Language challenges for students with mathematical difficulties – An overview on research results and instructional approaches

Susanne Prediger, Kirstin Erath, & Elisabeth Moser Opitz

Source. Prediger, S., Erath, K., & Moser Opitz, E. (2018). Language challenges for students with mathematical difficulties – An overview on research results and instructional approaches. Online-Manuscript available under http://www.mathematik.uni-dortmund.de/~prediger/publikationen.htm. A slightly modified and shortened version of this manuscript will appear as Prediger, S., Erath, K. & Moser Opitz, E. (2019). The language dimension of mathematical difficulties. In A. Fritz, V. Haase, P. Räsänen (Eds.), International Handbook of math learning difficulties: From the laboratory to the classroom (pp. 437–455). Cham: Springer. doi.org/10.1007/978-3-319-97148-3

Abstract. Students' mathematical difficulties are often connected to language factors, as various empirical studies from different disciplinary perspectives have shown. In this article, we discuss the language dimension from different perspectives and with respect to different groups of students. In Section 1, differences between everyday and academic language on word, sentence and text / discourse level, and their implications for mathematics learning processes are discussed. On this base, we describe how language factors affect the achievement of specific groups of students: second language learners, students with learning disabilities in mathematics and reading, and students with specific language impairment. In Section 2, important language dimensions for the mathematics learning focusing on the use of language as a learning medium and discourse practices are presented. Section 3 outlines some instructional approaches and explains why those instructional approaches seem to be most effective for supporting mathematics learning which provide learning opportunities especially for the mathematically most relevant discourse practices: explaining meanings of mathematical concepts and operations and for describing general pattern.

In this article, we briefly (1) report on theoretical backgrounds and empirical studies showing strong connections between language factors and mathematics achievement (as a *result* of learning), (2) explain in which way language is relevant in the *processes* of learning, and (3) present instructional approaches for enhancing students' language proficiency for supporting the learning of mathematics.

1. Language factors on different levels and their connection to mathematics achievement

Many empirical studies have shown that students' mathematical difficulties are often tightly connected to social factors and language factors (Secada, 1992), where also the social factors often trace back to language factors: "It is now well accepted that the chief cause of the achievement gap between socio-economic groups is a language gap." (Hirsch, 2003, p. 22). But what exactly does *language* mean in these contexts? Whereas Hirsch mainly refers to reading difficulties, other studies show language factors on word, sentence and text / discourse level, and all of them can contribute to mathematical difficulties.

1.1 Differences between everyday and academic language on word, sentence and text / discourse level

Language gaps concern not only multilingual learners, but also monolinguals who are fluent in the everyday language. This phenomenon can be explained by the difference between everyday language and academic language, which goes back to Cummins' (2000) distinction between Basic Interpersonal Communication Skills (BICS) and Cognitive Academic Language Proficiency (CALP). Whereas students from socially privileged families acquire CALP in their families, socially underprivileged or immigrant students sometimes have less access to it, although it corresponds to a crucial register (Halliday, 1974) in the context of schooling – in textbooks, exams, and learning tasks (Schleppegrell, 2004).

Table.1. Typical features of the school academic and technical register (compared to everyday register)

Word level (lexical features) • less familiar words with distinct meanings • more complex word structures, nominalizations • relevance of other types of words (e.g. prepositions)
Sentence level (syntactical and morphological features) • more complex syntactical structures like prepositional phrases, more complex subordinate clauses • impersonal constructions like passive voice construction • more subtle and precise use of morphological distinctions and connectives
Text level • specific text genres which are not used in the everyday practices (e.g. protocol, report, interpretation) • more explicit and more complex markers of cohesion and coherence • …
Discourse level • specific discourse practices (e.g. explaining-why, arguing, ….) with different norms of explicitness than in everyday practices • more abstract and more general talk • ….

Researchers in education and linguistics have characterized the specificities of the school academic language similar to the technical languages of different scientific disciplines (Schleppegrell, 2004; Bailey, 2007; Snow & Ucelli, 2009; Jorgensen, 2011). Table 1 summarizes typical features on the word, sentence, and text level (referring to written texts) as well as the discourse level (referring to oral discourse practices and their structures beyond the sentence level, see Section 2 for more details on the discourse level).

The school academic register as well as the technical register of each subject has its own characteristics and challenges. Morgan, Craig, Schütte, and Wagner (2014, p. 845) emphasize that "language has a special role in relation to mathematics because the entities of mathematics are not accessible materially". Therefore, communication about mathematics requires symbols, drawings, and mathematical language and demands a high precision and abstractness, which are characterizing features of the academic language.

1.2. Disentangling language obstacles on word, sentence, text, and discourse levels and their connection to mathematics achievements

Different research groups have developed different approaches to disentangle language factors and their connection to mathematics achievement: whereas some researchers exclusively refer to tests and differences on the item level (for all students or for those with mathematical learning difficulties), other researchers (mainly mathematics education researchers and linguists) also investigate the learning processes themselves (see Section 2).

1.2.1 Obstacles on the word level

Linguistic structure of number words. Among young children, the linguistic structure of the number words has an influence on their acquisition of numbers. In a study of Miura, Okamoto, Kim, Steere, and Fayol (2003), first graders from Asian countries who spoke languages which are organized so that numerals are congruent with the base ten number system had higher counting competences and a better understanding of the base ten number system than children from countries with less congruent number words (e.g. France, Sweden). Moser Opitz, Ruggerio, and Wüest (2010) found similar results for Turkish speaking kindergarten children (a language with number words congruent with the base number system) who had a higher counting competence than Italian and Albanian speaking children. In many languages (e.g. Arabic, Czech, Danish, Dutch, German) pupils especially struggle with the challenge of inversion by transcoding Arabic numbers from and into number words (e.g., Zuber, Pixner, Moeller, & Nuerk, 2009). In the inversion property, the order of basic lexical elements in their syntactical organization is inverted in symbolic and verbal notation (e.g. in German, the number word for 32 is "zweiunddreissig" [two and thirty], Zuber et al., 2009). Klein et al. (2013, p. 4f.) conclude that the numerical development is moderated by language, especially with regard to two digits numbers. Therefore, it seems to be important to explicitly discuss the irregularity of number words in the respective languages when working with young children.

Mathematical vocabulary. Different categorisations of mathematical vocabulary are possible, also depending on the linguistic feature of a specific language. Riccomini, Smith, Hughes, and Fries (2015, p. 238) distinguish (for English) the following categories: "(a) meanings are context dependent (e.g., foot as in 12 inches vs. the foot of the bed), (b) mathematical meanings are more precise (e.g., product as the solution to a multiplication problem vs. the product of a company), (c) terms specific to mathematical contexts (e.g., polygon, parallelogram, imaginary number), (d) multiple meanings (e.g., side of a triangle vs. side of a cube), (e) discipline-specific technical meanings (e.g., cone as in the shape vs. cone as in what one eats), (f) homonyms with everyday words (e.g., pi vs. pie), (g) related but different words (e.g., circumference vs. perimeter), (h) specific challenges with translated words (e.g., mesa vs. table), (i) irregularities in spelling (e.g., obelus [÷] vs. obeli), (j) concepts may be verbalized in more than one way (e.g., 15 minutes past vs. quarter after), and (k) students and teachers adopt informal terms instead of mathematical terms (e.g., diamond vs. rhombus)".

Studies give evidence that such kind of lexical features can affect mathematics achievement. Haag, Heppt, Roppelt, and Stanat (2015) showed that increasing the difficulties of lexical features (e.g. more general and specialized academic vocabulary) in test items increases the item difficulties. Schindler, Moser Opitz, Cadonou-Bieler, and Ritterfeld (forthcoming) found a significant correlation between students' mathematical vocabulary and arithmetical competence in a sample of fifth graders. With regard to different categories of mathematical vocabulary, the findings are inconsistent: Schindler et al. (forthcoming) report that mathematical terms which have different meanings in the everyday language and the academic language (e.g. difference, product) and terms which are used in both languages with the same meaning (e.g. square, rectangle) seem to be especially difficult for fifth graders. In contrast, this does not seem to apply for tenth graders who seem to master polysemy without problems (Prediger, Wilhelm, Büchter, Gürsoy, & Benholz, 2018).

1.2.2 Obstacles on the sentence and text level

On sentence and text level, several factors have an impact on mathematics learning. Reading difficulties on the sentence and text level have often been shown to influence the students' test achievement results (Abedi & Lord, 2001; Hirsch, 2003).

Syntactical complexities on the sentence level have to be taken into account for test and learning situations: Complex prepositional clauses (Jorgensen, 2011), conditional clauses, the use of nominalization (e.g. double – doubling; Schleppegrell, 2004) and complex issues of cohesion on the sentence and the text level. In addition, text length and the number of noun phrases can appear as significant predictors for low achievement in mathematics for third graders (Haag et al., 2015), but not necessarily for tenth graders (Prediger et al., 2018).

However, it is important to realize that especially students with low language proficiency do not only encounter reading obstacles: A differential functioning analysis on a high stakes test in Grade 10 has shown that all items with which students with low language proficiency had specific difficulties were items with high conceptual demands, not high reading demands (Prediger et al., 2018). Haag et al. (2015) investigated if the linguistic simplification of test items affects the mathematical performance of second language learners. They found only a small effect with limited practical relevance. These results hint to difficulties in the learning processes rather than only test biases (see Section 2).

1.3. Language factors in the achievement of specific groups

1.3.1 Second language learners

Since more than 25 years, research shows that language minority students often obtain lower test scores in mathematics than native speakers (Secada, 1992; Abedi & Lord, 2001; Haag, Heppt, Stanat, Kuhl, & Pant, 2013). They are often disadvantaged in school if their first language does not correspond to the language of instruction (Schleppegrell, 2004; Barwell, 2009). Whereas some researchers have investigated the language gaps between the students' home language and the language of instruction in more cognitive terms, e.g. problem of interferences between both languages for expressing number names (Krinzinger et al., 2011), other researchers have focused on cultural aspects and issues of identity and agency which apparently underprivilege language minority students more than the pure cognitive and communicative aspects (Norén, 2015; see Barwell et al., 2016, for an overview). That is why many researchers plead for avoiding deficit perspectives on multilingual students (see Moschkovich, 2010; Barwell, 2009).

Additionally, it is important to note very explicitly that the risk factor of second language learners is not their multilingualism itself, because multilingualism can also provide cognitive benefits (e.g. shown by Cummins, 2000; Kempert, Saalbach, & Hardy, 2011, see Barwell et al, 2016 for an overview). In contrast, the major risk factor seems to be the proficiency in the language of instruction: Paetsch, Felbrich, and Stanat (2015) found a significant relationship between reading comprehension, vocabulary, and mathematics achievement of second language learners (similarly Abedi & Lord, 2001). Also in a survey of tenth graders, the language proficiency was the factor with the strongest statistical connection to the mathematics achievement, stronger than multilingualism, immigrant status or socio-economic status (Prediger et al., 2018). That is why we agree to Moschkovich who claims that "studies should focus less on comparisons to monolinguals and report not only differences between monolinguals and bilinguals but also similarities" (Moschkovich, 2010, p. 11).

However, second language learners should not only be regarded as emergent language learners in the language of instruction. Their home language can be a resource which – if activated appropriately – allows a second access to mathematics (Planas, 2014; see Section 3.5).

1.3.2 Students with learning disabilities in mathematics and reading

Even if problems with learning mathematics are widely known, researchers have not yet been able to agree on a single definition (e.g., Gunn & Wyatt-Smith, 2011; Scherer, Beswick, Deblois, Healy, & Moser Opitz, 2016). In addition, different terms (learning disabilities, learning disorders) are used to describe the affected students, and different diagnostic approaches may lead to different results (Branum-Martin, Fletcher & Stuebing, 2012). Without discussing this issue here, and also without discussing genetic factors (e.g. Petrill et al., 2012) or neurobiological underpinnings (e.g. Ahskenazi, Black, Abrams, Hoeft & Menon, 2013) we use the term "learning disabilities" for referring to students with significant and long lasting learning problems. For many years it was assumed that such problems in learning mathematics and reading are isolated impairments, and even nowadays, the ICD-10 manual defines comorbid learning disorders in mathematics and reading as a “poorly defined residual category” (Deutsches Institut für Medizinische Dokumentation und Information, 2015).

This often led to the consequence that affected students had access to support *either* in mathematics *or* in reading. However, empirical evidence shows that significant problems with reading and of mathematics often co-occur (Dirks, Spyer, van Lieshout, & de Sonneville, 2008). According to Mann Koepke and Miller (2013), 17% to 66% of pupils with learning disabilities in mathematics also have reading disabilities. The research from Willcutt et al. (2013) gives evidence that significant problems with reading and mathematics are distinct, but related disorders that often co-occur because of shared neuropsychological weaknesses in working memory, processing speed, and verbal comprehension. Moll, Göbel, Gooch, Landerl, and Snowling (2016) found only verbal memory as a shared risk factor of pupils with reading and mathematics disorders and divergent other factors for reading impairment (slow verbal processing speed) and mathematics impairment (limitations in temporal processing, verbal and visuospatial memory). Fuchs, Geary, Fuchs, Compton and Hamlett (2016) conclude that pathway to calculation and word-reading outcomes are more different that alike.

These findings have important implications: First, it has to be acknowledged that a considerable part of students meets substantial problems in both domains, reading or math, and therefore need special support. Second, it has to be considered that the different cognitive profiles of students require interventions tailored to the special needs of the affected students.

1.3.3 Students with specific language impairment and their mathematics learning

The relationship between specific language impairment (SLI) and mathematics development is only scarcely investigated. The few results, which are available, show that students with SLI have lower mathematical achievement in some areas, compared with other students. According to Fazio (1996, 1999), children with language impairment do understand the process of counting objects and the cardinality principle. However, the have difficulties acquiring the number sequences correctly. Other authors (Donlan, 2003; 2015; Donlan, Cowan, Newton & Lloyd, 2007) report difficulties of SLI-students in the production of number words, in calculation and understanding place value. In a study of Ritterfeld et al. (2013), students which attended a special school for children with SLI and followed the normal curriculum, had a lower achievement in mental calculation than students in regular classrooms. However, they did not use more problematic counting strategies than pupils without SLI. Alt, Arizmendi and Beal (2014) discuss multiple possible problem sources for the aforementioned difficulties of pupils with SLI: The manipulation of mathematical symbols, the use of working memory for patterns, and the combination of complex linguistic syntax plus mathematical symbols. Durking, Mok and Conti-Ramsden (2015) investigated the relationship of language factors and IQ in the core subjects language, science and mathematics in a sample of students with SLI. Achievement in mathematics was predicted by IQ, but not by language factors. Other authors (Röhm, Starke, & Ritterfeld, 2016; Nys, Content, & Leybaert, 2013) assume that deficits in working memory – especially in the phonological loop – influence the mathematics learning processes of students with SLI.

To sum up, SLI is risk factor for a successful mathematical development, which seems to be caused by other factors than language. Schröder and Ritterfeld (2015) argue that that these SLI-students are in need for qualitatively enriched interactions with their teachers for achieving a successful participation in mathematical conversations. In this way, they promote to transcend the word and sentence level and work on the discourse level also for the students with most serious language challenges.

2. Language dimensions in learning processes

2.1 Language as a learning medium, learning prerequisite and learning goal

Students with low language proficiency experience difficulties not only in test situations, but – more importantly – in the learning situations themselves. This relates to the role of language as a *learning medium* in classrooms (Lambert & Cobb, 2003; Morgan et al., 2014): Language in mathematics classrooms is at the same time a medium of knowledge transfer and discussion (*communicative* role of language) and a tool for thinking (*epistemic* role of language, Morek & Heller, 2012; Pimm, 1987). Research in mathematics education repeatedly emphasizes the intertwinement of language and mathematical thinking for all students, but especially for students still acquiring the language of instruction (e.g., Moschkovich, 2015). As not all students have the same level of academic language proficiency, the learning medium turns into an unequally distributed *learning prerequisite*. In order to compensate differential learning prerequisites more explicitly, language must be an explicit *learning goal.* Lampert and Cobb (2003, p. 237) stress that „Learning to communicate as a goal of instruction cannot be cleanly separated from communication as a means by which students develop mathematical understanding" linking the language as a learning goal to language in its epistemic role.

2.2 Discourse practices as a construct to capture language demands on the discourse level

Whereas the lexical and syntactical features of academic language on the word and sentence level have been discussed in Section 1, this section focuses on the discourse level which has been shown to be crucial for the meaningful learning of mathematics (e.g., Bailey, 2007; Moschkovich, 2015; Erath, Prediger, Heller, & Quasthoff, submitted), especially for language learners.

Discourse is a construct frequently used in mathematics education (e.g., Bailey, 2007; Barwell, 2012; Moschkovich, 2015; Erath et al., in press) which is tied to different linguistic theories that can be united under the term 'discourse analysis'. For example, Barwell (2012) refers to discursive psychology and conversation analysis for defining discursive demands; Moschkovich (e.g., 2015) refers to sociolinguistics in order to conceptualize academic literacy in mathematics for English Learners; Erath et al. (in press) introduce the theory of Interactional Discourse Analysis in order to contribute to an empirically grounded theorization of academic language proficiency on the discourse level. Here, we refer to the definition of discourse practices from Interactional Discourse Analysis since it is compatible with other definitions used in mathematics education. In addition, it allows to differentiate between different subcategories of discourse. Explanations, arguments, descriptions etc. are defined by the task they solve in a speech community (e.g., a mathematics classroom):

"oral discursive practices are defined as multi-unit turns which are interactively co-constructed, contextualized and functionally oriented towards particular genres (Bergmann & Luckmann, 1995) such as narration, explanation or argumentation. By making use of conventionalized genres, discourse units in their joint achievement in interaction rely on patterns available in speech communities' knowledge to, e.g., convey or construct knowledge (explanations) or negotiate divergent validity claims (argumentation)." (Erath et al., in press, p. 4)

In this perspective, *discourse practices* are patterns that can be observed repeatedly in a speech community (e.g., Cobb, Stephan, McClain, & Gravemeijer, 2001). Specifying the relevant discourse practices hence allows the researcher and designer to specify relevant language demands, on the discourse and also the word and sentence level.

2.3 Discourse practices and discourse competence in mathematics classrooms

In mathematics classrooms, the most important discourse practices comprise (Prediger, 2016):

- reporting on procedures

- explaining the meaning of concepts and operations
- arguing about the validity of a claim
- describing patterns in a general way.

However, these most important practices appear with different frequencies in different classrooms (Erath et al., in press) and in addition, pattern are specific for different mathematics classrooms as Erath (2017) shows for the case of explaining. Furthermore, it can be theoretically and empirically shown that talking about conceptual knowledge (that Hiebert, 1986 defined in contrast to procedural knowledge) is tightly linked to the shared discourse practice of explaining in whole class discussions (Erath, 2017) and moderated small group work (Erath, submitted).

Discourse analytic constructs allow mathematics education researchers to understand how students' low language proficiency is intertwined with restricted mathematical learning opportunities. In this context, learning is conceptualized as "a process of enculturation into mathematical practices, including discursive practices (e.g., ways of explaining, proving, or defining mathematical concepts)" (Barwell, 2014, p. 332; Vygotsky, 1978). When learning mathematics is linked to participation in classroom interaction, this specifically explains limits in acquiring conceptual knowledge in discourse practices like describing general pattern or explaining meanings of concepts (e.g., Moschkovich, 2015; Erath et al., in press): Empirical studies on whole class discussions and moderated small group work (e.g., Erath, submitted; Erath et al., in press; Barwell, 2012) show that students with low language proficiency rarely participate in the conceptually interesting moments of discourses, often due to missing linguistic resources on the discourse level.

In Interactional Discourse Analysis, these resources are specified by the concept of discourse competence which distinguishes three facets (Quasthoff, 2011):

- *Contextualization competence* refers to a student's ability to recognize if for example a question requires an answer with a half a sentence or a longer utterance (i.e. a discourse unit). For a discourse unit, a student needs to recognize if a narration, an explanation or an argument is required.
- *Textualization competence* refers to a student's ability to structure the discourse unit according to the different requirements of explanations, arguments etc. In the case of explanations in mathematics classrooms, this for example involves state the procedure of an algorithm stepwise.
- *Marking competence* refers to a student's ability to use language means that make the chosen discourse practice and the related textualization recognizable for the public. Language means for argumentations are for example 'because' or 'for this reason'.

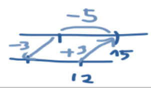
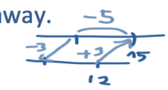
These three subcompetences are required by students for participating in discourse practices in mathematics education (further potential mathematical challenges are not explicated here): They need to "recognize *contextually* when to place which genre (e.g., to explain instead of telling narratives), master the genre's *textualization* patterns (e.g. explicating general procedures step-by-step) and use a genre-specific lexical and grammatical repertoire for *marking* the genre (e.g. 'that's why', 'consequently' for explaining-how)." (Erath et al., in press, p. 5).

2.4 General and topic-specific lexical means for different mathematical discourse practices

Beyond the *general* discourse competence and language means for *generally* marking a specific discourse practices, topic-specific language means are required for each mathematical topic. We illustrate the differences for the case of a whole class discussion on flexible subtraction strategies (Grade 3-5) in which a student, Kevin, has suggested to calculate 12 – 5 by the auxiliary task 15 – 5 – 3: "If this were 15, then it would be 10, and then make 3 less". As his ideas are not immediately understandable to the whole class, the teacher asks to explain Kevin's ideas (Prediger 2016). Table 1 shows three different discourse practices that might follow and the different general and topic-specific language means required for the discourse units.

The examples in Table 1 specifically illustrate the difference between a formal, technical vocabulary (subtracting, adding, result) and meaning-related vocabulary (giving away, borrowing, ….). A second example can be given for percentages: the formal vocabulary comprises concepts like, rate amount, base. To enable students to construct meanings for these concepts, it is important to engage them in the discourse practice of explaining meanings. For explaining, topic-specific meaning-related vocabulary is required such as "the old price", "the new price", "discount as a share of the new price", etc. (Pöhler & Prediger, 2015). Scaffolding these meaning-related phrases allows students with low language proficiency to participate in these kinds of discourse practices.

Table 2. General and topic-specific lexical means for different mathematical discourse practices – examples for the auxiliary strategy 15 – 5 – 3 for calculating 12 – 5

Discourse practice	**General and topic-specific language means required for the discourse unit**	**Concrete discourse unit and underlying lexical means**
... reporting on procedures	Topic–independent phrases: • sequential clauses (first, ... then, ...) • final clauses (in order to ...) Topic-specific phrases: • formal technical vocabulary • for communicating about flexible strategies, also visualizations	For 12-5, I calculate the auxiliary tasks 15-5=10. In order to reach 15, I add 3. After that I subtract 5 and then, the borrowed 3. Thus, the result is 10-3= 7. 
... explain the meaning of concepts and operations	Topic–independent phrases: • interpretative phrases (I imagine it like that…., this means…, this represents ...) Topic-specific phrases: • meaning-related phrases • visualizations	I imagine it like this: I have 12 candies and give 5 away. Before that, I borrow 3, and I have to give them away. 
... describe the general pattern	Topic–independent phrases: • generalizing phrases (for every, always, whenever, abstract forms) • functional relations (if-then-relations, the higher, the smaller...) Topic-specific phrases: • meaning-related phrases	For every subtraction task in which the first number is increased by 3, the difference also increases by 3. If the first numbers is held constant for subtraction tasks, then we have: The higher the second number, the smaller the difference.

3. Approaches for fostering students' language proficiency in mathematics

As language has turned out to be an important factor for different groups of students at risk (see Section 1), fostering students' academic language proficiency is demanded all over the world. The Council of Europe claims it as a major approach for achieving more equity (Thürmann, Vollmer, & Pieper, 2010), and many empirical studies show that it might positively influence the mathematics learning (see below). However, the practices of fostering language (e.g., DfEE, 2000) are often criticized for being too restricted to vocabulary training (Moschkovich, 2013) without taking into account the discourse level. In this section, we give some examples on more enhanced instructional approaches which focus on the discourse level and embed vocabulary work into this focus.

3.1 Enhancing discourse practices – qualitative output hypotheses

Given the empirical results on the discourse level (see Section 2), enhancing discourse practices like explaining, arguing etc. are important instructional approaches (Moschkovich, 2015). This is in line with the principle of pushed output formulated in second language learning (Swain, 1995), according to which language learning requires the enforcement of oral and written language production. Pushing output can be reached by suitable tasks and activity structures, and can be accompanied by materialized scaffolds. This may include language frames and teachers' continuous micro-scaffolding moves during the interaction (Bakker, Smit, & Wegerif, 2015).

Schröder and Ritterfeld (2015) emphasize the significance of enhancing discourse practices also for students with SLI on three dimensions: the dimension of the linguistic-interactive requirements and their supportive function, the didactical dimension of using materials and visualizations and the dimension of mathematical knowledge resp. knowledge acquisition.

3.2 Enhancing conceptual knowledge – relating registers and representations

In mathematics education, pushed output can be successfully combined with continuous activities of relating registers (the everyday, academic and technical register) and (symbolic, graphical, ….) representations forward and backward, rather than sequencing through them once (see Fig. 1 from Prediger, Clarkson, & Bose, 2016; similar in Moschkovich, 2013).

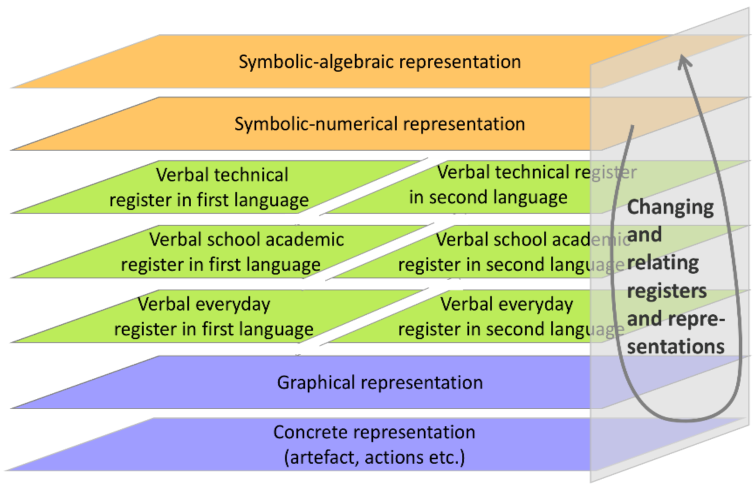


Figure 1. Design principle of relating registers and representations (Prediger et al., 2016)

These activities of translating between registers and finding coherences or differences offer very suitable, mathematically rich opportunities for verbalizing, explaining and arguing. At the same time, they have proven powerful for enhancing conceptual understanding, a critical point in the learning of students with low language proficiency (see Section 1 and 2).

Empirical evidence for the efficacy of this design principle has been provided, e.g. in a control trial in Grade 7 on an intervention designed for enhancing students' conceptual understanding of fractions based on this design principle (Prediger & Wessel, 2013). Qualitative analysis suggest that connecting the registers, languages and representations might even be more productive than purely switching between them.

3.3 Specifying mathematical and language goals – the SIOP model (ibid.)

One of the most widespread approaches for supporting language learners in subject matter education stems from the sheltered instruction observation protocol (SIOP-model, Echeverria, Vogt, & Short, 2010). The SIOP model starts from specifying mathematical learning goals, deriving the discursive demands and then figuring out the language objectives for realizing these discursive demands (Fig. 2). The necessary words and phrased are offered in language frames and trained by initiating the necessary discursive practices.

Empirical evidence for the efficacy of this model has been offered in various control trials (for a summary see Short, 2017).

Type of Language Objective	Algebra Example
Academic Vocabulary: key terms needed to discuss, read, or write about the lesson's topic (subject-specific, general academic, or word parts)	Students will define and give examples of positive and negative slope.
What it means instructionally	Teacher uses a concept definition map with class to define slope, call attention to related words (e.g., increase, decrease, vertical, horizontal), and elicit real-life examples of slope.
Language Skills and Functions: skills students will use in the lesson (e.g., read for main idea) or the specific purpose for using language (e.g., to compare, to persuade)	Students will orally justify the slope of a line between two points.
What it means instructionally	Teacher demonstrates how to find slope using a geoboard and offers language frames to justify the determination, such as "The slope is positive/negative __ because ...".
Language Structures: grammar or language structures in the written or spoken discourse of the lesson	Students will use if-then statements to describe what happens to a line when the slope changes.
What it means instructionally	Teacher teaches (or reviews) how to form two types of if-then sentences: 1) when the if clause comes first and 2) when it comes in the second half of the sentence. Teacher points out use of present tense in the *if* clause and future tense in the *then* clause.

Figure 2. Categories and Examples of Language Objectives in the SIOP-Model (Short, 2017, p. 4246)

3.4 Combining conceptual and lexical learning trajectories – macro scaffolding

Gibbons' (2002) approach of macro-scaffolding suggests to combine the conceptual and lexical learning opportunities in well-sequenced trajectories. Table 2 gives an example for a macro-scaffolding approach for the mathematical topic of percentages: The conceptual learning trajectory is sequenced in six steps starting from students' resources in informal thinking to informal strategies, formal procedures and their flexible use. Each step requires other discourse practices and different vocabularies which are sequenced in the lexical learning trajectory. Empirical evidence has been given that this intertwinement can be effective for mathematics learning (Smit, 2013; Pöhler & Prediger, 2015 with quantitative evidence in Pöhler, Prediger, & Neugebauer, 2017).

Table 3. Combining conceptual and lexical learning trajectories – macro scaffolding example for percentages (Pöhler & Prediger, 2015)

Levels	Conceptual learning trajectory: Mathematical conceptions	Lexical learning trajectory through different vocabularies
Level 1: Informal thinking starting from students' resources	Constructing meaning for percents by representing and estimating rates	Intuitive use of students' resources in everyday register, no offer of new lexical means
Level 2: Informal strategies and meaning-related vocabulary	Developing informal strategies for determining rates, amounts and later bases	Establish basic meaning-related vocabulary in the academic school register for constructing meaning for rates, amounts, bases in context
Level 3: Procedures for standard problem types	Calculating amounts, rates and later bases	Introduce formal vocabulary in the technical register
Level 4: Extending the repertoire	Widening to other problem types: change and comparison	Enrich the basic meaning-related vocabulary to more complex problem types
Level 5: Identification of different problem types	Identifying problem types of (non-) standard problems (in diverse contexts)	Explicit use and training of formal and basic meaning-related vocabulary
Level 6: Flexible use of concepts and strategies	Cracking more complex context problems flexibly (in non-familiar contexts)	Introduce extended reading vocabulary for various non-familiar contexts

3.5 Including home languages – activating students' multilingual repertoires

With respect to the group of multilingual students whose home language differs from the official language of instruction (which is the majority worldwide), another important instructional approach addresses the inclusion of home languages in order to activate the complete multilingual repertoire for mathematics learning.

The demand for including all multilingual resources has often been formulated, and qualitative empirical insights have been offered for its benefits (Setati, 2005; Barwell et al., 2016), such as participating in mathematical discourses activating everyday out-of-school experiences (Planas, 2014) or upgrading resources for meaning-making processes (Clarkson, 2006; Norén, 2015) and increasing agency (Norén, 2015). However, the quantitative evidence for its efficacy for mathematics learning is still too weak (Reljić, Ferring, & Martin, 2015). Therefore, further research is required for strengthening the quantitative evidence.

4 Conclusion

Language is a major *learning medium* used for communicative and epistemic purposes in mathematics classrooms. Hence, language proficiency is an important *learning prerequisite* without which participation in classrooms tends to be restricted. This applies to different groups of students:

- second language learners
- students with learning disabilities in mathematics and reading
- students with specific language impairment
- and monolingual students (often of low socio-economic status) which have not yet had sufficient learning opportunities especially for the academic language.

For all these students with limited language proficiency, language thus has to become a *learning goal*, also in mathematics classrooms (Lambert & Cobb, 2003).

The language dimension is crucial for students with difficulties in mathematics not only in test situations but particularly *during the whole learning process*: Students with low academic language proficiency regularly meet challenges on *word, sentence, text, and discourse levels*.

Therefore, instructional approaches to support language learners should not isolate the word level from the discourse level. Instructional approaches seem to become most effective for supporting mathematics learning when they provide learning opportunities especially for the discourse practices of explaining meanings of mathematical concepts and operations (Setati, 2005) and for describing general pattern. The lexical support of meaning-related vocabulary is therefore equally important to the formal technical vocabulary and specifically fruitful when offered in structured phrases rather than isolated words (Moschkovich, 2013; Prediger & Wessel, 2013).

References

Abedi, J., & Lord, C. (2001). The language factor in mathematics tests. *Applied Measurement in Education, 14*(3), 219-–234.

Alt, M., Arizmendi, G.D. & Beal, C.R. (2014). The relationship between mathematics and language: Academic implications for children with specific language impairment and English language learners. *Language, Speech, and Hearing Services in Schools*, 45, 220-233.

Ashkenazi, S., Black, J. M., Abrams, D. A., Hoeft, F., & Menon, V. (2013). Neurobiological underpinnings of math and reading learning disabilities. *Journal of Learning Disabilities*, 46(6), 549–569.

Bailey, A. (2007). *The language demands of schools. Putting academic English to the test*. New Haven, CT: Yale.

Bakker, A., Smit, J., & Wegerif, R. (2015). Scaffolding and dialogic teaching in mathematics education. *ZDM - International Journal on Mathematics Education, 47*(7), 1047–1065.

Barwell, R. (Ed.) (2009). *Multilingualism in mathematics classrooms*. Bristol et al.: Multilingual Matters.

Barwell, R. (2012). Discursive demands and equity in second language mathematics classroom. In B. Herbel-Eisenmann, J. Choppin, D. Wagner, & D. Pimm (Eds.), *Equity in discourse for mathematics education. Theories, practices, and politics* (pp. 147–163). Dordrecht: Springer.

Barwell, R. (2014). Language background in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 331–336). Dordrecht: Springer.

Barwell, R., Clarkson, P., Halai, A., Kazima, M., Moschkovich, J., Planas, N., ... Villavicencio, M. (2016). *Mathematics education and language diversity: The 21st ICMI Study*. Dordrecht: Springer.

Branum-Martin, L., Fletcher, J. M., & Stuebing, K. K. (2012). Classification and identification of reading and math disabilities: The special case of comorbidity. *Journal of Learning Disabilities*, (46), 490–499.

Clarkson, P. (2006). Australian Vietnamese students learning mathematics: High ability bilinguals and their use of their languages. *Educational Studies in Mathematics, 64*(2), 191-215.

Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. (2001). Participating in classroom mathematical practices. *The Journal of the Learning Science*, *10(1&2)*, 113–163.

Cummins, J. (2000). *Language, power and pedagogy*. Clevedon: Multi lingual Matters.

Deutsches Institut für Medizinische Dokumentation und Information (2015). *Umschriebene Entwicklungsstörungen schulischer Fertigkeiten*. http://www.icd-code.de/icd/code/F81.2.html [25.12.2016].

DfEE - Department for Education and Employment. (2000). *The National Numeracy Strategy: Mathematical Vocabulary*. London: Dep. for Education and Employment. http://www.belb.org.uk/Downloads/num_mathematics_vocabulary.pdf [23.1.2016]

Dirks E., Spyer G., van Lieshout E.C.D.M., de Sonneville L. (2008). Prevalence of combined reading and arithmetic disabilities. *Journal of Learning Disabilities, 41,* 460–473.

Donlan, C. (2003). The early numeracy of children with specific language impairments. In A. J. Baroody & A. Dowker (Eds.), *The development of arithmetic concepts and skills* (pp. 337–358). Mahwah: Routledge.

Donlan, C. (2015). Linguistic factors in the development of basic calculation. In S.J. Chinn (Ed.), *The Routledge International Handbook of Dyscalculia and Mathematical Learning Difficulties* (pp. 346–356). London: Routledge.

Donlan, C., Cowan, R., Newton, E. J., & Lloyd, D. (2007). The role of language in mathematical development: Evidence from children with specific language impairments. *Cognition*, 103, 23–33.

Durkin, K., Mok, P.L.H. & Conti-Ramsden, G. (2015). Core subjects at the end of primary school: identifying and explaining relative strengths of children with specific language impairment (SLI). *International Journal of language & communication disorder,* 50, 226-240.

Echevarria, J., Vogt, M. E., & Short, D. (2010). *The SIOP model for teaching mathematics to English learners*. Boston: Pearson.

Erath, K. (2017). *Mathematisch diskursive Praktiken des Erklärens. Rekonstruktion von Unterrichtsgesprächen in unterschiedlichen Mikrokulturen*. Wiesbaden: Springer.

Erath, K. (submitted). Talking about conceptual knowledge. Case study on challenges for students with low language proficiency.

Erath, K., Prediger, S., Heller, V., & Quasthoff, U. (in press). Learning to explain or explaining to learn? Discourse competences as an important facet of academic language proficiency. *Educational Studies in Mathematics,*

Fazio, B. B. (1996). Mathematical abilities of children with specific language impairment: A 2-year follow-up. *Journal of Speech and Hearing Research*, 39, 839–849.

Fazio, B. B. (1999). Arithmetic calculation, short-term memory, and language performance with specific language impairment: A 5-year follow-up. *Journal of Speech, Language, and Hearing Research, 42*, 420–431.

Fuchs, L. S., Geary, D. C., Fuchs, D., Compton, D. L., & Hamlett, C. L. (2016). Pathways to Third-Grade Calculation Versus Word-Reading Competence: Are They More Alike or Different? *Child Development*, 87(2), 558–567.

Gibbons, P. (2002). *Scaffolding Language, Scaffolding Learning. Teaching Second Language Learners in the Mainstream Classroom*. Portsmouth: Heinemann.

Gunn, S. & Wyatt-Smith, C. (2011). Learning Difficulties, Literacy and Numeracy: Conversations across the Field. In C. Wyatt-Smith, J. Elkins, & S. Gunn (Eds.), *Multiple Perspectives on Difficulties in Learning Literacy and Numeracy* (pp. 17–48). Dordrecht: Springer.

Haag, N., Heppt, B., Roppelt, A., & Stanat, P. (2015). Linguistic simplification of mathematics items: effects for language minority students in Germany. *European Journal of Psychology of Education, 30*(2), 145–167.

Haag, N., Heppt, B., Stanat, P., Kuhl, P. & Pant, H. A. (2013). Second Language Learners' Performance in Mathematics: Disentangling the effects of academic language features. *Learning and Instruction, 28*(1), 24–34.

Halliday, M. A. K. (1974). *Some aspects of sociolinguistics. Interactions between linguistics and mathematical education symposium*. Paris: UNESCO.

Hiebert, J. (Ed.). (1986). *Conceptual and procedural knowledge. The case of mathematics*. Hillsdale: Lawrence Erlbaum.

Hirsch, E. D. (2003). Reading Comprehension Requires Knowledge - of Words and the World. Scientific Insights into the Fourth-Grade Slump and the Nation's Stagnant Comprehension Scores. *American Educator, 4*(1), 10–44.

Jorgensen, R. (2011). Language, Culture and Learning Mathematics: A Bourdieuian Analysis of Indigenous Learning. In C. Wyatt-Smith, J. Elkins, & S. Gunn (Eds.), *Multiple Perspectives on Difficulties in Learning Literacy and Numeracy* (pp. 315–329). Dordrecht: Springer.

Kempert, S., Saalbach, H., & Hardy, I. (2011). Cognitive Benefits and Costs of Bilingualism in Elementary School Students: The Case of Mathematical Word Problems. *Journal of Educational Psychology, 103*, 547–561.

Klein, E., Bahnmueller, J., Mann, A., Pixner, S., Kaufmann, L.., Nuerk, H.-C., & Moeller, K. (2013). Language influences on numerical development—Inversion effects on multi-digit number processing. *Frontiers in Psychology, 4*, 1–6.

Krinzinger, H., Gregoire, J., Desoete, A., Kaufmann, L., Nuerk, H.-C., & Willmes, K. (2011). Differential language effects on numerical skills in second grade. *Journal of Cross-Cultural Psychology, 42*(4), 614–629.

Lampert, M., & Cobb, P. (2003). Communication and learning in the mathematics classroom. In J. Kilpatrick & D. Shifter (Eds.), *Research companion to the NCTM Standards* (pp. 237–249). Reston: National Council of Teachers of Mathematics.

Mann Koepke, K. & Miller, B. (2013). At the intersection of math and reading disabilities: Introduction to the special issue. *Journal of Learning Disabilities, 46*, 483–489.

Miura, I. T., Okamoto, Y., Kim, C. C., Steere, M., Fayol, M. (1993). First Grader's cognitive representation of number and understanding of place value: Cross-national comparisons – France, Japan, Korea, Sweden and the United States. *Journal of Educational Psychology, 85*, 24 – 30.

Moll, K., Göbel, S.M., Gooch, D.; Landerl, K. & Snowling, M. J. (2016). Cognitive Risk Factors for Specific Learning Disorder: Processing Speed, Temporal Processing, and Working Memory. *Journal of Learning Disabilities, 49*, 272–281.

Morek, M., & Heller, V. (2012). Bildungssprache. Kommunikative, epistemische, soziale und interaktive Aspekte ihres Gebrauchs. *Zeitschrift Für Angewandte Linguistik*, 67–101.

Morgan, C., Craig, T., Schütte, M., & Wagner, D. (2014). Language and communication in mathematics education: an overview of research in the field. *ZDM - International Journal on Mathematics Education, 46*(6), 843-853.

Moschkovich, J. (2010). Recommendations for Research on Language and Mathematics Education. In J. Moschkovich (Ed.), *Language and Mathematics education* (pp. 1-28). Charlotte: Information Age.

Moschkovich, J. (2013). Principles and Guidelines for Equitable Mathematics Teaching Practices and Materials for English Language Learners. *Journal of Urban Mathematics Education, 6*(1), 45–57.

Moschkovich, J. (2015). Academic literacy in mathematics for English Learners. *Journal of Mathematical Behaviour*, *40*(A), 43–62.

Moser Opitz, E., Ruggerio, D. & Wüest, P. (2010). Verbale Zählkompetenzen und Mehrsprachigkeit: Eine Studie mit Kindergartenkindern. *Psychologie in Erziehung und Unterricht*, *57*, 161–174.

Norén, E. (2015). Agency and positioning in a multilingual mathematics classroom. *Educational Studies in Mathematics, 89*(2), 167-184.

Nys, J., Content, A., Leybaert, J. (2013). Impact of language abilities on exact and approximate number skills development: Evidence from children with specific language impairment. *Journal of Speech, Language, and Hearing Research, 56*, 956–970.

Paetsch, J., Felbrich, A., & Stanat, P. (2015). Der Zusammenhang von sprachlichen und mathematischen Kompetenzen bei Kindern mit Deutsch als Zweitsprache. *Zeitschrift für Pädagogische Psychologie, 29*(1), 19–29.

Petrill, S., Logan, J., Hart, S., Vincent, P., Thompson, L., Kovas, Y., & Plomin, R. (2012). Math fluency Is etiologically distinct from untimed math performance, decoding fluency, and untimed reading performance: Evidence from a twin study. *Journal of Learning Disabilities*, 45(4), 371–381.

Pimm, D. (1987). *Speaking mathematically. Communication in mathematics classroom*. London & New York: Routledge/Keagan Paul.

Planas, N. (2014). One speaker, two languages: Learning opportunities in the mathematics classroom. *Educational Studies in Mathematics, 87*(1), 51-66.

Pöhler, B. & Prediger, S. (2015). Intertwining lexical and conceptual learning trajectories - A design research study on dual macro-scaffolding towards percentages. *Eurasia Journal of Mathematics, Science & Technology Education, 11(*6), 1697–1722.

Pöhler, B., Prediger, S., & Neugebauer, P. (2017). Content- and language integrated learning: A field experiment for percentages. In B. Kaur, W. K. Ho, T. L. Toh, & B. H. Choy (Eds.), *Proceedings of the 41st Annual Meeting of the International Group for the Psychology of Mathematics Education (PME 41)* (Vol. 4, pp. 73-80). Singapore: PME.

Prediger, S. (2016). Wer kann es auch erklären? Sprachliche Lernziele identifizieren und verfolgen. *Mathematik differenziert, 7*(2), 6-9.

Prediger, S. & Wessel, L. (2013). Fostering German language learners' constructions of meanings for fractions – Design and effects of a language- and mathematics-integrated intervention. *Mathematics Education Research Journal, 25*(3), 435-456.

Prediger, S., Clarkson, P. & Bose, A. (2016). Purposefully relating multilingual registers: Building theory and teaching strategies for bilingual learners based on an integration of three traditions. In R. Barwell, P. Clarkson, A. Halai, M. Kazima, J. Moschkovich, N. Planas, M. Setati-Phakeng, P. Valero, & M. Villavicencio (Eds.), *Mathematics education and language diversity: The 21st ICMI Study* (pp. 193–215). Cham et al.: Springer.

Prediger, S., Wilhelm, N., Büchter, A., Gürsoy, E., & Benholz, C. (2018). Language proficiency and mathematics achievement – Empirical study of language-induced obstacles in a high stakes test, the central exam ZP10. *Journal für Mathematik-Didaktik, 39*(1 Suppl.), 1-26. doi.org/10.1007/s13138-018-0126-3.

Quasthoff, U. (2011). Diskurs- und Textfähigkeiten. Kulturelle Ressourcen ihres Erwerbs. In L. Hoffmann, K. Leimbrink, & U. Quasthoff (Eds.), *Die Matrix der menschlichen Entwicklung* (pp. 210–251). Berlin: de Gruyter.

Reljić, G., Ferring, D., & Martin, R. (2015). A Meta-Analysis on the Effectiveness of Bilingual Programs in Europe. *Review of Educational Research, 85*(1), 92–128. doi:10.3102/0034654314548514

Rhöm, A., Starke, A., & Ritterfeld, U. (2017). Die Rolle von Arbeitsgedächtnis und Sprachkompetenz für den Erwerb mathematischer Basiskompetenzen im Vorschulalter. *Psychologie in Erziehung und Unterricht, 64*(2), 81-93. doi:10.2378/peu2016.art26d

Riccomini, P. J., Smith, G. W., Hughes, E. M., & Fries, K. M. (2015). The Language of Mathematics. The Importance of Teaching and Learning Mathematical Vocabulary. *Reading & Writing Quarterly, 31*(3), 235–252.

Ritterfeld, U., Starke, A., Röhm, A., Latschinske, S., Wittich, C. & Moser Opitz, E. (2013). Über welche Strategien verfügen Erstklässler mit Sprachstörungen beim Lösen mathematischer Aufgaben? *Zeitschrift für Heilpädagogik, 64*(4), 136–143.

Schindler, V., Moser Opitz, E., Cadonou-Bieler, M. & Ritterfeld, U. (forthcoming). Überprüfung und Förderung des mathematischen Fachwortschatzes der Grundschulmathematik – Eine empirische Studie.

Schleppegrell, M. J. (2004). *The language of schooling: A functional linguistics perspective*. Mahwah: Lawrence Erlbaum.

Schröder, A. & Ritterfeld, U. (2015). Children with Specific Language Impairment (SLI) Need Qualitatively Enriched Interactions to Successfully Partake in Mathematics Education. *International Journal of Technology and Inclusive Education, 4*, 574–582.

Scherer, P., Beswick, K., Deblois, L., Healy, L. & Moser Opitz, E. (2016). Assistance for Students with Mathematical Learning Difficulties? *ZDM - International Journal on Mathematics Education, 48*, 633–649.

Secada, W. G. (1992). Race, ethnicity, social class, language and achievement in mathematics. In D. A. Grouws (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 623–660). New York: MacMillan.

Setati, M. (2005). Teaching mathematics in a primary multilingual classroom. *Journal for Research in Mathematics Education, 36*(5), 447 - 466.

Short, D. J. (2017). How to Integrate Content and Language Learning Effectively for English Language Learners. *EURASIA Journal of Mathematics, Science and Technology Education, 13*(7b), 4237-4260.

Smit, J. (2013). Scaffolding language in multilingual mathematics classrooms. (PhD-Thesis), Freudenthal Institute, Utrecht.

Snow, C. E., & Uccelli, P. (2009). The Challenge of Academic Language. In D. R. Olson & N. Torrance (Eds.), *The Cambridge handbook of literacy* (pp. 112–133). Cambridge: Cambridge University Press.

Swain, M. (1995). Three functions of output in second language learning. In G. Cook & B. Seidlhofer (Eds.), *Principle and practice in applied linguistics: Studies in honour of H.G. Widdowson* (pp. 125–144). Oxford: Oxford University Press.

Thürmann, E., Vollmer, H., & Pieper, I. (2010). *Language(s) of schooling: Focusing on vulnerable learners*. Straßbourg: Council of Europe.

Vygotsky, L. S. (1978). *Mind in society. The development of higher psychological processes*. Cambridge: Harvard University Press.

Willcutt, E.G., Petrill, S.A., Wu, S., Boada, R., DeFries, J.C., Olson, R.K. & Pennington, B.F. (2013). Comorbidity between reading disability and math disability. *Journal of Learning Disabilities, 46*, 500–516.

Zuber, J., Pixner, S., Moeller, K. & Nuerk, H. C. (2009). On the language specificity of basic number processing: Transcoding in a language with inversion and its relation to working memory capacity. *Journal of Experimental Child Psychology, 102*, 60–7.