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Designing Tasks for Engaging Students in Active Knowledge Organization

Bärbel Barzel[1], Timo Leuders[1], Susanne Prediger[2], Stephan Hußmann[2]

[1]*Institute for Mathematics Education (IMBF), University of Education Freiburg*
[2]*Institute for Development and Research in Math Education (IEEM), TU Dortmund University*

Mathematical tasks aim at supporting students to engage in a range of mathematical activities with specific didactical goals. Task design has to take into account the specificity of these different didactical goals (e.g., exploration, concept formation, practising skills). In this study, we focus on tasks intended for the didactical aim of mathematical knowledge organization ("organizing tasks"). In our learning pathways, phases of knowledge organization usually follow a phase of open exploration, of constructing individual concepts; they aim at regularizing and systematizing the students' singular ideas and results. Because such organizing processes conducted in whole-classroom discussion often fall short of engaging every single student, our design research study sets out to develop task formats that promote adequate cognitive activities and formats for this organizing phase. The article describes the efforts in constructing and evaluating organizing tasks and presents – as a result of our study – a conceptual framework for the delicate balance between individual engagement and convergence.

Keywords: organizing tasks, organizing knowledge, regularizing and systematizing

The challenge of organizing knowledge

The construction of mathematical knowledge is a multistep process of "organizing fields of experience" as Freudenthal pointed out (1973, p. 123). In subsequent work of the realistic math education approach (de Lange, 1996), examples of organizing processes are abundant, comprising "horizontal" and "vertical" mathematization (Treffers, 1987, p. 247). Although these ideas lie at the heart of most contemporary approaches for mathematics education, it is undeniable that in mathematics classrooms such organizing processes can rarely be found explicitly (Hiebert et al., 2003). However, there is a need for development of activities which promote systematizing, regularizing, and preserving the results of exploration with several goals:

- structuring the singular and divergent results and connecting them to other facets of knowledge (*systematizing*),
- transforming results into regular and consolidated mathematics (Brousseau, 1997, calls this phase the "institutionalization", we call it *regularizing*);
- writing down in a form that is accessible later (*preserving*).

When and how do such procedures of organizing knowledge occur? When observing German mathematics classrooms, we often find phases of discovery and individual problem solving. For the subsequent phases of regularizing, we actually encounter two different types of classroom procedures.

In the first type of procedure, the teacher conducts a whole-classroom discussion in a Socratic dialogue, collects and evaluates students' contributions, and leads the class to an organized and structured knowledge. This procedure requires whole classroom conversation techniques and proves precarious with respect to the cognitive activation of every single student. In the second type of procedure, the teacher may avoid whole-classroom discussion and instead refer students to the information boxes in the textbook where the correct mathematical concept or result is stated. This ensures a common basis, but there is a danger that the individually constructed knowledge cannot be integrated in this "ready-made-mathematics".

What are alternative options for supporting students in organizing their knowledge, in mastering the step from the singular and individual knowledge to the regular and commonly accepted mathematical knowledge while preserving students' engagement? In our design research study, we developed approaches and tasks on the basis of three premises: (1) Teachers predominantly work with textbooks, so the organizing tasks should be embedded into a comprehensive textbook curriculum; (2) Students must be actively involved in the learning processes; (3) Teachers are not supposed to give up their role in moderating the process of organizing with the whole class. We consider the communication processes within the class as extremely important for attaining a high level of mathematical insight.

Hence, our goals are to construct tasks that support students in actively organizing their knowledge and simultaneously support teachers in guiding this process in an effective manner. We call this task type "organizing task" or, when we need to avoid the misunderstanding of organizing as a purely external, administrative activity, as "knowledge organizing tasks". The guiding questions for our study are:

Q1. Specification of learning goals:
What elements of knowledge have to be organized and preserved?

Q2. Types of Tasks:
Which types of tasks can support students' active knowledge organization?

Q3. Principles for the Task Design:
Which principles guide the construction of organizing tasks?

The framework: design research for a middle school curriculum

The study on organizing tasks is embedded in the long term design research project KOSIMA (2006-2016, cf. Hußmann et al., 2011). It is briefly presented here with its methodological framework and the conceptual framework for the design products.

Methodological framework of the long-term design research project

The project KOSIMA (Hußmann et al., 2011) follows the methodology of Didactical Design Research (Gravemeijer & Cobb, 2006; McKenney & Reeves, 2012) with its dual aim of *designing teaching-learning-arrangements* for a complete middle school curriculum (grades 5 to 10 of German Realschule, Gesamtschule, Sekundarschule) and *empirically researching* the teaching-learning-processes and their conditions. The developed curriculum is published as the textbook *Mathewerkstatt* from 2012 to 2017 (Barzel et al., 2012 ff.) and a comprehensive teachers' manual.

All teaching-learning-arrangements of the textbook-to-be are developed in iterative cycles of design, evaluation (by expert discussions and classroom experiments), and redesign. Whereas the design and evaluation steps of the project refer to the entire implementation of the textbook, the deeper research is organized in several smaller design research studies that necessarily have to address more narrow research questions. These studies use different concrete research methods and designs (e.g., intervention studies in quasi-experimental designs, design experiments in laboratory settings with up to four cycles, e.g., Leuders & Philipp, 2012; Prediger & Schnell, 2013). An overall evaluation of summative effectiveness commenced in August, 2012. Results of the quasi-experimental intervention with pre-post-test over two years can be expected in 2014.

The community involved in these processes is a large group of people from different backgrounds who collaborate fruitfully:

- researchers (the four authors of this paper, being the editors of the textbook and leaders of the design research project, supported by many PhD students and student researchers)
- authors of the teaching-learning-arrangements (about 20 experienced reflective practitioners, together with the editors)
- the publisher (with 2-4 copy editors who finalize the design products)
- project teachers (about 10 teachers with their classes, who teach with the curriculum and the textbook material continually in their regular classes).

Conceptual framework for the design product

The design of the middle school curriculum is guided by certain design principles. We only state those which are relevant for the focus of this paper, that is for designing organizing tasks (cf. Hußmann et al., 2011; Prediger et al., 2011). Following socio-constructivist theories of learning, we emphasize the importance of students' active engagement and sense-making by starting from meaningful context problems, and developing conceptual understanding (Leuders et al., 2012, following Wagenschein, 1977 and Freudenthal, 1973). For realizing these principles, every teaching-learning-arrangement (each for 2-6 sessions) is structured into four main phases: activation, exploration, organization of knowledge, and practice.

Activation of students' experiences. For including students' pre-instructional experiences into the learning pathways, every arrangement is situated in an everyday context that allows problems for ready-made mathematics to be reinvented and students to construct meanings for the intended mathematical topics.

The following is an example referred to throughout this article. "Constructing Packages" provides a context for students to think about solid figures and their characteristics, such as parallel and perpendicular lines. To solve problems related to this topic, students put themselves in the role of a package designer who has to create new packaging for a toy. In considering a good design for a package, a designer has to think about what criteria are relevant to create a good package-box. Apart from price, other possible criteria are look, stackability, and ease of construction. Stackability specifically leads to the necessity of having packages with parallel and perpendicular lines.

Exploration. In this extensive phase, the problems are operationalized into open and rich exploration tasks (Flewelling & William, 2001; Freudenthal, 1973). They allow students to actively and collaboratively re-invent ideas, concepts, procedures and relations in the sense of horizontal mathematization. Due to the openness and student-centricity of this phase, it often results in a large diversity of individual ideas, strategies, solutions, findings, pre-concepts, etc. The concrete design principles for exploration tasks that were developed or refined during the design research project are not reported here. Looking at our example in this phase: Students assume the role of the package-designer and they actually construct a package-box for a specific item, e.g. a toy. During this process of construction, many students realize that the “box must be straight and precise, not awry or askew”. These experiences prepare the systemization of the concept of *perpendicular and parallel* in the next phase.

Organization of knowledge. The goal of the subsequent phase of organization (in German “ordnen” which also means “ordering”, “arranging”) is to establish a shared understanding of the core concepts, theorems and procedures and to preserve and document this understanding in the self-written “knowledge-storage” (in German “Wissensspeicher”).

The packaging example highlights the kinds of activities required in this phase. Exploration often leads to a great diversity – in our example a lot of different boxes, ideas, images, and students’ thoughts. All of these products and ideas have to be shared and compared in order to systematize the new knowledge. To stimulate students´ mental processes, we created focused cognitive activities (called *acquisition activities*) according to the new knowledge.

The type of acquisition activity depends on the types of knowledge. For the concept of perpendicular and parallel, one can differentiate between the following aspects, which have to be learned:

- Students must learn the technical terms “perpendicular” and “parallel”. These words are names and just conventions, which cannot be discovered or explored. Students have to be informed about these new words. This has to be done in a language, which can be easily understood by students and with words which link to the experience of the exploration phase (see the first lines of the task in Fig 2).
- Another convention in our example is the marking of a right angle with a dot (a convention that differs throughout several countries). This is information that students must learn (see picture on the left side in Fig. 2).
- Students have to learn to recognize perpendicular and parallel lines. Task 2b (in Fig. 2) involves concretisation and distinction of the new concept. Pupils have to recognize which pairs of lines show parallel or perpendicular ones.
- The extent to which individual students can verbalise the new concept varies and can be described by the extrema: On the one side students formulate a definition by themselves, a very ambitious task; on the other side students have to copy a given definition (see Fig 3). Choosing a correct definition from several given ones is an activity in between these extrema, and this is demonstrated in our example. (See (2b) in Fig 2.)

When inspecting regular lessons and textbooks, we have only rarely encountered tasks that were constructed for supporting such organizing processes (e.g., Swan, 2005). That is why we had to conduct a design research study for specifying principles for a systematic construction and composition of organizing tasks. The results are reported in the section on Findings.

Practice. In the fourth phase of practice (in German “vertiefen” = deepen, intensify), the students are supposed to render their knowledge and skills more stable and flexible by repeated practice and transfer. Our design principles for these tasks refer to didactical approaches of productive exercises, structured tasks and reflective practising (Büchter & Leuders, 2005; Watson & Mason, 1998; Winter, 1984; Wittmann & Müller, 1990).

It is important to note that the construction of “organizing tasks” as a “carrier of the organizing” phase has only become indispensable within our didactical approach that distinguishes these phases explicitly to give students space for more individual mathematical activities. Within a more integrated approach, concept exploring and organizing activities could be combined or integrated more flexibly.

Methods of the design research study on organizing tasks

The construction of "organizing tasks" and the development of an overarching didactical approach for the principled construction of the tasks are embedded in the larger design research project depicted previously. For the concrete study, the cycles of constructing and evaluating organizing tasks were passed through topic by topic, each in four cycles (mostly including further microcycles).

For each topic, the *first and second cycle* of design, evaluation, and redesign is conducted by expert discussions. After specifying the goals of the learning arrangements, the writing team (authors and one editor) suggests a first draft for the formulation of target knowledge that students are supposed to save in their "knowledge storage". After discussing the selection and priorities by the editor team with respect to didactical-conceptual considerations, the (exploration and) organizing tasks are formulated, discussed and further developed. The details of formulation are edited by experienced copy editors of the publisher who optimize readability and coherence. The *third cycle of evaluation* is conducted in classroom experiments with 3-10 teachers in their regular classrooms. The data base for the investigation consists of teachers' written and oral feedback, scans of students' written texts for tasks, knowledge storages and classroom assessments as well as some videos of classroom interaction and design experiments in laboratory settings on selected tasks. For the current study, the qualitative data analysis is conducted with respect to connections between forms of tasks and a) students' engagement within the processes, b) the convergence of the processes and c) the results of organized knowledge as stored and especially as performed in the assessments. In the *fourth cycle of redesign*, with theoretical feedback authors, editors and copy editors again are involved in finalizing versions for the textbook and the teachers' manual. The final version is ready for widespread implementation and effective everyday use.

During these four-step processes for many different mathematical topics, we iteratively accumulated the reflections and experiences, generalized from the specific topics, and developed a conceptual framework for organizing task design. Whereas the answers for research question Q1 on the relevant kinds of knowledge were mostly generated in the first and second steps of *theoretical and conceptual evaluation*, answers for Q2 and Q3 on effects of forms of tasks mostly rely on the *empirical investigation,* involving deeper insights into the mechanisms of the teaching-learning processes as well as practical experiences on robustness for different classroom conditions. Out of the whole conceptual framework for designing organizing task, we present two major aspects in the next section.

Findings

Specification of learning goals: modes and facets of knowledge

The didactical base of a systematic task design is the exact specification of the intended learning goal, here concretely the mode of knowledge that is supposed to be systematized, regularized, and preserved. For concretizing this specification process for each mathematical topic, we developed a conceptual framework of different knowledge elements as printed in Figure 1 (Prediger et al., 2011).

The horizontal dimension follows the classical distinction (Hiebert & Carpenter, 1992) among knowledge about facts, concepts (e.g., numbers, operations, relations), and connections as codified in theorems (*conceptual knowledge*) on the one hand, and the knowledge about mathematical and technical procedures (*procedural knowledge*) on the other hand. We added *metacognitive knowledge* which includes problem solving strategies or steps in modelling processes.

	Which part? (Facet of Knowledge)				
What? (Modes of Knowledge)		**Explicit Verbalisation**	**Concretisation & Distinction**	**Meaning & Connections**	**Conventions**
	Conceptual Knowledge				
	Concepts	definitions	examples/ counterexamples	mental models / representations	technical terms
	Connections	theorems	examples/ counterexamples	(visualized) explanation/ proof	names of theorems, conventional rules
	Procedural Knowledge				
	Mathematical procedures, algorithms	instructions	conditions for applicability, special cases, knowledge about errors	mental models / reasoning as link to conceptual meaning	non-justifiable specifications
	Technical procedures	instructions	realization, conditions		non-justifiable determinations
	Metacognitive Knowledge				

Fig. 1. Conceptual framework for specifying learning goals (Prediger et al., 2011)

During the design research process, we realized the importance of specifying a second (here vertical) dimension that we call *facets of knowledge*: Whereas a piece of knowledge is often only represented by its *explicit verbalisations* (in definitions, theorems or instructions for procedures) or underlying *conventions* (like the technical terms), didactical research has often shown that knowledge acquisition must also comprise *concretization and distinction* (like examples and counterexamples for concepts, cf. Winter, 1984 and Fig. 2 Task 2a) or knowledge of possible errors in procedures (cf. Vollrath, 2010)) and *meanings and connections* to other elements of knowledge, given by visual representations and mental models, explanations and preformal proofs.

As mathematical understanding is conceptualized as individual construction of relations, dependencies, or connections between mathematical ideas, procedures and concepts (Hiebert & Carpenter, 1992), we promote that the organizing tasks and the knowledge storage must always include various facets of knowledge. It is the first step of the construction process of an organizing task to specify which cells of Fig. 1 shall be addressed in the task. These modes and facets of knowledge are then subject for initiating the processes of *systematizing* (= connecting different facets of knowledge systematically), *regularizing* (= transforming individual constructions from the exploration phase into regular mathematical concepts, connections and procedures), and *preserving* (= documenting facets in the knowledge storage so that they can be recalled some months later).

Types of tasks: initiating acquisition activities in a balance between students' engagement and convergence

As made explicit previously, students must be actively involved in the processes of systematizing, regularizing, and preserving knowledge. The *necessity of active engagement* is explainable within the socio-constructivist framework and reconstructable in the empirical investigations, because simple inputs (e.g., teachers dictating the information) were not suitable for activating students' mental processes.

That is why for each piece of knowledge (cells in Fig. 1 selected for a specific topic) that is to be systematized and preserved, a focused cognitive activity (*acquisition activity*) must be initiated that supports students' active acquisition of this piece of knowledge.

Fig. 2 shows some formats for how such activities could be initiated. First the technical terms (here parallel and perpendicular) are given - in the frame and context of the experiences and discoveries which have been done before (here the context of exactly folding straight boxes). In 2a), examples and counterexamples are to be identified. In 2b), students shall choose between possible definitions. Independent personal definitions would produce again very divergent solutions, but finding a correct and fitting one among some examples allows an active engagement with higher convergence.

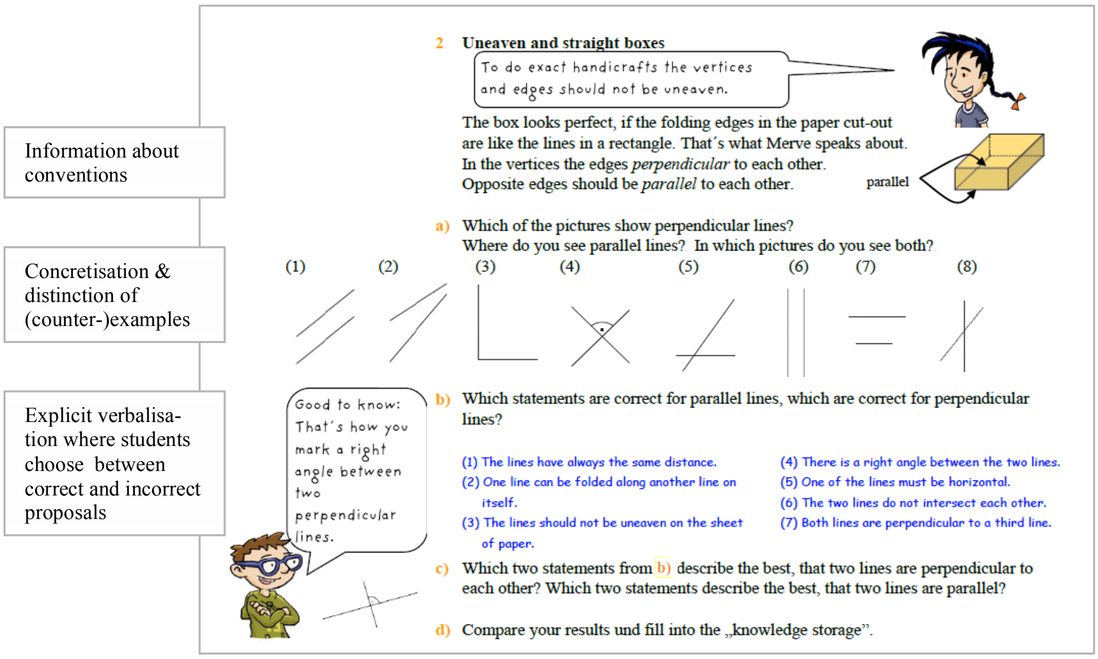

Fig. 2. Counter-/examples in 2a), Finding a correct definition in 2b) (Barzel et al., 2012)

The classroom experiments showed clearly that the balance between students' engagement and convergence of the process is delicate. If the openness of the activity is high, students can intensively engage with the content; like in the exploration tasks, they develop very divergent ideas and entries for the knowledge storage. These divergences produce either the need for the teacher to moderate the processes in funnel-like patterns (Bauersfeld, 1988), or to give individual feedback to each individual attempt to write a knowledge storage, which is too much work for each task.

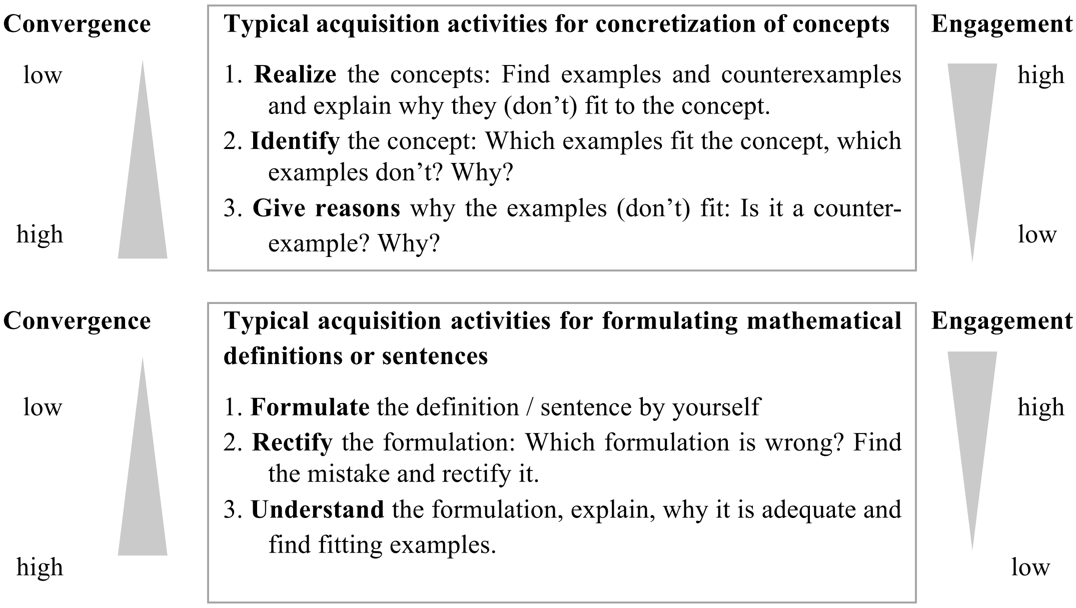

Fig 3. Range of acquisition activities in the balance between convergence and engagement

If tasks are optimized only with respect to convergence, it might risk students not being engaged enough. The adequate balance between convergence and engagement depends on the concrete topic and the concrete piece of knowledge.

The result of the generalization process was the specification of a range of acquisition activities for each piece of knowledge, as illustrated for two exemplary pieces of knowledge in Fig. 3.

Discussion

Due to the fact that our design research in the KOSIMA project results in a new textbook, the design part of the work itself has important impact on *several communities*. First of all, it affects the *students*, who get the chance to develop and reinvent mathematical concepts by relating relevant contexts with individual, sustainable conceptions. The developed learning arrangement offers structured tasks so that the teacher can moderate the learning processes and can support students in organizing their knowledge. It has to be mentioned that the *Mathewerkstatt* is one of only a few textbooks in Germany that are put to the test and fully revised and reviewed by sample classes for guaranteeing usability by *teachers*.

The research that was conducted in the iterative interplay with design, evaluation, analysis and revision of the learning arrangements showed the potential of the structure of activation – exploration – organization – practice. The insights gained into the deeper structures of the initiated learning processes allow us to contribute also to *didactical theory*. A further evaluation on the generated learning progress was started in August 2012 for a two year study.

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