WORKING GROUP 11
Different theoretical perspectives and approaches in research in mathematics education

CONTENTS

DIFFERENT THEORETICAL PERSPECTIVES AND APPROACHES IN RESEARCH IN MATHEMATICS EDUCATION

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The program committee assigned a very wide theme to this working group: different theoretical perspectives and approaches to research in mathematics education. In order to keep the work of the group focused and coherent, we published a somewhat narrower call for papers and, before the conference, decided, on the basis of the papers accepted, to concentrate discussion on research paradigms and/or theories within the context of their effect on empirical research. Specifically, we encouraged the working group participants to concentrate on one or more of the following:

1. The influence of different theories on data analysis by:
   a) considering a given set of data or phenomena through different theoretical lenses and analyze the resulting differences;
   b) analyzing the interactions of two or more theories as they are applied to the same empirical research study.
2. The relationship between theory and empirical research by:
   a) analyzing how a specific research paradigm influences empirical research and,
   b) exemplifying how empirical studies contribute to the development and evolution of theories;
3. The relationship between research and practice by analyzing how research influences practice and vice versa.

The over-riding theme during the group discussions turned out to be the need for a convergence in research, whether or not such convergence was desirable and possible, and, if so, how it may be achieved. In its research stance, mathematics education is multi-disciplinary, in the sense that researchers from different research communities - psychology, sociology, anthropology, mathematics, linguistics, and epistemology - contribute to it. It is also multi-disciplinary in the sense that though the theoretical frameworks built and used by the community of mathematics education researchers are strongly influenced by theoretical constructions and approaches initially developed outside the field, they progressively become genuine constructions of mathematics education.

As a consequence, it is not easy for researchers in mathematics education, even if they restrict themselves to the learning and teaching processes in mathematics, to delimit the pertinent objects for their research after taking into account the diversity of the determinants for these processes. Choices at this level result also from theoretical choices, and from the basic principles underlying the researcher's theoretical positions.

Beyond diversity emerging from multi-disciplinarity, there is also a more intrinsic diversity linked to the diversity of educational cultures, and to the diversity of the institutional characteristics of the development of the field of mathematics education in different countries or global areas. This diversity is both a source of richness for the field – its helps us to question what we often tend to consider as the normal or only way of thinking about or acting upon educational systems – and a source of fragility for research if we don't make specific efforts to counterbalance the difficulty that stem from communication. This is all the more so since the theoretical explosion we see today, the inflation of terms and notions, goes beyond what can be seen as a logical consequence of the sensitiveness of mathematics education to cultural differences.

Although there was a general, if cautious, agreement that convergence in research would be beneficial, the view that diversity implies richness, and should therefore be maintained, was also expressed. Indeed, Cestari, Daland, Eriksen & Jaworski[1] implicitly contributed to this view by presenting a developmental research paradigm. However, it was agreed that to be too general could run the risk of losing the specificity of mathematics education, including the requirement that research in mathematics education should deal in an essential way with mathematics.

The question thus arose whether or not there is a default research style or even a "mathematics education research paradigm" that can identify research in mathematics education. Kaldrimidou and Tzekaki gave hints of what we, as a community, may need to think about in generating such a paradigm and developing an all-embracing theory. It was a difficult conception to consider and no consensus was drawn, partly because of general problems of communication, linguistically, methodologically, and philosophically.

For example, research paradigms emphasized on one hand the social context and institutional practice (Bosch, Chevallard & Gascón) and on the other cognition (Poynter & Tall), but the two positions hardly converged. These two presentations soundly illustrated the degree within which the basic principles underlying a theoretical position shape, what we consider to be, deserving research agendas in mathematics education. From the perspective of Bosch et al., a basic assumption is that the key to understanding the teaching and learning processes in mathematics lies in institutional practices; the mathematical thinking of individuals is tightly shaped

[1] In this overview, we will frequently refer to the contributed papers that follow the overview. Such reference will be made simply by author names.

by these. What is known and how it is known is, in a sense, a by-product of these practices — a learners knowledge reflects what it is the institutional practices allows them to know and learn. Thus, investigating and establishing theories in the cognitive development of individuals is of minor interest for research that wants to understand the life of mathematics in educational systems.

Poynter and Tall, on the other hand, placed an emphasis on the cognitive growth of individuals, in an attempt to develop theory derived from the way in which individuals engage in mathematical activity. Though Tall and Chevallard agree that things, which look complex, may have a pattern that suggests that theory may be developed, they do not look for explanations in the same way. However, they agree on the importance of the mathematical component within their analysis, but once more they diverge because they are not led by the same intention.

Most researchers don't adopt such radical positions. The most prevalent practice is that of cross-breeding theories in varying degrees. Such cross-breeding often involves theories that are not exactly of the same nature and do not possess the same detail. This makes it possible to see them as either closely related or simply complimenting each other. Within then the working group several examples illustrated this feature. Some of the contributions suggested how context — including social context — and cognition might be brought to interact more closely. Bloch's work, for example, introduces semiotics into the theory of didactic situations. The integrated use of theories associated with cognitive and social perspectives was demonstrated by Bikner-Ahsbahs, whose contribution suggested how this might be done in one area of study. Kidron's contribution carries the implication that the relationship between the strengths associated with theory derived from a social context and theory derived from a cognitive one may be mediated by a theory outlining cognitive construction for abstractions in context, whilst Arzarello and Olivero indicated how a combination of theories on a larger scale could possibly work. Particular frameworks are most clearly seen in approaches to data collection and analysis but a comparative analysis of data that emphasizes different disciplinary frameworks can be illuminating (Lenfant, Roditi & Artigue; Leron).

A further difficulty in comparing, connecting or even unifying theories is presented by the fact that there exist different levels of theories. Researchers use theoretical frameworks as paradigms, perspectives, background theories, foreground theories, empirically grounded theories or local theories, to mention a few. Often, the theoretical level on which a researcher operates is implicit rather than explicit. Nevertheless, in this overview, we refer to all of the above by the collective term "theories".

Finally, even researchers who are quite explicit about the theoretical frameworks they use, are usually not explicit about, and can even be unaware of the assumptions underlying their theoretical approach. One possible exception to this lack of explicitness is the contribution by Wilhelmi, Godino & Lacaste. The approaches

discussed above, namely whether knowledge is constructed individually or socially, is one important example for underlying and often unquestioned paradigms. Other underlying assumptions concern ontological or epistemological questions such as the nature of mathematical objects, or how we can perceive the world by means of empirical research. If underlying assumptions are unclear, or even contradictory, there can be little hope of comparing theories, and even less for integrating them.

On the unifying side, all working group participants appeared to aim to make a difference in the quality of learning as a result of their research. This difference was explicit within the several papers that constructed theory from practice (Cestari & al.; Ferrara, Robutti & Sabena; Assude, Paquelier & Sackur) and the way in which theory could be transformed into practical use (Ejersbo & Leron).

The group-work provided an opportunity to examine ways in which theories that were new to individuals interacted with those that were known. It was an opportunity to restructure personal opinion. The meeting thus provided opportunities to become aware of and compare theoretical standpoints.

In conclusion, the central term that emerged from the working group was *networking*. The overall conclusion was that because of the reasons cited above, there was no expectation that theories would be integrated into a "grand unified theory" in the near future. In fact, even in such a long established science as physics, the desire to integrate physical theories dealing with forces at different orders of magnitude have met, so far, with only partial success. Therefore, though we should maintain high hopes for future integration, we should also be realistic. If we can develop and maintain a certain degree of networking between some of the advocates of the different theoretical stances that are currently evident within mathematics education, this will constitute an important step on the path towards establishing mathematics education as a scientific discipline.

The idea of networking theories thus appears as more realistic than integration. On the other hand, as a research community, we need to be aware that discussion between researchers from different research communities is insufficient to achieve networking. Collaboration between teams using different theories with different underlying assumptions is called for in order to identify the issues and the questions. Such collaboration could take the form of separately analyzing the same data and then meeting to consider and reflect upon each other's analysis. The project presented by Cerulli, Pedemonte and Robotti is a start in this direction. It aims at building an integrated frame for research and design on technology-enhanced learning. While this may be too ambitious a goal for the present, the project strategy is interesting. Beyond finding tentative integrative lines by reading and analysing research work, each of the six teams involved in the project will analyse a piece of software produced by another team and build an experiment around it, relying on the team's own theoretical frames, thus allowing later comparison with the analysis and experiments built by the team having produced the software.

In order to promote the networking of theories, we suggest two foci of discussion for the theory working group at the next CERME conference. First, it would be useful to make explicit the level at which a theory operates. This might be helpful in assessing the possibility of comparing, networking or integrating theories. Second, in any attempt to network theories, it is crucial to have an awareness of the underlying assumptions of each theory. Only on the basis of such awareness, can a discussion on the possible coherence of underlying assumptions begin to take place so that a common language supporting such networking can be developed. We therefore recommend that a second aim of the theory working group at the next CERME conference would be to work in teams with the objective of identifying and making explicit the underlying assumptions of some current theories. Finally, and possibly more importantly, we reiterate that collaborative work between teams using different theories is necessary for substantial progress towards networking theories, not only during but also in between conferences.