DIFFERENT THEORETICAL PERSPECTIVES IN RESEARCH
INTRODUCTION TO THE PAPERS OF WORKING GROUP 11

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INTRODUCING THE CHALLENGE OF DIVERSITY

One of the characteristics of the European Community in Mathematics Education is that we operate with a large diversity of different theories, research paradigms and theoretical frameworks. Like in a nutshell, this heterogeneity was present in the Working Group 11: 30 researchers, coming from 15 countries, discussed 18 different contributions in which in sum 16 different theoretical frameworks were explicitly stated (cf. Figure 1).

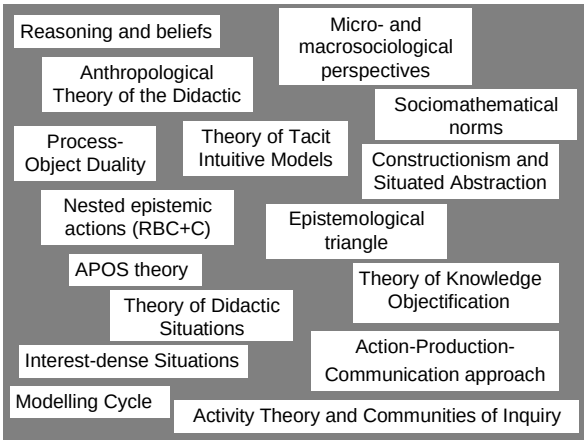

Figure 1. Research frameworks present in WG11

The large diversity of theoretical frameworks already starts with the heterogeneity of what *is called* a theoretical framework or a theory by different researchers of different traditions. In the list of explicitly stated theories, there are research paradigms, basic and comprehensive general theories as well as local conceptual tools, with different scopes and backgrounds. It affects also the way each theory conceptualizes and questions mathematical activities and educational processes, as well as the type of results it can provide. This diversity represents a challenge for the community for different reasons:

- Problems of communication: researchers from different theoretical frameworks sometimes have difficulties to understand each other in depth because of their different backgrounds, languages and implicit assumptions.
- Problems of integration of empirical results: as in the cartoon shown below, researchers with different theoretical perspectives consider empirical phenomena (see Figure 2) from different perspectives and hence come to very different results in their empirical studies. How can the results from different studies be integrated or at least understood in their difference?

- Problems of scientific progress: in the long run, improving mathematics classrooms depends in part on the possibility of a joint long term progress in mathematics education research in which studies and conceptions for school *successively build upon empirical research*. But how to do that when each study uses a different theoretical framework that cannot be linked to others? The incommensurability of perspectives produces sometimes incompatible and even contradictory results which not only impedes the improvement of teaching and learning practices, but can even discredit a research field that may appear as being unable of discussing, contrasting and evaluating its own productions.

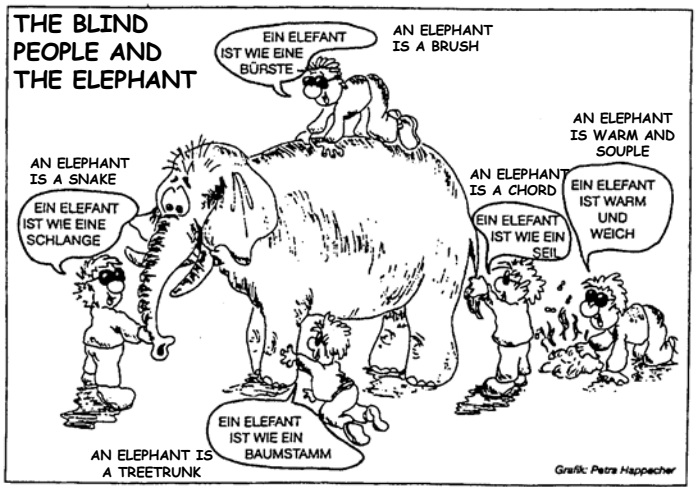

Figure 2. A metaphor for research from different perspectives

Although these aspects clearly show that the diversity of theoretical frameworks is a challenge for a community which intends to have communication and progress between researchers of different theoretical frameworks, we started the Working Group from the assumption that we do not aim at a progressive unification of all frameworks since we consider the *variety of frameworks to be a rich resource* that is absolutely necessary in order to handle the complexity of mathematics teaching and learning (see Bikner-Ahsbahs & Prediger 2006). This is also emphasized by one conference contribution:

> "Simultaneous exploitation of […] approaches is especially valid […] when the didactical phenomena occurring in the mathematics classrooms [appear] so complicated with respect to personal, social and epistemological aspects. A multiple approach seems necessary and the researchers have to build up the concrete connections that would make these different tools compatibles." (Kaldrimidou *et al.*[1])

LEARNING TO COPE WITH DIVERSITY

Hence, the proposal of learning how to deal with the diversity, complexity and richness of European theoretical perspectives in mathematics education is a task our community cannot postpone much longer. This necessity was at the origin of Working Group 11 at CERME 4, where a group of researchers lead by Tommy Dreyfus, Michèle Artigue, Mariolina Bartolini Bussi, Eddie Gray and Susanne

[1] All papers cited without reference nor year follow as contributions to these Working Group proceedings.

Prediger met around the discussion on research paradigms within the context of their effect on empirical research. One of the most important directions that emerged was the idea of *networking*:

"If we can develop and maintain a certain degree of networking between some of the advocates of the different theoretical stances that are currently evident within mathematics education, this will constitute an important step on the path towards establishing mathematics education as a scientific discipline." (Artigue et al. 2006, p. 1242)

Anyway, bringing to the fore the project of comparing, networking or (partially) integrating theories as an inevitable task does not make it less difficult. As it was pointed out:

"It is crucial to have an awareness of the underlying assumptions of each theory. Only on the basis of such awareness, can a discussion on the possible coherence of underlying assumptions begin to take place so that a common language supporting such networking can be developed." (Artigue et al. 2006, p. 1243)

Taking into account this discussion at CERME 4 in year 2005, the succeeding Working Group 11 at CERME 5 proposed to follow these orientations by focusing on two axes:

Theme 1: Deepening our insight into the underlying assumptions, relationships and differences of theories or approaches in mathematics education;

Theme 2: Handling the diversity of theories in our field of research in order to better grasp the complexity of learning and teaching processes.

More concretely, papers were asked to provide a "piece of answer" to the following starting questions:

Theme 1: Functions of a given theoretical framework

1.1. How do specific theories allow (re)formulating problems about the considered reality?

1.2. What specific methods, methodologies and heuristics are developed because of the use of a theory?

1.3. What are the consequences of the use of a specific theoretical framework on the interpretation and formulation of results of an empirical research?

1.4. How do empirical studies contribute to the development and evolution of theories?

Theme 2: Interactions of two or more theories

2.1. How do specific theories allow (re)formulating problems about the considered reality?

2.2. What methods, methodologies and heuristics are used to compare, develop, combine, integrate or complement different theories?

2.3. What consequences the interaction of theories has on the research, for instance in the formulation and approaching of problems?

2.4. What can be said about the issue of mutual consistency of different theories?

Some of the approaches are specifically treated in the papers related to theme 1: the theory of knowledge objectification (Radford), social practice theory and communities of inquiry (Goodchild, Jaworski), the nested epistemic actions (RBC+C) model for abstraction in context (Dooley) and the anthropological theory of the didactic (Rodríguez *et al.*, Wozniak).

The other appearing theoretical frameworks were compared, contrasted or combined with different purposes. Thus, Arzarello *et al.* combine and try to complement the Anthropological Theory of the Didactic with the Action-Production-Communication approach. Bergsten contrasts the way APOS theory, the anthropologic theory of the didactic and research on reasoning and beliefs approach to the problem of teaching limits of functions. Gellert compares the micro- and macro-sociological perspectives in mathematics education research through the analysis of a short transcript of 6th graders' collaborative problem solving. Kaldrimidou *et al.* use the sociomathematical norms and the epistemological triangle to analyse the mathematical knowledge under construction in two secondary school lessons. Kidron *et al.* focus on how the theory of didactic situations, the nested epistemic actions and the theory of interest-dense situations take into account social interactions in learning processes. Maracci approaches students' difficulties when solving vector space problems comparing two frameworks: the theory of tacit intuitive models and the theory of process-object duality. Finally, Cerulli *et al*. present a methodology for integrating research teams based on different theoretical perspectives: theory of didactics situations, anthropological theory of the didactic, socio-constructivism and activity theory, constructionism and situated abstraction. Petrou approached the connecting task by combining different research methods for her empirical questions.

Although we cannot summarize the whole process in the working group, we want to give insights in some important aspects and questions that arose in the discussion.

SOME ARISING ASPECTS AND QUESTIONS

The problem of the hidden assumptions

One of the most important obstacles for communication between different theoretical frameworks is the fact that each theory is connected to more or less explicit assumptions on epistemological, methodological, philosophical and sometimes psychological questions.

Nature of mathematical knowledge

By reminding of the work of Hans Georg Steiner (1987), Günter Törner and Barath Sriraman emphasized that among all the grounded assumptions of any theoretical approach, those related to the nature of mathematical knowledge may

appear as most fundamental (see Törner/Sriraman 2007 which is the elaboration of their preconference contribution). Even if the chosen mathematical epistemology is a crucial element of any approach, it is also important, as Steiner pointed out, to conceive them not as a "credo" or "norm" to follow, but as scientific models that are to evolve and be modified according to their productivity in explaining didactic phenomena. The degree of elaboration of specific epistemological models (or mathematical philosophies) also seems to differentiate mathematics educational approaches from those, more general, coming from psychological or social perspectives. Not only Törner and Sriraman pleaded for this level of comparison between theories, also other studies (Bergsten, Kaldrimidou *et al.*, Rodríguez *et al.*) showed that the assumptions on the nature of mathematical knowledge appears to be one key point for the analysis of similarities and differences between approaches.

The individual/social interplay and the challenge to constructivism

Another important level of basic assumptions concerns the nature of knowledge and learning, is it individually or socially constituted? At CERME 4, this difference was discussed as an important aspect which might even make integration of theories impossible (see Artigue *et al.* 2006). This year, we saw attempts of new conceptualizations of the individual-social interplay: Doodley presented a study on distributed knowledge construction without social learning theories, Kidron et al. presented a joint work with the comparison of different roles that the social interaction can play within three different theories, and Radford challenged constructivism by designing a "cultural theory of learning".

Extending theories to higher levels

Jaworski's and Goodchild's papers are good examples of how to extend the scope of a given theory to embrace a wider and more complex phenomenon such as the relationships between teaching/learning practices and research. While Goodchild focuses on the use of an activity theory perspective to analyse the learning process of a didacticians' team in a developmental research project, Jaworkski extends the social practice theory to analyse the practice of teaching as learning in practice.

In another framework that also considers teaching and learning mathematics as social practices of communities, Wozniak illustrates how the analysis proposed by the anthropological theory of the didactic extends the scope and nature of the studied phenomenon. Considering the teaching of statistics in France and the difficulty of its diffusion in compulsory education, the analysis highlights general restrictions coming from different levels of determination like the status of statistics within mathematics, the "reclusion" of statistics in professional education, and, more generally, the negative consideration of statistics in past times in the French society.

The problem of non-isomorphism between research questions in different frames

The anthropological theory of the didactic is also used in the paper by Rodríguez, Bosch and Gascón to contrast how the classical problem of teaching metacognitive strategies in mathematics can be formulated in terms of, on the one side, the passage from point-levels to local or regional levels of mathematical praxeologies and, on the other side, the different division of responsibilities between teacher and students as it is classically stated by the current didactic contract. Thus, a problem that has been historically approached from cognitive perspectives can be converted into a problem of the conditions and obstacles for didactic and mathematical praxeologies at school. This conversion changes the problem into a non-isomorphic one.

The "incommensurability" of perspectives can be made clearer when several perspectives upon a same practical question are taken into consideration. This is also shown by Bergsten who compared three different perspectives considering the questions approached, the methods and empirical evidence used, and the conclusions and implications stated in each case.

How different theories approach similar data

Studies considering different approaches to similar questions or data bring more evidence to the considerations above. In the case of sociological perspectives in mathematics education, Gellert's work consists in comparing the validity and relevance of analyses coming from a macro and a micro level when applied to a unique set of data. Different interpretations and understanding of the same account rise immediately: the micro-sociological analysis describing the emergence of a convincing argument while, at the macro-sociological level, differences between situations and their recognition by the students (then using everyday knowledge or a logic-mathematical thought) have a prominent role.

NETWORKING STRATEGIES

Theoretical approaches can be connected in multiple ways and degrees. From the extreme of mutual ignorance (or a relativist "laissez-faire") to the extreme of complete integration, almost all positions can be considered in between. It seems to exist a general agreement in the European community that, even if theoretical diversity is more a richness than a nuisance, we should advance towards a more consequent coordination of frameworks. But at the same time, we plead for respecting the pluralism of autonomous theories as a rich resource.

The working group started to discuss the wide spectrum of strategies for connecting theoretical approaches between the two extreme strategies – the laissez faire on the one hand and the unification on the other hand. We considered the

contributions as first attempts of connecting theories with different strategies like comparing, contrasting, coordinating or combining.

Understanding each other is the first strategy on which we spend a lot of time and the first part of the call for papers (see above).

The most modest but already ongoing strategy is *comparing* theoretical approaches and their impact on research processes. A comparison can start from the theoretical base, but also from a piece of data or a problem that has to be conceptualized as a research problem in different theoretical perspectives. A comparison can also lead to competing between different theories with respect to specified research interests.

Other contributions had their focus on a specific empirical research question and use different theoretical lenses for a deeper understanding of a concrete phenomenon, in these cases, *combining or coordinating* strategies can be discussed.

Integrating or synthesizing strategies aim at a further development of theories by putting together a small number of theoretical approaches into a new framework. In order to avoid building inconsistent theoretical parts without a coherent base, these strategies can only be applied to approaches with compatible (but not necessarily equal) backgrounds.

A shared experience for comparing:
"From a teaching problem to a research design"

Prediger and Ruthven designed one common experience for comparing theoretical approaches by considering their meaning for practically oriented empirical research. The main question was "How does a theoretical basis chosen for a study influence the nature of the purpose, questions, methods, evidence, conclusions, and implications of the study?" Whereas most presented comparisons started with a given piece of data and analysed it through different lenses, Prediger and Ruthven wanted to start earlier in the research process by focussing on the way an ordinary teaching problem is conceptualized in terms of the theory and how a research design is made out of it. The proposal was to follow a set of frameworks during the whole process of conceptualising a practical problem, transforming it into a more focused research question, developing a research design and forecast the kind of results that could come up. The answers given by eight teams of researchers were analysed through two main axes: the major intention of research (improve understanding of phenomena *versus* improve teaching and learning practices) and the level of analysis: micro, meso or macro (cf. Prediger/Ruthven).

Different networking strategies for connecting theoretical approaches

Some papers offer first suggestions of how to combine theoretical approaches. Maracci's combined the local conceptual frameworks for explaining students'

difficulties with linear algebra, namely Fischbein's theory of tacit intuitive models and Sfard's theory of process-object duality. Arzarello et al. combine bigger frameworks, the anthropological approach and the APC-space theory, to analyse the same subject, the "ostensives" or "semiotic tools" (oral words, written symbols, graphical objects, gestures, etc.) used in mathematical activities. Each approach produces different complementary insights on the same phenomenon and the resulting analysis is so enriched by the combined approach.

A paradigm of the necessity of networking theories for the needs of the research is the case described – and analysed – by the TELMA network project (Cerulli *et al*.). A group of six European research teams interested in the Technology Enhanced Learning in Mathematics (TELMA) had to develop a methodology for integrating their research approaches to favour the construction of a shared scientific vision, the development of common project and the building of complementarities and priorities in the considered research area. Their experience based in a cross-experimentation brings some important guidelines to the "dealing with diversity", some of which has been considered before: the "making clear and communicating the implicit" – related to the "hidden assumptions" commented below – and the differences in the conception of the experiments – as the collective work leaded by Prediger and Ruthven also illustrates. Furthermore, what the TELMA project shows is how the interaction of approaches in the design of teaching experiences and their putting into practice in classrooms often reveals the limitations of the theoretical frameworks – "what they do not say" – and appears as an excellent way for their future developing.

Another basic work on connection theoretical approaches is offered by Kidron *et al.* who make a reflected first attempt of networking theories. Starting from three different theoretical approaches (represented of the Theory of Didactical situations, the theory of interest-dense situations and the theory of abstraction in context with its RBC-model) they focus on one crucial aspect, namely the role of social interaction as the core for comparing and contrasting each pair of theories. Especially instructive is the question of what each of the theories has to offer for the others.

Networking in Different Profiles

By comparing these different first networking strategies, the *aim of networking* turned out to make a big difference. Whereas for example Kidron *et al.* search for a general development of their theories, Cerulli *et al.*, Kaldrimidou *et al.*, and Maracci start from an empirical phenomenon or a practical question with the aim of developing or understanding it better connecting different perspectives (Cerulli *et al.*) or local conceptual tools.

Bottom-up development	Top-down development
• Aim: understanding a concrete empirical phenomenon • Ongoing growing of the bulk of theories • Focus on the research question for a concrete empirical phenomenon • Trial & Errors • Dialectic among theories • Endogenous development (i.e. driven by the concrete study)	• Aim: use or development of a given set of theories • Different theories on the table from the beginning • Focus on the relationships among theories • Deductive approach • Networking / Combining / Integrating • Exogenous development (i.e. driven by the general interest)

Figure 3: Comparing different profiles

As a consequence of these different aims, the development of networking follows completely different profiles as sketched in Figure 3. Whereas the top-down profile starts with different theoretical frameworks from the beginning, the bottom-up profile searches for new theoretical tools only if the others turn out to be insufficient. With the focus on the research question, the process of combining theories follows the logic of trial and error with output for understanding the empirical phenomenon as the measure for suitability. In contrast, the top-down profile with its focus on relationships among theories follows a deductive approach. In this sense, the TELMA project offers a new profile while starting from a given set of theories but aiming at the development of a concrete empirical research. Although we only met these different profiles so far, it is still an open question whether others profiles are possible and fruitable which switch between the different columns.

OUTLOOK

Although we started a very interesting process, the discussion is far from being finished. Three big questions turned out to be crucial for the further work:

For the concrete networking, first appears the problem of how to link theories without getting contradictions and not destroying their internal coherence. A common supporting frame is necessary, at least to offer a location for the linking. Another possibility is to start the networking being located in a given "strong" perspective and developing it in order to "incorporate" the new approaches. Anyway, it seems clear that networking cannot be done in a "theoretical no-man's land".

The second question follows. How do the differences or similarities between theories influence the networking strategies? What shared backgrounds are necessary in order to ensure networking without loosing the rationale of each approach?

Finally, related to what we can call a "theory of networking theories", the question arises of what categories are needed to deal with different theories. Is it necessary to build up a common or shared background that may appear as a "neutral land" for the networking or, on the contrary, it is important to maintain a multiple reference system where each theory may appear as a potential chief supporter of the whole construction?

The ambitiousness of these questioning obviously exceeds the scope of a working group that only meets a few days in a two years period. The CERME Working Group on Theoretical Perspectives intends to start a long term project that might work also between the conferences, possibly by splitting into small groups. In each case, the aim is to establish a "reference place" for the study of theories, their differences, commonalities and connections, with the vision of a vivid and theoretically diverse but connected European community of research in Mathematics Education.

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