from linguistics, the specific need emerges to develop theories further for grasping the relation between mathematics learning and language. In spite of the maturity of some networking work in the broader field (Bikner-Ahsbahs & Prediger, 2014), our knowledge of the research community of mathematics education and language diversity suggests the need for further theoretical elaboration and discussion of
relationships between theories in order to proceed from imported theories to also include mathematics-specific and topic-specific contributions to theorizing.

Theory is a lens through which one views and understands events, behaviour and/or situations in a systematic way. Theory guides how we think about a situation and influences what we see and what we do not see when we analyse data or take instructional design decisions. As researchers in the area of mathematics education and language diversity, we know this very well and do not expect that further theoretical elaboration guides us to some uniquely correct theory. The theories we use determine the limits or boundaries of the distinct ways we talk about language and language issues, the basic questions we ask, the data we collect and the conclusions we draw from that data. These theories are sometimes difficult to compare because they use different constructs and methods to address different research problems. Importantly, theoretical frameworks guide practical judgement and steer new developments. They allow us as researchers to make links between the abstract and the concrete; the theoretical, the analytical and the empirical; thought and observational statements, etc. Being explicit about the theories we choose to use (and describing why we choose them over others) can assist those who might use our research to maintain a critical awareness of how the research is used, of the illusion of reaching a neutral, definitive observation language, and of the inexistence of a theoretical vacuum for the claims we make. These are central reasons for the significance of being explicit about our theories in mathematics education research on language diversity and being able to explain our choices and positions.

After this introduction, we compile the four concrete contributions and finish with some synthesising concluding remarks. As our colleagues explain and emphasize, the study, interpretation and building of theoretical lenses is a pervasive challenge in their empirical research in mathematics education and language diversity. Personal reviews of our own research over the years in the area are an excellent source of empirical evidence of how and why we choose some specific theories and conceptualisations, and what they allow us to see, say and do. This Research Forum is an opportunity to revisit these questions in terms of how they underpin our research but importantly in terms of how they collectively underpin research in the area. Thus, the process of addressing the two questions mentioned above begins with providing responses on the basis of personal research agendas and from here moves to the wider discussion of the theoretical challenges and insights that we share as part of the community of mathematics education researchers of issues of language and language diversity.

For this, we adopt and summarise four perspectives:

- Heteroglossia, stratification and mathematics (*Richard Barwell*)
- An ecological approach to academic literacy in mathematics in linguistically diverse settings (*Judit Moschkovich*)
- Ethnography and critical theory lens to look into language diversity as funds of knowledge in mathematics classrooms (*Arindam Bose*)
Tensions I

Throughout my career, I have adopted a broadly sociocultural, discursive perspective on the role of language in the learning and teaching of mathematics. I have generally focused on the way language is used in mathematics classrooms, particularly by students. This broad perspective includes some widely used assumptions, such as that language plays an important mediating role in the learning of mathematics, that language is a social phenomenon and therefore reflects collective ways of talking and thinking, and that language also reflects social and institutional structures, and so privileges or marginalises some participants. Learning mathematics is about learning to use the language of mathematics appropriately, where “appropriately” is situated, collective and in constant evolution.

In my initial work on language diversity in mathematics classrooms, including my collaborations with various colleagues (e.g. Barwell, 2009), it became clear to me that language diversity is itself diverse. I noticed that there was an increasing amount of research on language diversity in mathematics classrooms, that this work was conducted in a variety of sociolinguistic contexts, and that these contexts were in many ways very different. For example, the use of more than one language occurred in some situations but not in others. Despite this diversity, I began to see some common patterns. In many contexts, there was some kind of tension between the presence of multiple languages and the possibility of their use in mathematics classrooms. Several studies have reported situations in which only one language was supposed to be used, when in fact students or teachers or both made use of more than one language. Meanwhile in my own research in the UK, while there was increasing recognition that students’ proficiency in their home languages could be important for their schooling in English, I observed little opportunity for students to use languages other than English in mathematics. Nevertheless, I did observe occasions in which students’ home languages appeared to influence their use of English in mathematics (Barwell, 2005). I thus identified several tensions including: between home and school languages, between language policy and mathematics classroom practice, and between language for learning mathematics and language for getting on the world (Barwell, 2009).

Tensions II

In subsequent work, my thinking has been strongly informed by Bakhtin’s (1981, 1986) theoretical ideas about the dialogic nature of language. Why? Because Bakhtin’s ideas explain why the tensions I had started to notice arise and relate these tensions to a range
of other important phenomena that need to be understood, including the stratified and stratifying nature of language, and the role of the other in learning mathematics. From this perspective, the phenomenon of language use is organised around the kinds of tensions I had noticed. There is a deeper underlying tension between standardising or unifying on the one hand, and diversifying or heteroglossia on the other (Bakhtin, 1981). Languages depend on the idea that there are standard forms of grammar, vocabulary, pronunciation, genre, discourse and so on. This idea is common in mathematics: we speak of ‘formal mathematical discourse’, as though there is a clearly defined way of using language in mathematics. But while there are certainly forms of language use that are recognisably mathematical (e.g., “by treating rational numbers as ordered pairs, we can show that they are isomorphic to the natural numbers”), we must also recognise that mathematical language shows considerable variation (between sub-domains, in different contexts, for different audiences, and so on). Indeed, language only happens through a constant process of using language in new and original ways (including in adapting previous forms of language to develop new mathematics). This heteroglossia is apparent in any mathematics classroom, in the wide variety of ways in which students and teachers in fact use language(s) to do mathematics. The presence of standardisation and heteroglossia and the constant tension between them is an inherent feature of language. Moreover, this tension shapes every utterance (Bakhtin, 1981). Heteroglossia can be understood in terms of three overlapping dimensions: languages, discourses and voices (Busch, 2014). In my research, for example, I have described instances in which a second language learner discusses polygons with his teacher showing how the discussion is shaped by the student’s Spanish-inflected French (languages), non-standard mathematical expressions (discourses), and appropriation of the teacher’s words (voices) (all illustrative of heteroglossia), as well as by the teacher’s use of a more standard form of French, more formal mathematical discourse and her re-voicing of the student’s words (Barwell, 2016).

These ideas have made possible a more complete picture of language use and language diversity in mathematics classrooms. This perspective involves four principles: language is agentive; meaning is relational; language is diverse; and language is stratified and stratifying (Barwell, 2018). From this perspective, language itself makes a difference: participants respond to words, rather than to ideas or thoughts. Mathematical meaning arises from the relations between forms of language and is shaped by the inherent tensions in language. There is, for example, always a tension around multiple languages (there is scarcely anywhere in the world now in which multiple languages are not present). It is not sufficient only to pay attention to the use of multiple languages when it occurs, nor to focus research in contexts in which multiple languages are routinely used. In many contexts, multiple languages are present, a potential source of meaning in the mathematics classroom, but the prevailing combination of institutional and political forces means that they are not used. In a similar way, the interaction between languages, discourses and voices is highlighted as an important feature of mathematical meaning making. This perspective, applied in comparative ethnographic research, has highlighted a distinction between mathematics
classrooms in which language and its different forms is explicitly discussed and recognised (‘language positive mathematics classrooms’), and classrooms in which language is largely treated in implicit ways (‘language neutral mathematics classrooms’). The former seem to offer greater scope for supporting students to appropriate the contentions of mathematical discourse in the contexts in which they find themselves, while still recognising the diversity of their own languages, discourses and voices.

Superdiversity

In the context of the increasing diversity of diversity, referred to as superdiversity (Vertovic, 2007), in mathematics classrooms, most recently I have begun to work with theoretical ideas from the contemporary sociolinguistics of multilingualism (e.g. Blommaert, 2010). These ideas extend the preceding perspective of heteroglossia to theorise the stratified and stratifying nature of language. All instances of language use rely on what linguists call indexicality: for example, the use of certain words or ways of talking indexes mathematical activity. The ordered nature of indexicality is part of how we interpret beyond the meanings of individual words to understand, for instance, which of several adults in a classroom is the teacher. This indexicality is, however, also stratified, so that some forms or patterns of language use are perceived as more or less valuable, and are thus linked to issues of authority, control and marginalisation (Blommaert, 2010). These ideas explain why, for example, English is preferred as a language of instruction in many post-colonial contexts, even though this choice may disadvantage many students (e.g. Setati, 2008): English indexes social advancement, power and success. They also explain why some forms of mathematical discourse are privileged over others, and why some voices predominate in mathematics classrooms. Language diversity in mathematics classrooms is socially, institutionally and politically stratified. Contemporary sociolinguistic theory can help to untangle how this stratification shapes students’ learning in mathematics classrooms.

Superdiversity is a way of recognising the great complexity of multilingual mathematics classrooms, wherever they are and whatever their sociolinguistic profile. In such classrooms, students are learning language and learning mathematics. These processes interact and are shaped by the tension between uniformity and heteroglossia, a tension that leads to linguistic and social stratification. Learning mathematics and opportunities to learn mathematics are influenced by these linguistic processes. A key insight in Bakhtin’s work is that these processes are a fundamental feature of language. The question, therefore, is not how to solve the problem of multilingualism (why does no-one ever raise the problem of monolingualism), but how to work with the tensions in productive ways. My recent research suggests that the fostering of language positive classrooms, in which teachers and learners pay explicit attention to different features of language and its use may be a productive way forward.
AN ECOLOGICAL APPROACH TO ACADEMIC LITERACY IN MATHEMATICS IN LINGUISTICALLY DIVERSE SETTINGS

Judit Moschkovich, University of California Santa Cruz

Ecological approach

As a researcher in mathematics education, I specifically bring the lenses of the learning sciences and the field of mathematics education to my research. My particular focus is on mathematical thinking, learning and communicating in monolingual and bilingual settings, so I have had to read across several sets of research literature such as the work of Gee. While I remained grounded in my own field, I had to learn how to use theoretical perspectives from fields in which I had little training, such as linguistics, bilingualism and second language acquisition. My experiences of learning a second language in elementary school and later becoming an immigrant as an adolescent (and learning to live in both bilingual and monolingual modes) sparked my curiosity about bilingualism and second language acquisition. My commitment to improving the education of learners from non-dominant groups provided my motivation and sustains my dedication to tackling issues of language diversity in mathematics education. This is the more general context of my current work within an ecological approach.

Researchers in education have recently called for ecological approaches that integrate a dynamic view of cultural practices into the study of learning and development and document the resources in everyday thinking (Gutiérrez & Rogoff, 2003; Lee, 2008). These ecological approaches are based on the ecological framework proposed by Bronfenbrenner (1989), on seminal studies that examined cross-cultural learning and development and documented the complexity of reasoning in everyday settings (e.g. Lave, 1988; Saxe, 1991), and on studies of learning and development among youth from non-dominant communities (e.g. Lee, 2008; Nasir, 2000; Gutiérrez et al, 1999; Gutiérrez & Rogoff, 2003). The study of students’ mathematical reasoning practices and language diversity requires such ecological approaches not only because this work is cross-cultural, but also because it involves several interacting levels of analysis. These approaches provide theoretical notions and methods that simultaneously address the cognitive, domain specific, cross-cultural, and linguistic nature of mathematical reasoning in multilingual and multicultural settings.

My research has focused on examining bilingual students’ mathematical reasoning practices in middle school classrooms in the United States. Such studies needed to be carefully framed by theoretical notions that simultaneously address the cognitive, domain specific, cross-cultural, and linguistic nature of this work. I use an ecological approach to frame the complex endeavour of examining mathematical reasoning practices in settings with language diversity. This approach has three components. 1) An ethno-mathematical perspective grounds the analysis of multiple and hybrid ways to reason in the domain of mathematics. 2) Educational anthropology and cultural psychology ground the cross-cultural aspects of my work. 3) Research in
sociolinguistics, especially approaches to bilingualism and multilingualism, ground linguistic aspects of my work.

I summarize the three components of an ecological approach (Moschkovich, 2011) as using an ethno-mathematical stance, a naturalistic paradigm, and a situated view of language, that serve to simultaneously address the cognitive, domain specific, cross-cultural, and linguistic nature of this work. This ecological approach provides the central assumptions and overall framing to studies with learner populations who use one, two, or more languages in mathematics classrooms. The fundamental assumptions of an ecological approach include that context matters, that routine practices count, and that the cognitive, social, and linguistic dimensions of both individuals and contexts interact in important ways). These three components provide strategies for avoiding deficit models of learners such as providing a full picture of learners’ competencies and considering and valuing hybrid practices.

The three components of the ecological approach I use for my research are: a naturalistic paradigm (Moschkovich and Brenner, 2000), a situated view of language and Discourse (Gee, 1996 and 1999; Moschkovich, 2002), and an ethno-mathematical perspective on mathematical activity (D’Ambrosio, 1991). These components contribute several theoretical notions-- including culture, context, and mathematical activity-- and assumptions. These three conceptual frameworks provide an integrated ecological approach for analysing mathematical reasoning practices in the following ways: 1) A naturalistic paradigm considers the ecological validity of problems, tasks, and questions used to explore mathematical reasoning. 2) Situated views of language and discourse provide an ecological approach to language, in particular to the meaning of utterances, texts, and inscriptions used during mathematical reasoning. 3) Ethno-mathematics provides an ecological view of mathematical practices because it assumes that mathematical reasoning practices are multiple, heterogeneous, and connected to other cultural practices.

**Academic literacy in mathematics for bilingual and multilingual learners**

The second conceptual framework that I use in my work provides an integrated view of academic literacy in mathematics for bilingual/multilingual learners. Although there are many labels used to refer to students who are learning an additional language, I will use the term bilingual/multilingual learners.

The proposed definition of academic literacy in mathematics includes three integrated components: mathematical proficiency, mathematical practices, and mathematical discourse. The framework includes the following assumptions: the three components of academic literacy in mathematics are intertwined, academic literacy in mathematics is situated, and participants engaged in academic literacy in mathematics use hybrid resources. A sociocultural perspective of academic literacy in mathematics provides a complex view of mathematical proficiency as participation in discipline-based practices that involve conceptual understanding and mathematical discourse. The sociocultural perspective of academic literacy in mathematics described here builds on
previous work that appeared in several publications where I described a sociocultural view of mathematics learners who are bilingual and/or learning English (Moschkovich, 2002 & 2007a), of mathematical discourse (Moschkovich, 2007a), and of mathematical practices (Moschkovich, 2013). In Moschkovich (2008, 2009) I described how mathematical discourse is situated, involves coordinated utterances and focus of attention, and combines everyday and academic registers. The definition of academic literacy in mathematics used here (Moschkovich, 2015) brings together and builds on different aspects of those analyses. This sociocultural theoretical framework draws on situated perspectives of learning mathematics (Brown, Collins & Duguid, 1989; Greeno, 1998) as a discursive activity (Forman, 1996) that involves participating in a community of practice (Forman, 1996; Lave & Wenger, 1991), developing classroom socio-mathematical norms (Cobb et al., 1993, 2001), and using multiple material, linguistic, and social resources (Greeno, 1998). Mathematical activity thus involves not only mathematical knowledge, but also mathematical practices and discourse.

The view of academic literacy in mathematics presented here is different than previous approaches to academic language in several ways. First, the definition includes cognitive aspects of mathematical activity—i.e. mathematical reasoning, thinking, concepts and metacognition—but also sociocultural aspects—participation in mathematical practices—and discursive aspects—participation in mathematical discourse. Importantly for learners who are bilingual/multilingual, this integrated view, rather than separating academic language from mathematical proficiency or practices, takes the three components as working together. Separating language from mathematical thinking and practices can have direct consequences for this student population. Such a separation can make these students seem more ‘deficient’ than they might actually be, since they may not express their mathematical ideas through the language of instruction in the classroom, but may still be engaging in correct mathematical thinking and participating in mathematical practices that are language intensive---for example using objects or drawings to show a result, finding regularity in data, or using gestures to illustrate a mathematical concept.

This sociocultural perspective expands academic literacy in mathematics beyond simplified views of language as words. Simplified views of academic language focus on words, assume that meanings are static and given by definitions, separate language from mathematical knowledge and practices, and limit mathematical discourse to formal language. In contrast, the view of academic literacy in mathematics described here sees meanings for academic mathematical language as socio-culturally situated in mathematical practices and the classroom setting. A complex view of mathematical discourse also means that mathematical discourse draws on hybrid resources and involves not only oral and written text, but also multiple modes, representations (gestures, objects, drawings, tables, graphs, symbols, etc.), and registers (school mathematical language, home languages and the everyday register).

Beyond the assumption that mathematical activity is simultaneously cognitive, social, and cultural, a sociocultural perspective brings two other assumptions to a definition.
of academic literacy in mathematics. First, the focus is on the potential for progress in what learners say and do (Vygotsky, 1978), not on learner deficiencies or misconceptions. Second, participants bring multiple perspectives to a situation, representations and utterances have multiple meanings for participants, meanings for words are situated and constructed while participating in practices, and multiple meanings are negotiated through interaction.

Shifting from a simplified view of academic language as words to a view of academic literacy in mathematics that integrates mathematical proficiency and practices is crucial for the mathematics education of students from non-dominant communities and groups. Instruction for this student population needs to move beyond defining academic literacy in mathematics as low-level language skills (i.e. vocabulary) or mathematical skills (i.e. arithmetic computation) and use an expanded definition of academic literacy in mathematics to describe and prescribe instruction that supports academic literacy in mathematics. Such instruction (a) includes the full spectrum of mathematical proficiency, balancing computational fluency with high-cognitive-demand tasks that require conceptual understanding and reasoning; (b) provides opportunities for students to participate in mathematical practices; (c) allows students to use multiple modes of communication, symbol systems, registers, and languages as resources for mathematical reasoning, and (d) supports students in negotiating situated meanings for mathematical language that is grounded in mathematical activity, instead of giving students definitions divorced from mathematical activity.

ETHNOGRAPHY AND CRITICAL THEORY LENS TO LOOK INTO LANGUAGE DIVERSITY AS FUNDS OF KNOWLEDGE IN MATHEMATICS CLASSROOMS

Arindam Bose, Tata Institute of Social Sciences

I try to explore the role of language in fostering meaning making in mathematics classrooms and in learners’ work-contexts. I started by analysing multilingual mathematics classroom discourse to understand how languages are negotiated in student-teacher conversations under the assumption that language-use is a socially embedded process. The attempt was to comprehend in what different ways languages of learning and teaching (LOLT), home language and language of practice are mixed and switched to arrive at better clarity and understanding of the mathematical contexts and discursive practices (Bose & Choudhury, 2010).

The theoretical notion of funds of knowledge (Gonzalez, Andrade, Civil & Moll, 2001) developed by neo-Vygotskian theories largely frames my research. It helps me in understanding the potential language resource available in the community in the form of embedded mathematical practices in the work contexts that students are exposed to, and in illuminating the nature and extent of language negotiation at the interface of
knowledge drawn from cultural embeddings and different sites of mathematical learning. Funds of knowledge framework helps in understanding shared pattern of language and belief system to enquire into the connections between work practices and opportunities available for gathering everyday mathematical knowledge.

Complementarily, the lens of critical theory makes visible that funds of knowledge and language resources of people from underprivileged groups are often not leveraged in classroom pedagogy and that, hierarchical social structure (e.g. linguistic, caste, class division in Indian society) has bearings on academic achievements including mathematics learning (Bose & Kantha, 2014). The language of mathematics textbooks and the language of mathematics classroom practice often do not connect with the language resources and language repertoires of the learners. This disconnect becomes more apparent and causes debilitating academic effect particularly for learners from the non-dominant and underprivileged backgrounds. With the classroom space becoming more and more multilingual due to constant migration of population (often belonging to disadvantaged conditions) from one province to the other in search of better livelihood, classroom pedagogy arising from such disconnects becomes even more alarming given the bearings of the hierarchical social structure as described above. Therefore, it is necessary and important to understand the complexities and challenges arising in such classroom space in multilingual societies of the third world (like India and other similar countries) to find ways forward that can support mathematics teaching and learning.

India although has 22 official languages used in its different states/provinces, each state uses only one or two official languages and a few more languages are spoken there. English remains the subsidiary working language (associate official language) in most part of the country. Hindi is spoken as a mother tongue by a little more than 40% of India’s population but it is the official language of only 10 out of 29 states. Trilingual characteristic is a unique feature of most states (barring a few) which also prevails in most classrooms in different states. Although “three-language formula” is promoted by the national curriculum (NCERT, 2006, p. 12-14), often learners’ language repertoire and “spaces for participation” (Phakeng, Planas, Bose & Njurai, 2018, p. 292) remain unnoticed and unrecognised in the policy and in classroom practice. It is therefore of importance to explore and understand the interplay and complexities of languages of different forms (mother tongue or home language and languages of practice) and in different contexts. Ethnography as a lens offers a mediating tool to unpack not just some of the language practices in the multilingual contexts in the third world but also to look into the issues of rights, access and social justice concerns.

Following ethnography and critical theory lenses helps in pointing to the disconnection between students’ identities formed in out-of-school contexts as well as those formed during formal classroom learning. Formal mathematics learning in school facilitates shaping of students’ identity as learners. However, their exposure and experience in the out-of-school work-contexts help build their identities as knowers and learners as well as doers. Students draw out-of-school mathematical knowledge from their own
work practices or by observing others work, which helps them look at themselves as “knowers” of some body of knowledge that is valued in the community. The identity of a “doer” is shaped when children reflect on themselves as doing certain tasks in the out-of-school contexts, even if those tasks are fragmented, piecemeal or routine household chores. It can be argued that when classroom teaching practices acknowledge students’ language and knowledge resource and allow merging of their identities then such practices facilitate building powerful connections between out-of-school and school knowledge (of mathematics) and strengthen understanding. Merger or negotiation between students’ identities help in transfer of learning and therefore in enhancing learning opportunities. These processes happen through language negotiation and are part of the socio-cultural role of language in the social settings (Bose & Clarkson, 2016). Such bridging of the socio-cultural role of language and students’ identities goes beyond the contemporary research on connecting students’ identities with classroom norms and argues for bringing together students’ identities and funds of knowledge. Ethnographic exploration through extensive fieldwork in terms of a prolonged period of observation helped in understanding how the groups functioned and unpacked community’s language resource and settings.

Another aspect of my work has been to understand language policy formulations and ways in which the policies play out. Understanding language-in-education policy (LiEP) in cross-cultural contexts helped in analysing language complexities in multilingual mathematics classrooms in developing countries with similar socio-cultural-economic milieu by looking at similarities and differences of language practices in mathematics classrooms. My work has looked into India and South Africa’s contexts and explored how such practices shape learners' mathematical communication (Bose & Feza, 2018; Bose & Phakeng, 2017).

It is not just the use of learners' home language, LOLT and the language of mathematics or a mix of them that is critical for facilitating effective mathematical communication – necessary for developing sound conceptual understanding. Different languages function differently at the interplay with mathematical language depending upon the language’s intonation, syntax and diction. For example, in the case of South Africa and India, uniform policy formulation may not be effective in their multilingual mathematics contexts (Bose & Phakeng, 2017). As emerging economies, these two countries have to deal with learners' identities in the classrooms that emerge from the language settings. The socio-economic statuses of these two countries are vastly different from the developed countries or other developing nations. The languages that the learners use and how they use them during mathematics lessons often serve as an indicator of the social class they belong to or the social context they grew up in. Often learners' familiarity with everyday mathematical registers came from their exposure to the micro-enterprises around them. Their justification and reasoning revolved around their identities drawn from the work practices. In South African context, learners' identities emerge from their racial identities. However, there is a dearth of research that
explores links between identities and language use and how they influence learners' communication of mathematics.

There is an emergent possibility arising from the present studies to inquire into the connection between language, culture and mathematical cognition. Below is a list of possible areas in language and communication that can be explored in similar studies:

- How are communicative activity and mathematical thinking linked and in what ways does language negotiation support (or not) such activities in work-contexts and in mathematics classrooms?
- Different representations of mathematical concepts in different languages and their connection in building mathematical understanding drawing from out-of-school mathematics learning in multicultural and multilingual settings.
- Language negotiation at the interface of knowledge drawn from cultural embeddings and formal, academic knowledge – mutual impact on different sites of mathematical learning.
- A look at the curriculum and policy planning taking on board (or not) the connection between out-of-school mathematics learning and language diversity and ways in which such integration can be achieved.

THEORETICAL FRAMEWORKS IN TOPIC-SPECIFIC DESIGN RESEARCH ON SUPPORTING LANGUAGE LEARNERS

Susanne Prediger, TU Dortmund University

Theoretical backgrounds and need for theory building

The MuM-research group in Dortmund (currently consisting of four postdocs, one professor, five teachers and nine PhD students) works on designing and investigating topic-specific learning opportunities for monolingual and multilingual language learners in different research designs and within different theoretical perspectives. Four basic assumptions about relevant ingredients influence the project-specific choice of theoretical perspectives:

- the epistemic function of language and a functional perspective on language and mathematics learning
- the emphasis on the discursive level of language and their social embeddedness
- the need for design research to combine different theoretical frameworks
- the topic-specificity of language demands

The epistemic function of language and the functional perspective

Language is crucial for mathematics learning due to its function as a “tool for thinking”. The tight connection between language and thinking has been theorized by different theoretical approaches. It goes along with functional perspectives on language like in systemic functional linguistics (Halliday, 1985) or in functional pragmatics (Redder,
The different functional perspectives on language share their focus not on the forms of language alone, but on the interplay of forms and function (function in communication and thinking). The functional perspective is hence crucial for avoiding the treatment of language forms as an end in itself.

The emphasis on the discursive level

Language can be considered on the word level (comprising lexical features such as words and chunks), the sentence level (comprising syntactical features to structure the relation between the words), or on the discursive level on which to locate the more complex language features. The socially embedded more complex features on the discourse level comprise discourse practices like arguing, explaining, reporting, but also the sociomathematical norms which regulate which practice counts as valuable in a specific classroom culture. The relevance of the discursive level has been emphasized by many mathematics education researchers (e.g. Moschkovich, 2015; Barwell, 2012), even if different conceptualizations of discourse are applied in mathematics education research. For our work, the Interactional Discourse Analysis (IDA by Quasthoff, Heller & Morek, 2017) has proven insightful as it provides a powerful framework to link the social, interactional phenomena of discourse practices with the individual discourse competence of each student to participate in these joint practices (Erath et al., 2018).

The need for design research to combine different theoretical frameworks

The mentioned general (mostly linguistic) theoretical approaches provide very insightful topic-independent frameworks for grasping general language demands and for investigating the role of language for mathematics learning within a certain focus. However, each perspective is also blind for other aspects, and that is why different theoretical perspectives must be combined in order to grasp the complexity of language in mathematics teaching and learning. Especially for designing and investigating learning opportunities for language learners in mathematics classrooms, the linguistic frameworks must be combined with frameworks grasping the mathematical topic to be learned and typical learning pathways towards these topics. These topic-specific frameworks change for each project, e.g. functional relationships (Prediger & Zindel, 2017), percentages (Pöhler & Prediger, 2015), fractions (Prediger et al., submitted), or logical deductions (Prediger & Hein, 2017).

The topic-specificity of language demands requires more specific theory building

When supporting language learners in a language- and content-integrated way, there is not only a need to juxtapose theoretical frameworks for language with topic-specific theory on learning the specific topic. Instead, for each of the projects, there was a need to generate theory for disentangling the topic-specific language demands in more details. For this purpose, the combination of theoretical frameworks must be complemented by careful empirical reconstructions in order to identify the language demands and a theoretical framework in which they are captured best. So far, we have only found local solutions, but there is still a huge need for theory building which goes beyond importing linguistic theories to mathematics education. So the question is not only:
Which theories can be imported from other disciplines? But also: Which mathematics-specific and even topic-specific theories must we develop in order to find a sound theoretical base?

Design research for building topic-specific theories on relating registers and languages

Design research is a suitable research methodology in which we can design learning opportunities for language learners and then investigate the interplay of language demands and conceptual demands when studying the initiated learning processes for the aim of contributing to more integrated theory building (Prediger, 2019). For example, the project on functional relationships (Prediger & Zindel, 2017) developed a theory for capturing students’ processes of unpacking conceptual demands within the highly condensed concept of function. The project on percentages (Pöhler & Prediger, 2015) could successfully rely on a well-established learning trajectory towards percentages and combine it with matching language learning opportunities which focus on constructing meanings; the empirical trace-analysis of students’ language uses shows the complexities of moving from the informal resources to the meaning-related academic language and the technical language. The project on logical deductions (Prediger & Hein, 2017) drew upon mathematics education research on logical structures and identified (in Halliday’s 1985 framework) the interplay to the syntactical language demands for expressing these logical structures. The project on multilingual students’ learning of fractions (Prediger et al., submitted) showed how students’ learning pathways towards the part-whole concept can be shaped by translanguaging and connecting different language-related nuances. In all four projects, the instructional approach of macro-scaffolding (Gibbons, 2002) was elaborated in topic-specific ways. In all four projects, first theoretical contributions about the role of languages and registers in topic-specific learning pathways were constructed which require further elaboration and transfer to other topics (Prediger, 2019).

The common core is that all four projects provide concretizations for the often-repeated claim for building upon the students’ everyday experiences and informal language resources as well as for actively supporting their systematic relation to more formal language demands. However, the results also show the definite need to proceed in theorizing these processes in a much finer way on the micro-level.

REFLECTIONS, FUTURE ISSUES AND DIRECTIONS

In the Research Forum coordinated by Richard Barwell and Philip Clarkson fifteen years ago, some concluding remarks were importantly related to “where to look further” as well as to all that “remains to be done to take account of multilingualism at substantive, methodological or theoretical levels of our research” (Barwell & Clarkson, 2004, p. 252). The contributions to the current Research Forum reveal the vitality of the theoretical debate in the mathematics education research on language diversity, together with the emergence of newer strong foci such as content-specific approaches to multilingual mathematics teaching and learning, cognitive and sociocultural
integrated ways of addressing academic literacy in mathematics in linguistically diverse settings, ethnography-based models of curricular design, or the significance of the forces and tensions involved in language use over the course of mathematics teaching and learning. A look at the changing picture of the area of study throughout these years shows that some concepts are still used – e.g. academic literacy in mathematics – but in theoretically modified ways. This is consistent with the discussion in Planas and Schütte (2018) about the same terms taking on more or less different meanings and being used to indicate belonging to more or less different combinations of theories. The meaning of academic literacy in mathematics, for example, depends on the theories used to interpret it and actually varies in the understandings of language that articulate cognition and participation. This same phenomenon can be possibly made in relation to other areas of study as well as the need for being more explicit in determining the role of theory in research.

In the different contributions put together on the occasion of this Research Forum, some important theoretical challenges deal with ontological positions regarding the nature of language, of language negotiation, of mathematical reasoning, and the relationships between them. Not only is the relevance of such positions acknowledged, but the necessity of dealing with them in detail is also considered.

The four theoretical perspectives showcased in this Research Forum display several similarities and points of connection. In particular, they all adopt a view of language that positions variation, multiplicity and difference as valuable (rather than as problematic), whether in terms of sources of meaning, resources, or funds of knowledge. In this common orientation, language is understood to be a tool, a mediational means or a set of discursive practices or literacy practices through which or with which learners and teachers construct or create mathematical meaning. In all four approaches, language and mathematics are understood to be socially shared and reproduced. This position, which can be traced back to research in the 1980s (see Barwell, Moschkovich & Setati Phakeng, 2017, for a discussion) in turn leads to an assumption that pedagogical innovation in the face of language diversity must take this social dimension of language and mathematics into account.

While there is a shared ontological position in how the world is represented as socially produced, a number of differences (at least in emphasis) appear in how this position impacts the analytical proposals and methods for investigating language and mathematics in settings of language diversity and mathematics teaching and learning. Barwell’s theoretical proposal emphasises the dialogic nature of language, for example, whereas Moschkovich emphasises academic literacy. The two perspectives are similar and can be related, but the former leads to more attention on the (dialogic) relationships between forms of language (different languages, informal expression, genres), while the latter suggests more attention to relationships between modes (e.g. gestures, diagrams, spoken language). Similarly, Bose’s approach arguably has a more material perspective on mathematical knowledge (in the form of funds) and tends to highlight the different nature of knowledge in school and out of school. Prediger, meanwhile,
emphasises didactics, as developed through pragmatic cycles of task design, where a key relation is that between actual and desired mathematical practices (including language and discursive practices).

Didactical design research, critical ethnography, sociolinguistic indexicality and culturally situated cognition are some examples of the very different analytical proposals linked to the adoption of very similar positions regarding the nature of language, of mathematical reasoning and of their relationship. In this respect, we can also identify the fundamental question of the extent to which our analytical choices refer to the theories in use and under development. Given the strong theoretical dimension of any methodological proposal, it is important to acknowledge the diversity of analytical strands within very close views of language.

Not less significant and indeed very relevant to all this discussion is what can be planned next or, as posed by Barwell and Clarkson (2004, p. 252), “where to look further”. Among possible future issues and paths to consider we include those that are open and relevant to theory use and theory development, particularly to theory that offers a lens for making sense of, impacting and giving direction to research design and practice in mathematics education settings with language diversity. At least six unresolved questions are important here:

• Which aspects of learning and teaching mathematics in multilingual classrooms need most urgent attention?
• Which theoretical and empirical findings from outside mathematics education seem most relevant to research and practice in mathematics education?
• How can researchers decide which theoretical frameworks to use for their research?
• How can researchers develop expertise in theories that are outside of mathematics education?
• How can curriculum designers include attention to language when designing materials?
• Which kind of topic-specific theory elements are crucial for the foundation of curriculum design?
• How can teachers learn to include attention to language in their mathematics lessons?

It is not possible to think of all these questions and make them intelligible without theory, nor is it possible to investigate tentative responses without some clarity about how theory informs the process of inquiry. We hope that this Research Forum has given some insight into the relationship between different theoretical perspectives used in this area of study and the kinds of questions that researchers are working on.

**Collected references**

Bakhtin M. M. (1986). *Speech genres and other late essays.* Austin, TX: University of Texas Press.


barriers and creates space for marginalized learners (pp. 277-304). Rotterdam, Netherlands: Sense Publishers (Brill).


