Low achieving eighth graders learn to crack word problems:
a design research project for aligning a strategic scaffolding tool to students’ mental processes

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Abstract. Topic-specific didactical design research provides means not only to investigate how to learn but also what to learn, i.e. for specifying learning contents by analyzing students’ comprehension processes in detail. This important characteristic of didactical design research is exemplarily shown for students’ difficulties in finding symbolic expressions for word problems, which can often be traced back to deficits in their comprehension strategies. The article presents a design research project on strategic scaffolding for eighth graders with limited language proficiency for specifying fruitful comprehension strategies and enhancing their use. Intensive qualitative investigation of students’ processes was required to align the strategic scaffolding tool, the word problem cracker (WPC), to students’ mental processes. Four cycles of design experiments allowed iteratively developing a local theory of learning to write algebraic expressions for multi-step word problems.

1. Introduction

Many design research studies mainly focus on pedagogical questions. However, the methodology also proves to be fruitful for specifying topics to be learnt in deeper ways (e.g., what exactly should students learn in order to achieve a learning goal or to overcome a specific problem?). This article intends to show how the methodology of topic-specific design research can provide an instructive framework for empirical contributions to iteratively specifying learning contents, here specifically for comprehension strategies for low achieving students. For unfolding iterativity, the article accounts for intermediate steps between cycles.

For this purpose, we exemplarily report on a design research study starting from a well-known problem, students’ difficulties with word problems. Typical obstacles have been identified in the comprehension processes from the text to mathematical expressions, and some explanatory findings have been gained about underlying misleading strategies. Based on this large body of descriptive and explanatory findings, the presented study aims at contributing to the theoretical instructional knowledge on which comprehension strategies are most fruitful and how students’ use of these strategies can be enhanced.

As most research on comprehension obstacles refers to elementary arithmetic word problems, it is less often connected to the discussion about algebraic word problems with multiple steps and variables (like the example printed in Fig. 1).

Starting from these different branches of literature on comprehension in general and on algebraic reasoning, this article reports on a design research project on how to foster students’ comprehension strategies for algebraic word problems, especially for low achieving students with low language proficiency. The design research study contributes to specifying fruitful comprehension strategies as relevant learning content and to elaborating the design principle of strategic scaffolding (Hanaffin et al. 1999) according to which students can acquire new strategies when being temporarily supported by a scaf-

Open air bath problem with varying rainy days
The open air bath in a small town is open seven days a week when the weather is good. During rainy days, it stays closed. The circulation pump for the water runs 12 hours on open days.

Find an algebraic expression for calculating the operation time of the pump for a varying number of rainy days per week. The expression is supposed to have the structure.

Adile’s solution:

Fig. 1 Multi-step algebraic word problem and typical erroneous student solution
folding tool. The article shows how several cycles of design research were required to adapt the tools found in literature to the specific structure of multi-step algebraic word problems (as in Fig. 1) and to take into account interindividual differences in students’ mental processes, in the comprehension processes as well as in the learning processes.

The study is presented in five steps: Section 2 discusses the state of research on the content to be learnt, comprehension strategies for algebraic word problems. Section 3 explains the design principles of strategic scaffolding and the design challenges and research questions. The in-depth analysis of students’ processes on the micro-level is necessary, (a) for understanding students’ comprehension processes as the learning content in a topic-specific design research, and (b) for gaining a local instructional theory for strategic scaffolding. Section 4 reports on the methodological framework for reaching these goals in four design experiment cycles. Section 5 presents the design outcome of the project, the so-called word problem cracker and its backgrounds, and Section 6 offers selected iterations of empirical insights into the initiated comprehension and learning processes while enhancing students’ strategies.

2. Theoretical background for specifying the learning content: strategies for cracking word problems

2.1 Typical general obstacles when dealing with word problems

Students’ difficulties with mathematical word problems have often been described for different age and achievement levels. Within 40 years of empirical research, mainly three different sources of difficulties have been identified (cf. Reusser 1997; Prediger 2010; Duarte et al. 2011 for overviews):

- **Conceptual obstacles:** The difficulty of a word problem can depend on the underlying semantic problem structure. Empirical findings specify more or less accessible structures which are connected to students’ access to different basic models (in German Grundvorstellungen, vom Hofe et al. 2006; Stern 1993). Instructional approaches starting from these obstacles focus on the development of students’ basic models, i.e. their conceptual understanding of the mathematics.

- **Habitual obstacles of superficial modeling:** Many students got acquainted to superficial solution strategies, for example by applying unsuccessful key word strategies (Nesher & Treuhal 1975) or solving even senseless word problems (Verschaffel et al. 2000). As these habitual obstacles have been traced back to problematic classroom practices, one important branch of the discussion on instructional consequences for elementary schools has focused on increasing the authenticity of word problems and the intensity of reflection of the context (ibid.).

- **Comprehension obstacles:** Grasping the semantic structure of a problem involves mathematical comprehension as well as reading comprehension. Even for students who principally have acquired the necessary basic models and overcome also the habitual obstacles, word problems often pose further obstacles in the first steps of the modeling process (cf. Borromeo Ferri 2006; Galbraith & Stillman 2006), namely the transition from the word problem text to a situation model and to a mathematical model (e.g., Reusser 1994, 1997; Kintsch & Greeno 1985). As successful comprehenders seem to apply other strategies than non-successful students (Franke & Ruwisch 2010; Aebli et al. 1986), instructional approaches for overcoming these obstacles should focus on enhancing these successful comprehension strategies.

Although all three kinds of obstacles are interconnected, their mutual relevance seems to depend on age and language proficiency level, mathematical topic, and problem structure of the involved word problems (Reusser 1997, Duarte et al. 2011). Although all three are important in our context, this paper mainly con-
tributes to the third line, the *comprehension strategies in the interplay between reading and finding mathematical relations*. Whereas the research on comprehension obstacles mainly focused on one step arithmetic word problems, we investigate how the findings transfer to multi-step algebraic word problems and language learners.

### 2.2 Specific obstacles for algebraic word problems

Besides the general literature on word problems, algebraic word problems have been investigated in the algebra education perspective (cf. Stacey, Chick, & Kendal 2004 for a wide overview). In the algebra perspective, difficulties with algebraic word problems have been mostly traced back to *conceptual challenges* with relevant basic models giving meanings for variables (Usiskin 1988, p. 11ff): (a) variable as unknown, as used in problem solving contexts which lead to equations, (b) variable as generalizer, as used to study and generalize relations between quantities, or (c) the variable in the context of the study of relationships among quantities in a functional covariation perspective. Finally, the variable (d) also appears in contexts of symbolic manipulations as a meaningless sign. (c) and (d) belong to later steps in a long-term trajectory, thus are not addressed in our context where the focus is on (b).

One well-established generalization approach for developing students’ basic model of *variables as generalizers* is the figural pattern approach (e.g., Mason et al. 1985; and many others): Investigating growing patterns for figural shapes and/or number sequences (like in Fig. 2) leads to constructing the meaning of variables and algebraic expressions as symbolic means to express general patterns. In the example (in Fig. 2), $2+x\cdot6$ expresses the general rule for the numbers of dots in the shape at the $x$-th position (with \(\cdot\) signifying multiplication $\times$ in German).

#### Fig. 2 From patterns of figure shapes and numbers to the variable as generalizer – Overcoming conceptual challenges

In our design research project, the figural pattern approach was integrated in a remediating basic algebra course for low-achieving grade 8 students, and realized in three steps: The students find growing patterns for different sequences of shapes and numbers, then express these growing patterns informally (with pictures, tables, own verbal descriptions, or with quasi-variable numbers such as 42), and finally arrive at the variable as a means to express the general case (cf. Prediger & Krägeloh 2015).

Although this approach proved to be successful for acquiring the basic model “variable as generalizer”, it turned out to be not sufficient for finding expressions for word problems: Even when the low achieving students in the basic algebra course successfully learnt to find algebraic expressions for figural pattern, they still experienced *additional comprehension obstacles* in cracking algebraic word problems due to challenges in the text: although being able to algebraize the varying quantity, they struggle with reconstructing the relations. This important observation became the starting point for the research reported here.

These additional obstacles are partly related to a well-known challenge in algebraic reasoning: Whereas numbers and operations in figural pattern only refer to counting elements and numerical operations (with uninterpreted numbers or cardinalities), most algebraic word problems require *reasoning in quantities* and *quantitative operations* (relating quantities like prices, sizes or others, cf. Thompson 2011). Even if the
students in our basic algebra course were principally able to reason on general relationships among quantities, they rarely activated this mode of reasoning during cracking the algebraic word problems. Hence, overcoming comprehension challenges also requires to support reasoning in quantities (Thompson 2011).

In our instructional design, we combine the idea of constructing and focusing quantities with approaches for overcoming some of the above mentioned general comprehension obstacles. This approach relies on further starting points: a model for successful comprehension processes (Section 2.3), the specification of target comprehension strategies as relevant learning content strategies (Section 2.4), and approaches for scaffolding these strategies (Section 3).

2.3 Process model for treating word problems

Diverse alternative models exist for describing and structuring students’ mental processes while comprehending and solving word problems (overviews in Borromeo Ferri 2006; Reusser 1997). These models proved of value for locating students’ obstacles more precisely in the process (e.g., Galbraith & Stillman 2006).

In this article, we built upon ideas of Reusser’s (1990; 1997) model in Fig. 3 (its covered lower part hints to the article’s focus on finding the algebraic expression, without manipulation).

Although we do not share Reusser’s optimism of being able to isolate each step in the mostly holistic processes, his ideas are still useful for locating comprehension challenges in this combined model of language and situation comprehension components:

“solving mathematical word problems [is …] a strategic process from text to situation to equation, a progressive, elaborative, and incremental process of deepening problem comprehension, and of transformation of an initial textually based problem representation into an equation [or expression].” (Reusser 1990, p. 481)

Even if the processes take place inseparably, we refer to Reusser’s useful analytic (not chronological) distinction of steps from the episodic situation model (an often temporarily structured internal reconstruction of the situation in the text) to the episodic problem model (referring also to the question of the word problem) and then to the mathematical problem model (in which the situation is structured according to relevant mathematical relations). These distinctions help to locate low achieving students difficulties to find algebraic expressions from texts.

Thompson’s (2011) emphasis on the relevance of constructing and thinking in quantities before finding an expression can hence be located in Reusser’s mathematical problem model. This resonates with Reusser’s emphasis of iterativity instead of chronological linearity of the steps.

2.4 Specifying comprehension strategies as main learning content

For students to go through these mental processes successfully, they need no awareness of the process model, but need to activate (possibly unconsciously) adequate comprehension strategies. What is often
called “making sense” in a holistic perspective on successful students’ processes, is hence required to be decomposed in different strategies for the low achievers.

Various qualitative and quantitative investigations of students’ processes have been conducted in order to describe and understand how the students’ individual processes differ from ideal process models as those in Section 2.2. Focusing on the comprehension phases of the process model, the literature review allows to identify some frequent obstacles. In light of the constructive goal to foster students’ comprehension capabilities, we search for existing empirical findings on underlying individual strategies.

The most intensively researched obstacle concerns superficial comprehension when significant steps in the process model are shortened directly from the problem text to a mathematical model. Although these obstacles can often be traced back to habitual obstacles (see Section 2.1), it is worth to identify typical individual compensation strategies beyond the habitual level:

• Select some numbers and immediately start calculation (e.g. Verschaffel et al. 2000; Franke & Ruwisch 2010, pp. 79ff.)
• Focus only on key words (Nesher & Teubal 1975; Aebli et al. 1986)
• Select numbers and chose operation currently taught (e.g. Verschaffel et al. 2000).

As Verschaffel et al. (2000) have emphasized, students often experience their individual compensation strategies as successful in many simple arithmetic word problems if the instructional contexts allows it. However, more complex word problems call for overcoming these misleading individual compensation strategies and for replacing them by more suitable strategies.

Theoretically, we conceptualize the multi-faceted construct ‘strategy’ by adapting Ashcraft’s definition of strategy as “any mental process or procedure in the stream of information-processing activities that serves a goal-related purpose” (1990, p. 207). For two aspects of adaptation, we refer to the conceptualization that Threlfall’s (2009) suggested for children’s mental calculation strategies:

• First, rather than reserving the construct strategy (like e.g. for heuristic strategies) in a normative sense for target strategies that really can serve their purpose, we also use the word in a descriptive sense and subsume students’ individual, successful and unsuccessful strategies.
• Second, we follow Threlfall (2009) in not necessarily assuming a conscious rational choice of (preexisting) strategies, but often implicit spontaneous “reinventions” of strategies emerging in the comprehension process.

Less research studies have empirically reconstructed successful strategies, e.g. by systematically comparing successful and non-successful individual processes. However, the first design cycles allowed us to identify the following strategies as candidates for overcoming the mentioned obstacles, in general and specific for algebraic word problems:

• Standard reading strategy: find relevant information, which is often cited and fostered in classrooms, but doesn’t work in exclusive use.
• Focus on information together with their meaning (Aebli et al. 1986; Reusser 1994; Capraro et al. 2012); specifically for algebraic word problems, this means focusing on the meaning of variables (Usiskin 1988) and on quantities (Thompson 2011).
• Focus on relations connecting the information (Aebli et al. 1986; Jorgensen 2011; Kintsch & Greeno 1985, p. 112; Reusser 1994), specifically for algebraic word problems, this means focusing on relations among quantities (Thompson 2011; Cai et al. 2011).

Hence, the general literature on comprehension resonates with the literature on algebraic reasoning as a special case (cf. Stacey et al. 2004 and mainly Thompson 2011). However, we choose the more general wording for the two strategies in order to facilitate transfer e.g. to geometry problems.
Most research studies on general reading comprehension deal with one-step word problems, involving only one operation to be found (e.g., Reusser 1997; Kintsch & Greeno 1985). But additional obstacles appear for multi-step problems which require to combine several information by well-sequenced operations (shown for arithmetic problems e.g. Carpenter et al. 1980, p. 44). According to our experience, especially students with language difficulties often do not even start the comprehension process for multi-step word problems and deliver empty sheets because their fear of repeated failure leads to avoidance. A strategy for overcoming these affective obstacles is “just get started anyway”.

Even those who do get started, often do not accomplish the multi-step mathematization process properly; either they stop after one step or use a false order of operations. Beyond these error phenomena, two typical individual strategies can be reconstructed:

- Use numbers for operations according to their order in the text (Carpenter et al. 1980; Reusser 1997)
- Start with parts of the situation model and proceed spontaneously rather than planning the complete process in advance (as Pólya 1945 and others claim to be necessary).

The last mentioned individual strategy requires a more thorough analysis since on the one hand it seems to contradict the main idea of many process models with their strong emphasis on advance planning. On the other hand various studies have shown that even expert modelers do not plan the whole process in advance but iteratively switch between situation model, problem model and mathematizations. That is why Reusser (1994) included the iterativity between steps of successively constructing the situation model, problem model and mathematical expression. Rather than demanding a complete planning before mathematizing, he emphasizes the strategy of working systematically forward for decomposing the process in several steps so that every step only contains one operation.

However, working only forward without considering the final question risks not to reach the goal, that is why building an episodic and mathematical problem model in Reusser’s sense (cf. Fig. 2) includes the reference to the final question and the mediation between questions and results. For this, Aebli et al. (1986) underline the relevance of the classical heuristic strategy ‘working forward and backward’ (Pólya 1945) for sequencing the treatment of a multi-step problem into several steps.

To sum up, although the literature on successful comprehension strategies is limited (especially for multi-step problems), it resonates with the specific research on algebra. On this base, the following six target strategies are identified as relevant for successful comprehension processes for multi-step algebraic word problems:

(S1) basic reading strategy: find relevant information
(S2) focus on information with their meaning, e.g. quantities
(S3) focus on relations connecting pieces of information, e.g. relations between quantities
(S4) decompose in several steps and working forward
(S5) work forward and backward for decomposing in several steps towards a goal
(S6) get started anyway with first ideas for overcoming affective obstacles

As usual for a design research project, this specification of the learning content was not the starting point of the project, but one of its major outcomes that was refined during the iterative design cycles (see Section 4 to 6). Careful analysis of students’ processes were necessary for identifying exactly these six as the most relevant in our context. For facilitating readability, we nevertheless present the six target strategies already here in the theory chapter.
3. Theoretical background for the design approach: a scaffolding tool for cracking word problems

3.1 General approach of strategic scaffolding

The general notion ‘scaffolding’ was introduced by Wood, Bruner and Ross, who defined scaffolding as to enable

“a child or novice to solve a problem […] which would be beyond his unassisted efforts. This scaffolding consists essentially of the adult ‘controlling’ those elements of the task that are initially beyond the learner’s capacity, thus permitting him to concentrate upon and complete only those elements that are within his range of competence.” (Wood, Bruner, & Ross 1976, p. 90).

Scaffolding was first discussed only for tutor-learner (e.g. parent-child) interaction, and later transferred to support given in the design of the teaching learning arrangement (Hannafin et al. 1999; Lajoie 2005). In both cases, a scaffold is conceptualized as a “temporary framework to support learners when assistance is needed and is removed when no longer needed”, the so-called fading out (Lajoie 2005, p. 542).

Hannafin et al. (1999) distinguish four types of scaffolds for different learning contents: conceptual, procedural, metacognitive, and strategic scaffolding. For our learning content, strategies for comprehending algebraic word problems, we use strategic scaffolding, being defined as “guides in analyzing and approaching learning tasks or problems” by focusing “on approaches for identifying and selecting needed information” (Hannafin et al. 1999, p. 131, 133). The aim of strategic scaffolds is to “trigger a series of related strategies” (ibid, p. 134) and was first suggested by Aebli et al. (1986) for comprehension strategies for word problems.

As Collins, Brown, & Newman’s (1989) explain within their model of cognitive apprenticeship, teachers can introduce new strategies through modeling their use by thinking aloud. This allows the students to grasp the idea and purpose of the strategy. “The apprentice […] observes the master executing (or modeling) the target process […]. The apprentice then attempts to execute the process with […] scaffolding” (ibid, p. 456). Hence the authors follow an approach of “learning through-guided experience” (p. 457) and emphasize the importance of making explicit the usually implicit internal mental processes.

In spite of a huge variety of scaffolding approaches, two quality criterion are always emphasized as crucial: the final aim is fading out (Lajoie 2005), and supporting the students to reach the individual zone of proximal development is only possible with a high degree of adaptivity of the scaffolding (Renninger & List 2012). This raises issues of timing as well as of interindividual differences that challenged our design.

3.2 Existing scaffolding tools for cracking word problems

Several instructional approaches exist that intend to offer temporary guidance for students’ comprehension processes, even if not named strategic scaffolding by their designers. We briefly present three paradigmatic examples with respect to their fit to scaffolding multi-step algebraic word problems.

In many language-sensitive reform classrooms, general reading strategies are supported by scaffolds like “ask questions to the text”. These scaffolds can only partially support students’ comprehension process because they focus only on strategy S1 (as resumed in Section 2.4). If combined with prompts to draw connections between pieces of information, it can also address S2 and/or S3.

Many scaffolding tools in mathematics education research go back to Pólya’s (1945) early models for problems solving: 1. Understanding the problem, 2. Devising a plan, 3. Carrying out the plan, and 4. Looking back. For example, Mevarech et al. (2010) report on significant effects of their training program IMPROVE which offers metacognitive and strategic scaffolding through metacognitive questions and two heuristic strategies: finding analogue word problem structures (called connection) and drawing the relevant relations on an empty number line. IMPROVE addresses strategies S1, S2, and S3 for a specific type of
word problems (additive / subtractive single-step problems). However, multi-step problems and other structures are not covered.

Blum’s solution plan (cf. Fig. 4) reshapes Pólya’s (1945) early model in a simplified modeling process model (cf. Section 2.3), without explicit support for comprehending in the first stage. As the model is presented as cyclic, it might be used for multi-step problems, but does not explicitly address the process of decomposing into several steps (S4 and S5).

A scaffolding tool that seems to be well aligned to the focus of our paper, namely students’ mental processes while comprehending multi-step word problems, is Reusser’s (1994) computer assisted tool HERON (Fig. 5) which was designed as a didactical consequence of a thorough cognitive process analysis. It draws upon Aebl’s solution tree as a representation for decomposing multiple steps. Additionally, a focus on relations and meanings is set by asking not only for the numbers, but also for the quantities (e.g. length of scarf to be knitted) and measurement units (e.g. rows). Hence, this scaffolding tool has the potential to support the mentioned strategies S1-S5, especially with a focus on quantitative reasoning. But the tool has a quite narrow format that attempts to bring students onto the intended pathways rather than leaving space for students’ individual pathways.

Although for each of these models, there exist some quantitative results on effects of its use, the adaptivity to different pathways is worth to be considered in more empirical detail.

3.3 Requirements for a design research study

The literature review shows the necessity to conduct a design research study with the aim of deeper understanding of low achieving students’ successful and non-successful strategies during their comprehension processes, for the specific case of multi-step algebraic word problems asking for general expressions. Deeper empirical insights allow consolidating the specification of the learning content, namely appropriate target strategies for the specific problem type.

Furthermore, investigating the learning processes being shaped by scaffolding and fading out allows refining the instructional trajectories. Both aspects are subsumed in the following guiding questions:

(Q1) How are students’ comprehension processes shaped by individual and target strategies? And in which combination do the strategies support a successful comprehension of the multi-step algebraic word problem?

(Q2) How can the use of target strategies be supported by the scaffolding tool?

(Q3) How could the scaffolding tool take into account interindividual differences between students?
The last question was emphasized by Renninger & List (2012), assuming that adaptivity of scaffolding must leave space for different pathways.

4. Methodological framework

The design and investigation of a strategic scaffolding tool for comprehending multi-step algebraic word problems was conducted within the methodological frame of Topic-Specific Didactical Design Research as it also allows to specify the learning content more thoroughly.

4.1 Topic-Specific Didactical Design Research as methodological frame

Like other design research approaches, our framework of Topic-Specific Didactical Design Research (Prediger et al. 2012; Prediger & Zwetzschler 2013; following main ideas of Gravemeijer & Cobb 2006) relies on the iterative interplay between designing teaching-learning arrangements, conducting design experiments, and empirically analyzing the processes. Its four working areas are shown in Fig. 6 and exemplified in Section 5 and 6: Specifying and structuring the learning content, developing the design, conducting and analyzing the design experiments, and developing local theories on teaching and learning processes. As the framework is content-focused on topic-specific aspects, the specification and structuring of learning goals and contents are treated as one of four intertwined working areas.

Expected design outcomes comprise the specified and structured mathematical content (here target comprehension strategies), the refined design principles (here for strategic scaffolding) and the prototypic teaching-learning arrangement (the second part of our basic algebra course).

The research outcomes consist of empirical insights and contributions to local theories on learning and teaching processes of the treated topic (here the necessity of adaptive strategic scaffolding).

4.2 Design experiments as method for data collection

The design research project presented here deals with a teaching-learning arrangement designed for low-achieving eighth graders with limited German academic language proficiency. The scaffolding tool is established in a (partly remediating) basic algebra course (comprising 10 sessions of 45 minutes each) on conceptual understanding of variables and dealing with algebraic expressions.

In the overarching project (Krägeloh in prep.), we conducted six design experiment cycles with a total of 70 students in laboratory settings for covering a large variety of different individual learning trajectories. The comprehension strategies were treated in the last four cycles with design experiments series in laboratory settings with 20 pairs of students (i.e. n=40 students for this article), chosen according to low language proficiency and weak comprehension strategies. In these cycles, about 160 x 45 minutes were video-recorded and partly transcribed. Additionally, practicability of the learning arrangement for whole class
teaching was tested in two classes, from which mainly teachers’ reports and written protocols were analyzed.

The case study of this article focuses on data of the first time that cracking word problems was treated (cycle 3), and two later cycles (cycle 5 and 6), each with design experiment series of 10 sessions. Three pairs of focus students (14-16 years old) were selected from each cycle for a deep analysis of transcripts. This focus data was embedded in the complete video data of n=40 students and the written protocols from two further classes.

4.3 Methods for data analysis

For reconstructing students’ comprehension processes and their successive development, selected transcripts and students’ written products were qualitatively analyzed in depth.

• In Step I, a sequential analysis was carried out by systematic extensive interpretation (Beck & Maier 1994) for reconstructing the individual and interactional comprehension processes and the individual steps of making sense of information and relations in the text for the six focus students.

• In Step II, two qualitative coding procedures were conducted for the video data of 40 students on two tasks with respect to (a) the steps in Reusser’s process model, and (b) the obstacle the students meet (first column of Table 1).

• In Step III, systematic intra- and interindividual comparisons of successful and non-successful moments were conducted for the six focus students to reconstruct (c) the individual strategies for overcoming the obstacles (second column in Table 1, individual strategies were operationalized as a category beyond repeated similar ways of individual processing). And (d), we deduced target strategies by which the reconstructed obstacles could be overcome (third column) and searched for evidence in the video data of all 40 students how the second and third column are connected in the comprehension processes.

• Step IV considered the focus students’ processes in relation to the offered scaffolds and thereby focused on the relation between the third and fourth column of Table 1. The method of systematically contrasting selected moments of different processes allowed reconstructing typical pathways, obstacles, conditions, and means for the teaching-learning process which were then again compared to those of the 34 non-focus students.

5. Design outcome: The word problem cracker and its underlying target strategies

5.1 Learning content and scaffolding tool

The developed scaffolding tool had one main constraint: it should work for students working alone or in pairs with paper and pencil, i.e. without computer and without permanent one-to-one support by the tutor.
Table 1. Obstacles and underlying individual strategies, target strategies and design elements

<table>
<thead>
<tr>
<th>Typical obstacle in situation comprehension and mathematization</th>
<th>Possible underlying individual strategy or absent strategy</th>
<th>Target strategy for overcoming the obstacle</th>
<th>Element of the WPC for scaffolding the intended strategy use</th>
</tr>
</thead>
<tbody>
<tr>
<td>wrong situation or problem model due to superficial comprehension</td>
<td>• select some numbers and immediately start calculation</td>
<td>(S1) reading strategy: find relevant information</td>
<td>left column: identify information = numbers + meaning</td>
</tr>
<tr>
<td></td>
<td>• focus only on key words</td>
<td>(S2) focus on information with their meaning</td>
<td>middle and right column: write relations with meaning</td>
</tr>
<tr>
<td></td>
<td>• take operation currently taught</td>
<td></td>
<td></td>
</tr>
<tr>
<td>wrong situation model or problem model due to superficial comprehension</td>
<td></td>
<td>(S3) focus on relations connecting pieces of information</td>
<td></td>
</tr>
<tr>
<td>multiple steps in solution process not accomplished, e.g.</td>
<td>no planful sequencing, e.g.</td>
<td>(S4) decompose in several steps and work forward</td>
<td>row structure of the word problem cracker WPC: decompose into steps</td>
</tr>
<tr>
<td>• stopping after one step</td>
<td>• use numbers for operations according to their order in text</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• wrong order of numbers and operations</td>
<td>• start anywhere and rather than planning the process in advance</td>
<td></td>
<td></td>
</tr>
<tr>
<td>combining any information in multi-step word problem</td>
<td>• start to work forward without considering the goal</td>
<td>(S5) work forward and backward for decomposing in several steps towards a goal</td>
<td>row structure, but final question in the beginning</td>
</tr>
</tbody>
</table>

After four design experiment cycles, the complex scaffolding tool called ‘word problem cracker’ (WPC) received the form printed in Fig. 7. For explaining the different scaffolding elements, Table 1 gives an overview on six target strategies identified in Section 2.4 (third column), together with the obstacles and underlying individual strategies (first and second column).

The last column lists the scaffolding elements designed for supporting students’ strategy use. Their specific form will be explained in the empirical Section 6.

**Fig. 7** Word problem cracker with fictitious solution in its final form (Krägeloh in prep.)
5.2 Instructional trajectory for introducing and fading out the scaffolding tool

The remediating basic algebra course starts with developing a conceptual understanding of the variable as generalizer for figural pattern as sketched in Section 2.2. The second part of the course (8 units of 45 min. each) focuses on finding algebraic expressions for multi-step word problems, a new learning content for the students.

The word problem cracker (WPC) is introduced after having initially worked on three word problems without scaffolding but with generalizing in tables for connecting the reasoning in quantities to the generalization experiences. According to principles of cognitive apprenticeship (Collins et al. 1989), the tutor models the target strategy use in the WPC, then the students use it first with tutor’s help, then without. After executing the process with the scaffolding tool for some word problems without and with variables, the students successively fade out the WPC in their individual pace.

6. Snapshots from the design research process

Three episodes were chosen with respect to the research questions and the course of the design experiments: Episode 1 shows individual strategies as used to handle typical obstacles and why the target strategies are necessary for overcoming them. Episode 2 from an intermediate design experiment cycle illustrates how the scaffolding tool had to be widened to the variety of individual realizations of strategies. Episode 3 finally exemplifies how the final scaffolding tool could foster some students’ comprehension processes.

6.1 First Design Experiment Cycle: Reconstructing obstacles in students’ comprehension process

Episode 1: Meral und Adile

Meral and Adile are two 14 and 15 year old girls who had acquired conceptual understanding of variables as generalizers and learnt to interpret parentheses in expressions. Episode 1 shows how they are working on the problem from Fig. 1 by identifying the meaning of the variable and needing to decompose the problem model into two steps. For fostering their communication and flexible modification of structures, the structure of the expression and its elements 12, 7 and x are given on cards on the table.

Meral (M) first writes \(7 \cdot (x - 12)\), then corrects it to \(7 \cdot (12 - x)\). She tries to explain her second expression (later scratched out in the scan) for the tutor (T):

1 2 3

Meral

Adile

M
Ehm, I have 7 times 12 minus x?

T
Ok, why?

M
Ehm, because here, we have… Ehm… That is, no. If – no. For good [weather] it is open seven days a week.

[5 sec break] well how shall I explain that? [16 sec break]
[to Adile] You continue, I will read again.

A
Ehm. Well, for good weather, seven days a week are open. Then I have written 7 here. And ehm… ehm, well the pump for the water runs on open days for 12 hours. Hence 7 times 12.

Meral’s expression [2] is ordered according to the order of numbers in the text, so she seems to implicitly apply this compensation strategy without building a complete situation or problem model. Although trying hard, her explanation stagnates after reciting the first sentence about 7 days from the text. The relevant numbers were given in the text (target strategy S1), but so far, she neither focuses on the meaning of the underlying quantity (S2) nor on their relation (S3). She then reads the text again to find out more (#9).
Adile’s (A) explanation for her expression $7 \cdot (12 - x)$ in #10 shows that she has built a partial situation model without proceeding to the problem model. She makes successfully sense of the meaning of 7 and 12, but does not refer to the meaning of $x$ (partial use of S2) nor to the question of the task that includes the rainy days. The tutor therefore asks her for the $x$:

13 T Why minus $x$, then?
14 A Well, you have to with $x$ ...

Hence, Adile’s way of dealing with $x$ is not guided by a focus on meaning (S2), but only by the fact that $x$ has to be included anyway. Meral again modifies her expression, to expression [3], namely $7 \cdot x - 12$. She explains:

21 M Because, there we have. Eh, for good weather, it is open 7 days, and for rainy days it stays closed. That is why I have 7 times ehm. … No…. Yes. 7 times 12 and then minus $x$.

In her explanation, she only attributes a meaning to 7 whereas for 12 and $x$, no meaning is mentioned. In the following, the tutor prompts her to continue the explanation and she focuses on the meaning of 12:

25 M Eh, Well. 7 days it is open. For good weather.
26 T Mhm. For good weather.
27 M Yes and for rainy days it is closed – and …. ehm … yes the circulation pump or so
28 T Circulation pump, yes.
29 M It is 12 hours, though.
30 T Mhm.
31 M Thus, it runs 12 hours long. I don’t know, I have it anyway.

Although the prompts helped her to focus on the meaning of the information 7 and 12 (target strategy S2), the girls still do not focus on the relation between these quantities for making sense of the multiplication and subtraction (S3). Thus, the tutor wants to prompt this focus:

32 T Mhm [nodding]. And what do you calculate by the expression?
33 A … What?
34 T Well, what do you calculate when you write 7 times $x$ minus 12 or 7 times 12 minus $x$?
35 A 7 days. Though…. and here it will be 12 hours, won’t it?

As the girls still do not refer to the relation between the quantities until line 35, the tutor pushes them to agree between their two different expressions in the non-printed following lines. But the girls cannot overcome the obstacle of superficial modeling and finally agree on Meral’s expression $7 \cdot x - 12$ without referring to the meaning of the quantitative operation and relations given in the text.

This Episode 1 exemplifies student’s individual ways in the comprehension process as reconstructed for many of the 40 analyzed cases. Comparing different cases from the first design experiment cycle, we could identify the following individual compensation strategies as repeatedly appearing pattern:

- select some numbers and immediately start calculation;
- take operation as given (here in the pre-given expression structure);
- use operations and numbers according to their order in the text;
- start to work forward without considering the goal.

Being supported by the tutor, Meral and Adile can apply strategies S1 and S2 (find relevant information and focus on their meaning), but do not yet get access to a focus on relations between the quantities (S3) or to decompose the problem into two steps (S4) and to think from the final question (S5).
6.2 Next design experiment cycle: Reconstructing potentials and interindividual differences

Based on these observations on students’ (lacking or misleading) strategies, we developed a scaffolding tool not only for strategy 1S, but also for the S2, S3, S4 and S5. In its preliminary form (shown in Fig. 8), working forward and backward (S4 and S5) is supported by the first boxes, the decomposition into steps by the rows in the main part (S4, here not used by Nathalie).

The structure in columns supports the focus on the quantities and their relations (S2 and S3) by collecting the information together with its meaning in the left column and the relation between both in the middle column. Four formats are suggested in the literature for focusing on relations (S3):

- demand graphical representations, which mainly work for some students or a selected number of operations (e.g. when restricting to addition/subtraction like in Cai et al. 2011).
- formulate questions for the intermediate steps, e.g. “How many open days does the week have?” for Fig. 1 (Aebli et al. 1986). This is realized in the preliminary word problem cracker in Fig. 8. However, the analysis of students’ processes obliged us to open the format also to other individual preferences:
- name the numbers together with the involved quantities, e.g. “length of scarf” as in Fig. 5 (Reusser 1994, Thompson 2011)
- simply give the measurement units, e.g. days versus hours.

In the preliminary version in Fig. 8, the format of the middle column was fixed to intermediate questions. From the empirical reconstruction of the variety of students’ choices of formats, we became aware of the necessity of openness for different formats for guaranteeing adaptivity for individual ways of expressing the quantities. This insight is exemplified by Episode 2.

**Episode 2: Viviane and Nathalie – measurement units for focus on relations**

Viviane and Nathalie, the 14 and 15 year old girls, work on the word problem from Fig. 1 with the preliminary WPC in Fig. 8.

The girls do not use the row structure for decomposing the problem in several steps. Nathalie immediately takes two steps and writes the expression $7 \cdot (12 - x)$.

Based on a suitable situation and problem model (visible by the correct entry of the final question in Fig. 8 and her correct rewording of the situation), she starts explaining her expression during the writing. She adopts the order of reading from left to right to the expression, first without considering the parentheses:

1  
   N  Ok, 7 stands for 7 days in a week that it is usually open when the weather is good. Normally, when good weather by … ok……. Then … times parentheses opens …. no because of parentheses ….

The utterance “7 stands for” appears in her working on the interplay between left and right column (strategies S1 and S2), whereas she neglects the middle column in this moment. She writes the parentheses and corrects herself:

3  
   N  So, then in parentheses, there is then the 12 because of the pumps. Thus, these circulation pumps there, for the water, they ran at … at the open days they run for 12 hours.

…

5  
   N  Ok, then you have to minus x, well minus the days with rain always. …
calculate minus because then the pump is out of work and the bath is closed. Stop, stop, [changes the expression to \(12 \cdot (7 - x)\)] ok… yes. That’s how it is because we … because if … if yes, that is correct, that is correct. Because, look, if the 7 days, those on which it is usually open for good weather.

But now, when you calculate these minus the days then you have to calculate 7 minus x. Because, we do not know how many rainy days, that is the x, isn’t it? Because of the rainy days.

Although not referring to intermediate questions and explicitly written intermediate single-step expressions, Nathalie is able to correct her own expression when realizing that 7 and x have the same measurement units and thus have to be subtracted from each other. This individual approach ‘search for common measurement units’ substitutes the intermediate questions for her focus on relations of quantities (S3) and naturally decomposes the steps (S4). She continues:

13  N  Because these now, the days. These are the days that we have calculated then. After that, the result of the days times 12, then we have the hours how long the pumps, then, in the days, when the bath is open.

15  V  Yes, because … you do not know … well that is seven – seven days in a week. The bath is open and you do not know … ehm… when it rains, that’s why minus x. And the pump, it runs then though, always on the open days. That’s why times 12, yes.

Whereas Nathalie still only refers to the measurement units, Viviane explains by the involved quantities and their meanings. Although the intermediate question is still implicit, both girls together construct the meaning of the expressions \(7 - x\) and \((7 - x) \cdot 12\). When later asked why her initial expression \(7 \cdot (12 - x)\) was not correct, Nathalie explains that hours minus days wouldn’t make sense.

The short insight into Episode 2 exemplifies student’s individual pathways towards comprehension reconstructed for many students working with the preliminary word problem cracker. Comparing different cases in this design experiment cycle, we can identify the following strategies as repeatedly appearing pattern:

- focus on information and their meaning (S1 and S2), which is addressed by the initial boxes and the left column of the WPC;
- focus on relations between quantities (S3), but not necessarily by formulating intermediate question in the middle column. Although some students followed the scaffolding in the intended way, others, like Nathalie and Viviane, chose other formats for the relations (only very few chose graphical representations). We therefore decided to open the format in which the middle column could be used;
- like Viviane and Nathalie, many students in this cycle did not use the row structure for a stepwise construction of the expression (intended strategy S4), but chose to write a complete (possibly wrong) multi-step expression immediately. Other students didn’t write anything before they were sure. In order to lower these inhibitions, we invented the trial and error space. It supports the strategy S6 to simply start, but also scaffolds strategy S5, working forward and backward, for those who do not see the complete decomposition in steps from the beginning.

6.3 Final design experiment cycle: The word problem cracker at work

The adjustments of the scaffolding tool led to its final form as printed in Fig. 7 and Fig. 9. The last empirical snapshot, Episode 3, shows this scaffolding tool at work. Rather than presenting a pure success story, the episode was chosen to show that students always find their individual way of using it.
**Episode 3: Benjamin and Konstantin**

Benjamin and Konstantin, the 15 and 16 year old boys, work on a four-step word problem with high textual complexity (cf. Fig. 9).

Reading the problem, the boys immediately start to fill the boxes:

7  K  x stands for the visitors, guests
8  B  Yes [writes]
9  K  And the question we shall answer is the costs.

Hence, the tool scaffolds the focus on the meaning of the variable (S2) and the boys’ mental construction of quantities as well as the final questions as part of working forward and backward (S5).

Then, the boys fill the left column by combining two pieces of information:

11  K  Six euros [writes] times the number of persons, but this is x, isn’t it?
12  B  Yes, times, though
13  K  Six
14  B  Six euros times persons, ok. What do I get then? …. the costs of the pizza

These lines show the boys’ focus on meanings and relations of quantities (strategies S1, S2, and S3), especially by the question in #14. In the same way, they proceed with the subexpression 4 · x and their sum 6 · x + 4 · x (Fig. 9).

The trial and error space allows the students to immediately start and work forward consequently (S4 and S6). In their further pathway, they often ask, “What do we get? What does it mean?” with a consequent focus on quantities (until non-printed #82; result in Fig. 9). Interestingly, they switch between the order of columns quite flexibly, always starting with the easiest aspect, for example in #106 were Konstantin switches from the right column to the left to the middle column:

106  K  This would be 6 times … well, this here and this here [hints to costs for meals and drinks and bowling]. There we must first … mmhm … now we have to fill yellow and blue [i.e. the left and middle column] before writing the expression.

As the brief snapshots show, the WPC scaffolds Benjamin’s and Konstantin’s comprehension process with its row structure (decomposing, S4 and S5) and its column structure (focus on quantities and their relations, S2 and S3).

Untypically, they do not use the trial and error space, whereas other students tentatively write subexpressions in it. This can be traced back to their strong control by strategy S3. In later phases of the scaffolded learning process, this search is mostly internalized and the trial and error space rarely used in written form. It is typical that the boys do not repeat the steps made in the trial and error space if they are successful, but continue their way forward.

In sum, the strategic scaffold allows the formerly low achieving students to touch all relevant phases in the comprehension process and to mathematize by a complex expression.
6.4 Fading out after some experience

It takes a while for the students to get used to the word problem cracker with its complex structure and many strategies. That is why it is even more important to make sure that the learning process also comprises the fading out: Once the strategies are internalized, the scaffolding is reduced and processes can be conducted internally. This applies for the oral use of the trial and error space, but also the need to decompose in many steps: A case study (master thesis Schmidt & Kinkel 2014) showed how students reduce the complexity of the row structure in the WPC after some experience. They succeeded in composing more steps together at a time (cf. Fig. 10).

Also the left and middle column was not used anymore after some experience, many of the 40 students could then internalize the process of making sense of the quantities and relations. These learning pathways show how the scaffolding tool helps to become unnecessary.

![Row structure for one of the first word problems](image1)

![Row structure for one of the last word problems](image2)

Fig. 10 Change of row structure during a learning process – more steps at a time after some experience

7. Discussion and Outlook

“If you want truly to understand something, try to change it.” This aphorism attributed to Kurt Lewin (e.g. by Stam 1995, p. 31) applies to the didactical design research with its emphasis on specifying the learning content conducted here: only by trying to change students’ ways of comprehending word problems, we were able to identify the individual and the target strategies as the main contents to be considered. In this way, the project contributed to iteratively specifying learning contents, here required target comprehension strategies. The presented study exemplifies potential outcomes and typical limitations of many topic-specific design research studies.

7.1 Discussion of outcomes

The design research study on fostering low achieving eighth graders’ comprehension processes for multistep algebraic word problems by strategic scaffolding provides different research and design outcomes:

**Outcome 1.** For the local instruction theory of the learning content, Table 1 is a major outcome. Most of the general research literature on comprehending word problems provides descriptive knowledge by identifying typical obstacles (first column) or explanatory knowledge by reconstructing the individual strategies underlying the obstacles (second column). However, for fostering students, these individual strategies must be contrasted with successful target strategies (third column). Whereas most empirical studies on comprehending word problems have focused on unsuccessful strategies, this design research study integrates outcomes from the research on algebra learning with an emphasis on quantitative reasoning. The methodolog-
cal design allowed to move a step forward in integrating this two branches of literature and finding hints for successful strategies. The category of strategy (which is still rarely used in the discourse on word problems) seems to be promising for specifying and structuring the learning content, especially for students with low language proficiency. In this way, a substantial normative contribution to the local instruction theory could be made.

**Outcome 2.** Based on this specification and structuring of the learning content, a scaffolding tool was designed and iteratively refined for strategically scaffolding students’ use of target strategies. The empirical snapshots showed how aligning the scaffolding tool to the students’ processes depends on in-depth analysis of students’ processes. Some adjustments have been made from Episode 1 to 2, others from Episode 2 to 3. On the more general level, the analysis shows that a complex interplay of different strategies is necessary for a successful comprehension process, not a single strategy as often focused in other intervention programs or the algebra literature. The final scaffolding tool “word problem cracker” and the teaching learning arrangement (the instructional trajectory) in which it is introduced and later successively faded out is a major practical design outcome with a high relevance for classrooms.

**Outcome 3.** Beyond this practical design outcome, the project provided more generalizable insights into conditions and means for successful strategic scaffolding tools that have the potential to be transferred to other types of word problems and other types of strategic scaffolding:

- only an interplay of different strategies can enhance the complexity of comprehension processes
- adaptivity to students’ individual ways of thinking and pathways to comprehension is crucial, the process model underlying the scaffolding tool must therefore not be too narrow, and the tool itself must be open for different realizations. For the WPC, this referred to the order in which the students work with the columns as well as the format in which the middle column is filled. We deduce a general guideline: a tool for strategic scaffolding must be as narrow as necessary and as open as possible.

**Outcome 4.** Finally, the project offers descriptive insights into students’ learning processes while being fostered by strategic scaffolding (only briefly sketched here, cf. Krägeloh in prep. for more details). The introduction of a scaffolding tool requires not only the initial modeling, but also a persistent teacher guidance before students get acquainted with the tool. After this warming up phase, the tool scaffolds students’ processes successfully, even if used orally instead of written. After a while, students internalize the processes and strategies and the scaffold can be faded out. This happens at different moments for different elements of the scaffolding tool.

**7.2 Limited studies require continued design research**

For methodological reasons, the contributions to the instruction theory provided by design research studies like the presented are only local in two ways: local as intimately related to the specific teaching learning arrangement (here also the specific type of word problems), and local as only generated in case studies with restricted number of participants.

Although 40 students are already many for a detailed process analysis, the number is still too low for generalizing statistically. Like for other design research studies, an important continued research could thus extend the sample for the experiment and enrich the research by quantitative measures of learning progress. However, the presented study shows a typical phenomenon: such a quasi-experimental intervention study wouldn’t have been adequate before this in-depth analysis, which substantially shaped the content to be learnt (and hence every pre-post-test developed to measure the progress).

A methodological and theoretical challenge for each design research project is to provide germs of generalizable theoretical contributions. However, the generalization always requires further design studies on
the possibility of transferring insights from the specific local context to other students, conditions, and topics, in our specific case for other types of word problems. A next design research project can build upon the results, transfer them, e.g. to another type of word problem (e.g. involving algebraic equations, arithmetic multi-step problems, open-ended problems etc.) and then investigate whether other aspects are then required (e.g. other target strategies or other scaffolding elements). By cumulating over several transfer projects, contributions to more general theories can grow successively.

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