

TWO LANGUAGES – SEPARATE CONCEPTUALIZATIONS? MULTILINGUAL STUDENTS' PROCESSES OF COMBINING CONCEPTUALIZATIONS OF THE PART-WHOLE CONCEPT

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Different languages are said to provide slightly different conceptualizations for mathematical concepts, e.g. for the part-whole concept of fractions. But how do multilingual learners make use of these different conceptualizations in their individual conceptual pathways? This case study investigates how fourteen German-Turkish seventh graders develop their part-whole concepts in a bilingual design experiment. The qualitative analysis shows that they use the conceptualizations across both languages and merge them into a multi-faceted part-whole concept. These findings provide a topic-specific empirical elucidation for the general idea of dynamic and interconnected multilingual repertoires.

STARTING POINTS AND THEORETICAL BACKGROUNDS

Language-related conceptualizations and research needs for multilingual learners

The observation that languages sometimes provide different structures and conceptualizations has fueled controversial academic discourses on the so-called linguistic relativity hypothesis since von Humboldt and Sapir and Whorf (Lucy 1992; Gumperz & Levinson 1996). In mathematics education, the discourse led to empirical comparative studies that investigate if speakers of one language have advantages or disadvantages for their (habitual, not potential) mathematical thinking compared to speakers of other languages (e.g., Miura et al. 1988; Leung 2016). These comparative studies tend to adopt a monolingual perspective, assuming each student to be shaped predominantly by one language. However, for *multilingual* students, the question is not whether they are acquainted to one *or* the other conceptualization, but how the *interplay of different languages and conceptualizations* shapes their learning processes on the micro-level. This shift of research focus corresponds to the idea of dynamic and interconnected multilingual repertoires in multilingual communication rather than separate language proficiencies (Cummins 2000; House & Rehbein 2004; Lüdi 2006).

In this paper, we contribute to this research need by studying a case involving German-Turkish seventh graders' bilingual teaching and learning processes of the part-whole concept of fractions. The research question is: *How do learners in a bilingual teaching intervention adopt and combine (possibly language-related) conceptualizations of the part-whole concept across both languages?* After presenting the theoretical backgrounds and the methods, the qualitative analysis of 14 students shows that they relate several conceptualizations to each other in mostly fruitful ways.

Language-related nuances in conceptualizing the part-whole concept of fractions

In most countries, the part-whole concept of fractions counts as one major meaning of fractions, besides rates and ratios (Cramer et al. 1997). However, different languages seem to provide different nuances in the conceptualization of the part-whole concept, connected to the reading and writing order: In Western languages, fractions are read and written top down (3 *fifths* in English or 3 *Fünftel* in German), whereas in most Asian languages fractions are read and written bottom up (“five parts, take two” in Mandarin (Bartolini-Bussi et al. 2014), or *beşte üç*, “five-therein three” in Turkish).

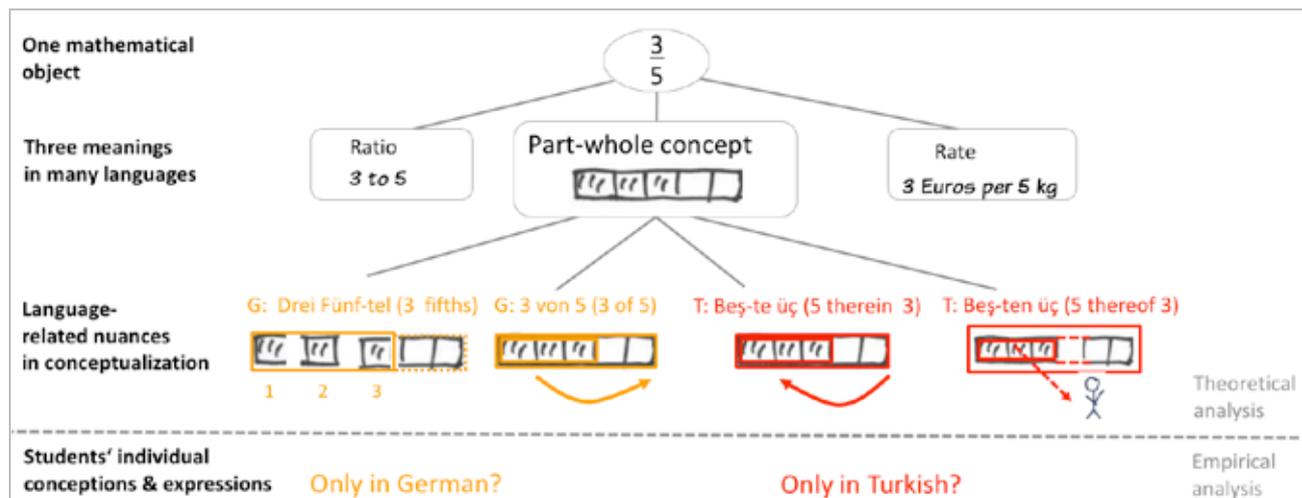


Fig. 1: Different conceptualizations in German (G, orange) and Turkish (T, red)

These language-related differences seem to not be restricted to single words but to more general habitual modes of thinking (Lucy 1996) and ways of linguistic action. This was confirmed in the case of geometric constellations by Leung’s (2016) analysis of how order differences can shape mathematical thinking: Leung distinguishes the typical Asian *analytic approach* (first consider the whole, then the pieces) from the Western *synthetic approach* (first consider the pieces, then the whole) and shows far-reaching consequences for students’ problem solving. In the case of fractions, the distinction of synthetic and analytic directly corresponds to the different conceptualizations in Turkish (with its Asian influences) and German (a Western language).

The German and English expressions “three fifths” also reveal a second nuance called quasi-cardinal conceptualization: considering a fifth as a unit and then counting the units (Cramer et al. 1997). The Turkish everyday language of German-Turkish immigrant students also provides a second expression, with another grammatical case: the ablative suffix *-ten* (used for movement away conditions) instead of the locative suffix of *-te* (used for static position conditions). In sum, Figure 1 shows four different nuances of conceptualizations for the part-whole concept.

So far, little is known how multilingual students with their access to both languages adopt and combine these different conceptualizations.

Alternative hypotheses for students' use of language-related conceptualizations

The idea of functional distinctions of languages for different purposes that underlies much research on code-switching (summarized in Barwell 2009) leads to *Hypothesis H1*: Multilingual students will learn all four conceptualizations and use each in the language in which it can be expressed best. The alternative hypothesis is shaped by the idea of dynamic and interconnected “multilingual repertoires” rather than separate languages (Lüdi 2006). On this basis, *Hypothesis H2* is that students adopt the conceptualizations across different languages.

METHODS OF THE LEARNING-PROCESS STUDY

Research context. The research question was pursued in a learning-process study that was part of the larger mixed-methods project MuM-Multi. The larger project combined a randomized control trial with German-Turkish seventh graders ($n = 139$) in a teaching intervention on fractions in groups of 2-5 students (Schüler-Meyer et al. 2017) with several in-depth case studies analyzing videos and transcripts with respect to the integration of verbal and nonverbal communication (e.g., Wagner et al. 2016).

Data corpus of the learning-process study. From the large video data corpus, we selected about 230 minutes of video material for the learning-process study presented here. We concentrated the analysis on seven tasks (treating the part-whole concept in contexts of comparing fractions) done by $n = 14$ focus students (who were sampled according to contrasting backgrounds in their German and Turkish language proficiency and pre-test results on conceptual understanding of fractions). All students spoke at least German and Turkish, all were educated in Germany without prior formal experience of learning mathematics in Turkish.

Methods for qualitative data analysis. The transcripts were analyzed with respect to students' conceptual development across languages. For this purpose, an analytic tool for fractions was adopted based on Vergnaud's (1996) theorems- and concepts-in-action. After sequencing the transcript, the individual theorems- and concepts-in-actions were extrapolated for each sequence and then related to the language-related socially shared conceptualizations from Figure 1, e.g., the individual theorem-in-action <For comparing fractions, I compare the size of the pieces> (which is only adequate for unit fractions) is shaped by the individual concept-in-action <fraction as size of the pieces> rather than <fraction as part-of-whole>. In the analytic schemes (e.g., in Figures 3 and 4), the individual conceptualizations are presented graphically to show proximity/variation to the socially shared language-related conceptualizations; the colors of the utterances signify the language used by students to express them. The color codes allow the relation between language used and language-related conceptualizations to be conveyed. Two episodes (chosen to show the phenomena in a nutshell) are analyzed in some detail. The result of the complete analysis is summarized in a table that reveals more global pattern with respect to Hypotheses H1 and H2.

EMPIRICAL RESULTS

The following two episodes show that both hypotheses can apply in the multilingual learning processes.

Episode 1: Ilknur and the complementary use of languages

In Session 2, Task 3, the students are asked to draw $1/2$, $2/3$, $3/4$, $4/5$, and $5/6$ in fraction bars (see Figure 2). One of the students, Ilknur, explains the task to her partner Akasya.

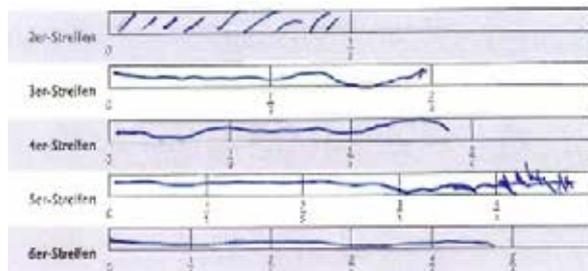


Fig. 2: Ilknur's fraction bars for Task 3

Turn Person	Original (Turkish in black, German in grey)	English Translation (from Turkish in red, from German in orange)
90 Akasya	Ich versteh das nicht!	I do not understand that!
91 Ilknur	Guck, ich zeig dir! Şimdik bak! Şöyle bakıyorsun. Burda ne yazıyor? Ein Halb! [...] O zaman bunu boyuyorsun.	Look, I show you! Now look! [points at the 2-element bar in the fraction bar table] You look this way. What does it say here? One half! [...] Then you have to color this one here [colors the first piece in the 2-element bar] But why?
92 Akasya	Niye ama?	Because it is one half [points at the fraction on the worksheet] and this also one half
93 Ilknur	Weil es ein halb ist und das auch ein Halb	[points at the first piece in the 2-element bar]
94 Akasya	Dann kann ich doch zwei Drittel hier kann ich doch auch das hier anmalen, wo ist zwei Drittel hier?	Then I can two thirds, here, [points at the share on the worksheet], I can yet color it also here, where is two thirds here? [looks irritatedly at the fraction bar table]
95 Ilknur	Bis zwei Drittel'e bakacaksın	Up to two thirds you have to look [points at end point of the second piece of the 3-element bar]
96 Akasya	Ah, zwei Drittel burda! Dann muss ich ja das hier alles anmalen oder was?	Ah, two thirds here! Then I must color all of this here or what?
97 Ilknur	Ja!	Yes!
98 Akasya	Und hier drei Viertel	And here three fourths [points at the end point of the third piece in the 4-element bar]

Ilknur explains how to draw the fractions in suitable bars of the bar table (see Figure 2). She starts using an analytic, localizing perspective where the whole is only implicit (*Buni boyuyorsun*, “this one here”) in #91. When her partner Akasya signals misunderstanding, Ilknur changes languages and, with it, perspectives (a strategy often found for teachers and students; see Wagner et al. 2016): Within a Turkish phrase, Ilknur switches to German, *bis zwei Drittel* (until two thirds), in #95 to express a quasi-cardinal view, counting the pieces of the bar from the left zero until the two-thirds point in the bar.

From the German preposition *bis* (“until”) we infer that using the German expression really corresponds to the conceptualization in her thinking at that moment. She repeats this conceptualization in #95 (*bis dahin*, “until here”) and later in #99 and #101. In #96, Akasya connects both conceptualizations by combining the German expression *zwei Drittel* (“two thirds”) with the Turkish *burda* (“here”) and *das hier alles anmalen* (“color all of this here”).

In sum, the analysis of Episode 1 (in Figure 3) shows a typical example of code-switching in Ilknur’s complementary use of languages that supports Hypothesis H1. Akasya’s reaction shows how combining two nuances of conceptualizations in two languages can enhance conceptual understanding, a phenomenon that could be found by means of deep linguistic analysis in various cases (Wagner et al. 2016).



Fig. 3. Analysis of Episode 1

Episode 2: Emir and the travel of conceptualizations from Turkish to German

In another group, the students work on the same Task 3, which asks them to draw 1/2, 2/3, 3/4, 4/5, and 5/6 in fraction bars (see Figure 2). Emir and Osman immediately draw one half, and the teacher asks them in Turkish where they see the half.

Turn	Person	Original (Turkish in black, German in grey)	English Translation (from Turkish in red, from German in orange)
27	Emir	Eh iki komplett	Err, two, complete [gestures with his pen to refer to the complete fraction bar]
28	Teacher	Mhm.	Mm-hmm [yes]
29	Emir	Bi tanesinde alyoz.	And one in it, we take.
....		[Students proceed with other fractions]	
47	Osman	Zwei Stück anmalen. Zwei Drittel, drei Viertel.	Coloring two pieces [marks two thirds in the 3-element bar] two thirds, three fourths
48	Osman	Drei Stück anmalen	Coloring three pieces
49	Ismael	Das gehört zu...?	That belongs to...?
50	Osman	Drei Stück an—warte—ja	Three pieces at—wait—yes [marks three fourths on the 4-element bar]
51	Emir	Und dann vier anmalen.	And then, color four.
...			
53	Osman	Vier Stück? Bei beş'li çubuk?	Four pieces? In 5-element bar?
54	Emir	Mhm, ja. Und bei 6er musst du fünf	Mm-hmm, yes. And in six, you must five
....		[Students finalize drawing and discuss what to write down for the question "What can you discover? Write down your observation."]	
67	Emir	Also, ich schreibe jetzt, dass unten [...] Das zwei- das von ein Zweitel ist eh- Einhalb ist zwei als Ganze und du nimmst eins.	Well, I will write that below [...] That two, that of one second, is Err, one half is two as the whole and you take one.

When asked to explain how to draw one half, Emir speaks in a mixed utterance, and refers to the Turkish *–therein* expression for the analytical conceptualization in Turkish (in #27 and #29). Again, we see an initial moment of code-switching. But then, Osman adopts the typical German synthetical conceptualization: He first names only the parts (“two pieces” in #47, “three pieces” in #48) and completes the whole only implicitly. In #53, he addresses the five-element bar as the whole explicitly (in Turkish). Emir, in contrast, keeps his Turkish reading order even when speaking German in #54. When asked to write down what they saw, Emir prefers to explain in #67 the Turkish *–thereof* conceptualization in German words.

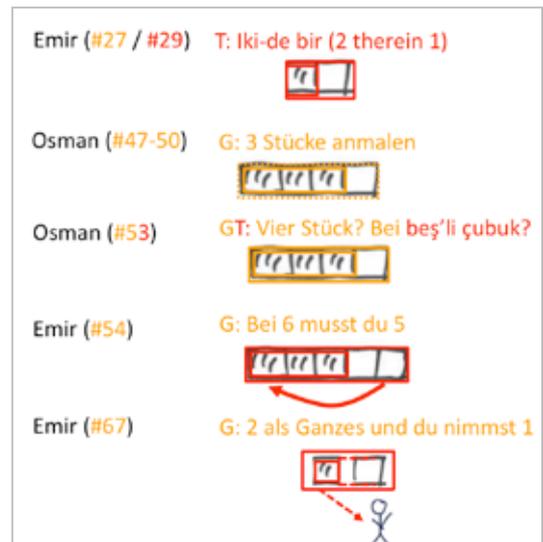


Fig. 4: Analysis of Episode 2

Hence, Episode 2 gives evidence for both Hypothesis H1 in the beginning and then later for Hypothesis H2. Figure 4 summarizes the analysis and shows the travel of conceptualizations from Turkish to German.

Overview on more cases and tasks:

Travel of conceptualizations through languages

Both episodes show phenomena that could also be found for the other cases of students and tasks. Table 1 shows a summary of the analysis of five of the 14 focus students. For each student, the sequence of uttered nuances of conceptualizations for the part-whole concept is ordered from up to down and is arranged in columns according to the socially shared conceptualizations to which the utterance refers. The transcript lines are shown in different colors for German (G) and Turkish (T) so that shifts of nuance become visible by placement along the horizontal axis and switches in language by changing colors along the vertical axis.

This summary illustrates that all students, not only Ilknur and Emir, start activating a nuance of conceptualization in one language, but when they use other languages, then also refer to the same nuance of conceptualization in the other language and in a mixed code. This might be different in other groups. What is made visible here for a sequence of four tasks seems to apply even more when considering several sessions of the intervention and more nuances of conceptualization than those presented in Figure 1.

	T: Drei Fünf-tel (3 fifth) 	T: 3 von 5 (3 of 5) 	T: Beş-te üç (5 therein 3) 	T: Beş-ten üç (5 thereof 3) 
Ilknur	Task 3, #84 (G) Task 3, #91 (GT) Task 3, #95 (T) Task 3, #99 (G)	Task 3, #177 (G) Task 4, #114 (GT)	Sess1, Task 7, #3 (GT)	
Akasya		Task 2, #81 (T) Task 2, #107 (G) Task 3, #96 (GT) Task 3, #100 (G) Task 3, #159 (G) Task 3, #173 (T) Task 4, #67 (G)	Sess1, Task 7, #10 (T)	
Emir		Task 1, #20 (G) Task 1, #43 (T) Task 1, #76 (G) Task 2, #15 (G) Task 3, #90 (T) Task 4, #26 (T)	Task 3, #29 (T) Task 3, #54 (G)	Task 3, #67 (G)
Ismail		Task 1, #81 (G) Task 2, #2-7 (G) Task 2, #31 (T) Task 3 # 39 (G) Task 3, #90 (T)	Task 4, #28 (T)	Task 4, #64 (T)
Osman		Task 2, #25 (G)	Task 3, #50ff. (GT)	

Table 1: Travel of conceptualizations through the languages for five focus students

DISCUSSION

Two languages, two separate conceptualizations? By investigating the learning processes of 14 seventh graders on their pathways towards the part-whole concept of fractions, we found moments, such as those in Episode 1, that conform with Hypothesis H1 in that language is used complementarily for different conceptualizations. These complementary uses seem to enrich the multi-faceted conceptual understanding. However, across all of the video material and transcripts of the 14 students, we find more moments that conform to Hypothesis H2: In their learning processes, most students activate different conceptualizations, and, in the long run, address them across both languages. The case studies presented and the larger analysis of the data provide evidence that the **travel of conceptualizations across languages** can enrich the conceptual understanding by merging the different conceptualizations into a multi-faceted part-whole concept.

Although these findings are still shaped by methodological limitations such as the limited number of focus students and the specific tasks, it is already an interesting contribution to the idea of dynamic and intertwined multilingual repertoires which resonates with Cummins's (2000) and Lüdi's (2006) arguments against considering multilingual learners as having separate language proficiencies that may work only complementarily. The findings correspond to previous findings from the same project that it is not the complementarity, but the *connection* of languages (like the connection of different representations) that can substantially enhance students' conceptual development (Wagner et al. 2016, Schüler-Meyer et al. 2017). The findings motivate further studies of the connections of languages as specific resources of multilingual.

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