Low entrance or reaching the goals? Mathematics teachers’ categories for differentiating with open-ended tasks in inclusive classrooms

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Open-ended inquiry tasks are considered a powerful approach for addressing the diversity of inclusive mathematics classrooms due to their potential for natural differentiation. However, this potential can only unfold when the teachers know how to work with the tasks. This article investigates teachers’ personal categories for differentiating with an open-ended task, especially with respect to providing support for students with special needs. In a qualitative case study, a category-eliciting activity was conducted within a professional development session. Data gathering comprised 14 secondary mathematics teachers’ and special needs teachers’ video-taped group discussions and written answers, which were analyzed qualitatively. The results show that most teachers’ ideas for support provided for the students with mathematical learning disabilities only addressed the low entrance, but not the core learning goals and the required basic conceptual knowledge.

Keywords: Inclusive education, natural differentiation, professional development, teacher knowledge.

Most German secondary schools only recently shifted to an inclusive system, so many secondary mathematics teachers and special needs teachers currently learn how to differentiate in inclusive mathematics classrooms. As effective PD programs need to take into account teachers’ typical starting points, we investigate teachers’ perspectives to deal with various differentiated teaching approaches, here specifically with respect to differentiating with open-ended inquiry tasks (Scherer, Beswick, DeBlois, Healy, & Moser Opitz, 2016).

The paper starts with presenting the teaching approach of open-ended tasks and its potential for natural differentiation (Scherer & Krauthausen, 2010). The necessary teacher expertise for differentiating with open-ended tasks is conceptualized in the framework of Bromme (1992). The qualitative case study based on this conceptual framework uses category-eliciting activities for pursuing the following research question in the empirical part of the paper:

Which categories and self-reported practices do teachers activate for differentiating in inclusive mathematics classrooms with an open-ended task? And how can this be supported by facilitation?

Background on classroom level:
Open-ended inquiry tasks for natural differentiation

Inclusive mathematics classrooms call for differentiated instruction with joint whole-class experiences and specific support for students with special needs (Lawrence-Brown, 2004; Tomlinson, Brighton, Hertberg, Callahan, Moon, Brimijoin, Conover, & Reynolds, 2003). One of the teaching approaches which have proven useful (Scherer et al., 2016), applies rich open-ended inquiry tasks with the potential of so-called natural differentiation:
Open-ended tasks with a low entrance and high ceiling provide the potential for natural differentiation if they allow for multiple representations, diverse solution pathways and different cognitive activities along the trajectory of discoveries (Scherer & Krauthausen, 2010).

One example for such an open-ended task for Grade 5 is printed in Figure 1. It aims at discovering the multiplicative structure of the volume of cuboids, and has a wide differentiating potential (see Figure 2). The task has a *low entrance* as all students succeed in finding at least one cuboid. The task’s *core goal* is to discover that counting in rows and layers leads to a multiplicative structure of the volume. The *high ceiling* for students with strong mathematical potentials covers the combinatorial challenge to find all cuboids, usually by considering the multiplicative decomposition of 24 in three factors.

**Open-ended task: Build many cuboids**

Here, you have 24 wooden cubes. Which cuboids can you build with them? Document all cuboids which you have found. How many do you find?

Possible differentiating prompts:
- Do you find one cuboid?
- Do you find many cuboids?
- Do you find all cuboids?
- How can you be sure that you have found all?

All possible cuboids:
1\(\times\)1\(\times\)24, 1\(\times\)2\(\times\)12, 1\(\times\)3\(\times\)8, 1\(\times\)4\(\times\)6, 2\(\times\)3\(\times\)4, 2\(\times\)2\(\times\)6.

![Figure 1: Open-ended task for discovering the multiplicative structure of volumes (Prediger, 2009)](image)

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**Diverse ways of using material and structures**
- longer or shorter use of hands-on material
- later or earlier discovery of structures
- more or less systematic search for all cuboids
- more or less strive for justifying completeness

**Possible steps in the trajectory of discoveries**
Step 1 Build with the cubes and count separately
Step 2 Build with the cubes and count rows and / or layers and sum up these partial results
Step 3a Build with the cubes, multiply for areas in each layer and add layers (partly use multiplicative structure)
(OR Step 3b Build the cuboid, decompose it in layers and discover their multiplicative structure)
Step 4 Build only parts and mentally imagine the rest and / or partly use multiplicative structures
Step 5 Mentally imagine the cuboid and use multiplicative structures for rows and layers
Step 6 Purely use multiplicative structures for rows and layers

**Multiple representations for the documentation**

<table>
<thead>
<tr>
<th>Graphical</th>
<th>Numerical</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Graphical Representation" /></td>
<td><img src="image" alt="Numerical Representation" /></td>
</tr>
</tbody>
</table>

**Necessary basic mathematical concepts**
- entry level: definition of cuboid as shape with rectangular faces
- access to goal: mental model of multiplication as counting in groups

![Figure 2: Differentiating potential of the open-ended cuboid task as empirically identified by Prediger (2009)](image)
Teacher expertise for differentiating with open-ended tasks

The degree to which rich and differentiating tasks are really productive for inclusive mathematics classrooms depends on the teacher’s expertise for really ensuring responsiveness, not only in a reactive way (i.e. ad hoc repairing occurring obstacles), but also proactively in preparing specific support for students in need for it (Tomlinson et al., 2003). Especially, low expectations have often been problematized (Büscher, submitted) as research shows that students with mathematical learning disabilities can reach more than often expected if responsive, proactive support is provided (Peltenburg, 2012). That is why the teachers must be prepared for these challenges. In order to base professional development offers on teachers’ starting points, the study presented in this paper aims at capturing the teachers’ current categories for differentiating with open-ended tasks.

Personal categories are a crucial part of Bromme’s (1992) conceptualization of teacher expertise. He defines teacher expertise as the teacher’s capability to cope with complex situations in subject matter classrooms, comprising (a) the teacher’s practices by which they cope with situational demands (the so-called jobs), (b) the orientations, e.g. the (content-specific or content-independent) attitudes guiding the prioritization and interpretation of the jobs and (c) the categories which implicitly or explicitly guide their perception and practices (see Prediger, 2019 for a more general discussion of this conceptualization). Especially, Bromme suggests a powerful “heuristic to search for the ‘natural’ categories in expert knowledge” (Bromme, 1992, p. 88, translated by the authors) by analyzing the situational demands with respect to the relevant practices and their underlying implicit categories.

Based on the large literature on differentiating in inclusive mathematics classrooms (Tomlinson et al., 2003; Lawrence-Brown, 2004; Scherer et al., 2016), we identified three sub-jobs for the larger job of differentiating with open-ended tasks:

(1) analyzing tasks with respect to students’ potentially diverse solution pathways and approaches
(2) identifying differentiating potential in a task and possible obstacles for reaching the core goals
(3) providing support for students with specific needs for reaching the core goals.

Job 1 is a preliminary job for Job 2 and 3, which is necessary in order to unpack the mathematical structure and the possible steps on the trajectory of discovery (see Figure 2). The differentiating potential identified in Job 2 can refer to these steps and the multiple ways or representations, but also to possible obstacles, mainly those lying in required basis concepts. The later is crucial for proactively providing support in Job 3, especially for students with specific needs (in our case, students with mathematical learning disabilities, with or without the official status of having special needs).

Taking into account that effective differentiation is always knowledge-centered (Tomlinson et al., 2003), the teachers should have a focus on the six steps of the trajectory of the discovery as well as on the required basic knowledge in understanding the underlying mathematical concepts (as Scherer et al., 2016 emphasize). In the concrete task, this concerns the concept definition of cuboid and, even more importantly, the mental model of multiplication as counting in groups which is required for the transition of counting separately (Step 1) to counting in rows or layers by multiplication (Step 2-4). Providing support for students with specific needs should not only guarantee the low entrance of the task, but also the possibility of reaching the core goal, here the multiplicative structure of the volume. At the same time, the high ceiling (finding all cuboids with 24 cubes) provides challenges for the students with strong mathematical potentials.
Methods of the qualitative study for investigating teachers’ starting points

Given that teachers’ expertise is often implicit in their practices, the research question (Which categories and self-reported practices do teachers activate for differentiating in inclusive mathematics classrooms with an open-ended task? And how can this be supported by facilitation?) was pursued in a qualitative study based on a category-eliciting activity.

Methods for data gathering by a category-eliciting activity on three jobs

**Sample.** The sample consisted of 14 secondary math teachers who participated in their first session of a volunteer professional development series on inclusive mathematics classrooms. They had between 2 and 20 years of experiences in math teaching. 8 of them held a teacher degree as mathematics teachers (PCK*SNK+, i.e. with a formal qualification in pedagogical content knowledge in mathematics education, abbreviated PCK+, but no formal qualification in special needs knowledge on specific needs of students with learning disabilities, abbreviated SNK-), 1 of them was special needs teacher without a degree in mathematics education (PCK-SNK+), and 5 of them were special needs teachers with a degree in mathematics education (PCK*SNK+).

**Category-eliciting activity.** For eliciting teachers’ personal categories and self-reported practices for differentiating with the open-ended task, the teachers’ activity in Figure 3 was structured according to the jobs introduced in the next section. The teachers were asked to write down their ideas and discuss them in small groups during the PD session. The group discussions were video-taped and partly transcribed.

**Methods for qualitative data analysis**

The qualitative data analysis was based on deductive-inductive procedures (Mayring, 2015), starting from the prospective analysis of differential potential and possible obstacles (see Figure 2), but open for teachers’ further personal categories. The articulated personal categories were inductively subsumed under the most relevant categories (results listed in Table 2). The elicited categories along the three jobs were compared between the three subsamples PCK*SNK+, PCK*SNK-, PCK-SNK+.

The video data was analyzed qualitatively with respect to the inductively developed categories and their emergence in the discussions. The video data was of specific importance for determining not only what teachers miss, but also how the facilitator could activate inert knowledge after a while.

**Insights into teachers’ practices and categories for differentiating**

**Main focus on low entrance in Step 1 instead of learning goals and basic concepts**

Table 1 shows exemplary answers written by three teachers, together with the category assigned to the answers in the data analysis. Table 2 embeds these three cases into the complete group of 14 teachers (if only 13 teacher occur, one has not answered this part).
Table 1: Examples of teachers’ written answers and the assigned categories

<table>
<thead>
<tr>
<th>Job 1: Analyzing the task</th>
<th>Melanie (PCK*SNK+)</th>
<th>Dieter (PCK-SNK-)</th>
<th>Hayat (PCK*SNK+)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Some kids just start → trial &amp; error</td>
<td>one self-confident student (most probably with no special needs) overtakes the task</td>
<td>Students find cuboids with same volume and same length</td>
</tr>
<tr>
<td></td>
<td>some find pattern and use them (de-composition of w × w in multiplication tasks with 3 factors)</td>
<td>attitude to work […]</td>
<td>Students find cuboids with different lengths and same volume</td>
</tr>
<tr>
<td></td>
<td>Step 1; Step 2-6 in one line</td>
<td>trial</td>
<td>Just start and build</td>
</tr>
<tr>
<td></td>
<td></td>
<td>affective/social factors, low expectations, implicit Step 1</td>
<td>Step 1; systematic or trial &amp; error</td>
</tr>
<tr>
<td>Job 2: Unfolding differential potential</td>
<td>Some kinds only find one cuboid by trial and error; others find many / all cuboids by mathematical thinking; commutativity (different cuboids by shifting factors)</td>
<td>Preset the layers</td>
<td>Students find some cuboids or only one</td>
</tr>
<tr>
<td></td>
<td>natural differentiation along the steps and diverse ways</td>
<td>make writing task more precise</td>
<td>Students find pattern</td>
</tr>
<tr>
<td></td>
<td>different modes of documentation</td>
<td>no natural differentiation but reduce complexity</td>
<td>natural differentiation along the steps</td>
</tr>
<tr>
<td></td>
<td>natural differentiation regarding documentation &amp; high ceiling</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Job 3: Planning specific support</td>
<td>for some students no support</td>
<td>Less cubes</td>
<td>Working in pairs</td>
</tr>
<tr>
<td></td>
<td>for students with physical disorders: larger cubes</td>
<td>present basic face? → kills fun</td>
<td>surface level (grouping);</td>
</tr>
<tr>
<td></td>
<td>support for articulation</td>
<td>reduce complexity; no basic concepts addressed</td>
<td>no basic concepts addressed</td>
</tr>
<tr>
<td></td>
<td>support for documentation surface level (material) no basic concepts addressed</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In **Job 1** (Analyzing the task), Dieter and Hayat only focused on Step 1 of the trajectory of discovery (see Figure 2), only Melanie also considered further steps (even if condensed in one line without unpacking the necessary steps). Within the group of all teachers (see Table 2), Melanie was an exception, whereas many teachers (6 of 13 teachers) only focused on Step 1 of students’ trajectory, but not how to reach the core goal of the task, discovering the multiplicative structure of the volume. Other categories teachers activated in Job 1 concerned social and affective factors, many of the teachers expressed low expectations like Dieter and two of them even expect that students get no possibility to build a cuboid.

In **Job 2**, it is interesting to see that Dieter (and with him three other teachers) could not identify the potential for natural differentiation and immediately started to provide specific support for weaker students. Those who identified potential for natural differentiation like Melanie and Hayat focused on the steps (7 of 13), the students’ diverse ways of using material and structures (2 of 13), or on multiple representations for documentation (2 of 13). Teachers with PCK*SNK+ could identify more potential for natural differentiation then the teachers with PCK-SNK-. The teacher with PCK-SNK- found none. The required basic concepts (cuboid, multiplying as counting in groups) were only addressed by one teacher in Job 2. With respect to **Job 3**, most teachers have some resources for planning support for students with mathematical disabilities, this is shown by the fact that only three teachers planned their support exclusively on surface levels of grouping strategies or taking larger wooden cubes.

However, as a consequence of the strong focus on Step 1 in Job 1, the planned support in **Job 3** mainly focused on Step 1/2 again. That means, 2 of 5 teachers with PCK*SNK+ and 4 of 8 with PCK-SNK- only provide support for guaranteeing the low entrance. By this focus, they miss the core
goal of the task, which was discovering the multiplicative structure of the volume. As a consequence of the limited view on the necessary basic concepts, only three teachers with PCK SNK+, provided support for overcoming limitations in the basic concepts, and none of PCK SNK- did. Among the three who took care of basic concepts, two made sure that the definition of cuboid as shape with rectangular faces is accessible also for students with mathematical learning disabilities, and only one single teacher’s support addressed the potentially missing mental model for multiplication as counting in groups.

### Table 2: Quantitative overview on elicited categories

<table>
<thead>
<tr>
<th>Job 1: Elicited categories for analyzing the task</th>
<th>Subsample PCK+SNK+(n=5)</th>
<th>Subsample PCK+SNK+(n=1)</th>
<th>Subsample PCK+SNK-(n=8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sole focus on Step 1</td>
<td>1 out of 4</td>
<td>1 out of 1</td>
<td>4 out of 8</td>
</tr>
<tr>
<td>Also focus on Step 2/3</td>
<td>0</td>
<td>0</td>
<td>2 out of 8</td>
</tr>
<tr>
<td>Also focus on Step 5/6 (not on Step 4)</td>
<td>3 out of 4</td>
<td>0</td>
<td>2 out of 8</td>
</tr>
<tr>
<td>Affective / social factors</td>
<td>0</td>
<td>1 out of 1</td>
<td>0</td>
</tr>
<tr>
<td>Low expectations on students</td>
<td>0</td>
<td>1 out of 1</td>
<td>3 out of 8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Job 2: Elicited categories for unfolding differential potential</th>
<th>Subsample PCK+SNK+(n=5)</th>
<th>Subsample PCK+SNK+(n=1)</th>
<th>Subsample PCK+SNK-(n=8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Natural differentiation identified along steps</td>
<td>3 out of 5</td>
<td>0</td>
<td>4 out of 7</td>
</tr>
<tr>
<td>Natural differentiation identified re diverse ways</td>
<td>1 out of 5</td>
<td>0</td>
<td>1 out of 7</td>
</tr>
<tr>
<td>Natural differentiation identified re representations</td>
<td>2 out of 5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>No natural differentiation identified</td>
<td>1 out of 5</td>
<td>1 out of 1</td>
<td>2 out of 7</td>
</tr>
<tr>
<td>Focus on necessary basic knowledge</td>
<td>1 out of 5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Affective / social factors</td>
<td>0</td>
<td>0</td>
<td>1 out of 7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Job 3: Elicited categories for planning specific support for students with mathematical disabilities</th>
<th>Subsample PCK+SNK+(n=5)</th>
<th>Subsample PCK+SNK+(n=1)</th>
<th>Subsample PCK+SNK-(n=8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No support required due to natural differentiation</td>
<td>1 out of 5</td>
<td>0</td>
<td>1 out of 8</td>
</tr>
<tr>
<td>Focus on basic knowledge</td>
<td>3 out of 5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Reducing complexity by presetting the first sub-step</td>
<td>2 out of 5</td>
<td>0</td>
<td>2 out of 8</td>
</tr>
<tr>
<td>Reducing complexity by reducing from 24 to 12 cubes</td>
<td>0</td>
<td>1 out of 1</td>
<td>1 out of 8</td>
</tr>
<tr>
<td>Support only for Step 1 / 2</td>
<td>2 out of 5</td>
<td>1 out of 1</td>
<td>4 out of 8</td>
</tr>
<tr>
<td>Support for Step 3</td>
<td>0</td>
<td>0</td>
<td>1 out of 8</td>
</tr>
<tr>
<td>Support for Step 4-6</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Support for documentation</td>
<td>3 out of 5</td>
<td>0</td>
<td>3 out of 8</td>
</tr>
<tr>
<td>Support only on surface level (grouping, material)</td>
<td>1 out of 5</td>
<td>0</td>
<td>2 out of 8</td>
</tr>
</tbody>
</table>

### Shifting teachers’ categories by prompts to further steps

In order to avoid a deficit-oriented inventorization of teachers’ perspectives, it is crucial to consider how the teachers’ activation of categories can be supported by facilitation in the PD session.

When the facilitator collected teachers’ ideas of the aspects of natural differentiation and for supporting students with mathematical learning disabilities after the first group discussions in the PD session, she became aware of the exclusiveness of teachers’ focus to the first steps of the trajectory. After acknowledging all teachers’ efforts for guaranteeing low entrance for everybody and collecting the diverse aspects of potential for natural differentiation, she shifted the focus to the central learning goal, emphasizing the multiplicative structures of counting in rows and layers for determining the volume. She sent the teachers back into their group discussions with the following prompt which started an interesting process of eliciting further categories:

143 facilitator: What do we do know [means for the students with learning disabilities] that they can really…? So what kind of support can we give so that they can learn - not only what to calculate, but also understand why to use multiplication?  

How to come to Step 2-5?
The levels while layering, there are the layers \textit{[shows the layers with her hands]}\textcolor{red}{\text{Step 2}}.

And then coloring, well anyway, really draw the picture as three-dimensional and mark the layers and reflect, how many cubes are in one layer\textcolor{red}{\text{Step 2+3a}}.

I mean, when you have worked adequately, then they know this, this with lying the dot plates \textit{[she refers to rectangular arrays such as printed here]}\textcolor{red}{\text{Basic concept multiplication}}.

I was just gonna say, the point system and with it, you can take the cuboids of one layer. \textcolor{red}{\text{Step 3a}}

And then coloring, well anyway, really draw the picture as three-dimensional and mark the layers and reflect, how many cubes are in one layer.\textcolor{red}{\text{Step 3a}}

Exactly, I have first..# \textcolor{red}{\text{basic concept}}

That is then, the remediation of the rectangle […] \textcolor{red}{\text{Step 3a}}

Area, isn’t it […] \textcolor{red}{\text{Step 4}}

And then the rectangle. \textcolor{red}{\text{Step 4}}

You have the dot frames and the rectangle – that is what I would remediate here, and then there are the layers, though. […] \textcolor{red}{\text{basic concept}}

And then, if you color that, you are, easy peasy, in multiplication. \textcolor{red}{\text{Step 4}}

You mean, decomposing it, actually, after it, decompose into the same forms. \textcolor{red}{\text{Step 3b}}

But then, of course, you are still in the hands-on, and you have to anyway, ehm. \textcolor{red}{\text{Step 5}}

Or you come into the multiplication. […] \textcolor{red}{\text{relevance of}}

If you have remediated it thoroughly, before. \textcolor{red}{\text{basic concept}}

The analysis of the transcript with respect to the implicitly or explicitly addressed categories from Table 2 shows that after the facilitator-initiated shift of attention from the low entrance to reaching the core goal, the group discussion at the observed table quickly turns to carefully thinking through all steps of the trajectory of discovering multiplicative structures (as marked next to the transcript). During this reflection, the teachers also identify the most important basic concept, the mental model of multiplication in the rectangular array (addressed by the teachers with different idiosyncratic terms – the dot plates, the point system, etc. – but exactly that meaning).

It is only now that the teachers discover an important form of differentiating with the open-ended task: When planning how to provide support for students with mathematical disabilities, the basic concepts required for reaching the learning goals of the task must be identified and possibilities for integrated remediations of these basic concepts have to be searched. In addition, teachers can express higher expectations.

**Discussion and Outlook**

Although the sample of 14 teachers is much too small to take the quantitative comparisons of teachers’ different backgrounds as statistically representative, these first insights into an ongoing project can already point to teachers’ resources and blind spots for differentiating with open-ended tasks:

- Open-ended tasks can only \textit{unfold their potential of natural differentiation} if teachers can unpack this potential. In our sample, 9 teachers started to unpack it, which is a good starting point.

- Their focus is mainly on the start of the trajectory of discovery, and as a consequence, the provided support for students with mathematical learning disability is concentrated on the first steps, but not on the steps towards reaching the learning goal. As a consequence, also the \textit{support is not yet concentrated on the core goals}.  


• The observation that only three teachers provide support for overcoming obstacles posed by potentially missing basic conceptual knowledge is a very strong concern as students cannot reach the learning goals without getting access to the basic mathematical concepts.

• Within the PD session, the facilitator succeeded to shift the participants’ focus from the low entrance to possible supports for reaching the core goals, so with adequate prompts, the teachers can activate categories in their PCK that help them. These include especially the basic concepts.

• Further questions: Which other blind spots for differentiation can be identified and how can they be processed?

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References


