ETC4
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‘Classroom-Based Research on Mathematics and Language’

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ETC4 Introduction

The research force and community behind ETC4 and the conference proceedings

Marcus Schütte and Núria Planas

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The ETC4, with the topic ‘Classroom-based research on mathematics and language’, took place from the 22nd to the 24th of March 2018 at the Technical University of Dresden in Saxony, Germany. Why this topic when there are so many others to chose from? And why the subthemes of ‘language of the learner’, ‘language of the teacher and the classroom’ and ‘language of mathematics’? To emphasize the importance of the conference and situate its topic and subthemes, in this introduction we give some information about its origins and development.

This conference was conceived in the collaborative context of the preparation of the chapter about mathematics education research on language for the ERME Handbook project, which has become reality in the form of the Routledge volume ‘Developing research in mathematics education. Twenty years of communication, cooperation and collaboration in Europe’. The survey work regarding two decades of ERME research in mathematics and language showed the enduring dominance of classroom-based research. Since the beginning of ERME in 1998, in the group entitled ‘Social interaction in mathematical learning situations’ and in the group at present, ‘Mathematics and language’ (TWG09), the study of language has mostly meant studying mathematics classrooms as dynamic environments with multiple student-teacher and student-student interactions, along with the study of processes and methods to explore the nature and role of these interactions. This was clearly the case for CERME10 in Dublin, where a large number of classroom-based papers were presented and discussed in the sessions of TWG09.

Together with the dominance of classroom-based work, a closer look at the contributions in CERME10 and former CERMEs reveals the prevalence of three major objects of study: language of the learner, language of the teacher and the classroom, and language of mathematics. Despite the continuity in the classroom-based orientation and in the prevalence of major objects of study, the ERME domain has changed in terms of complexity in the theories and frameworks implied. In order to understand the contemporary domain and move the discussion forward, the aim of ETC4 was planned to facilitate and allow plenty of time for careful revision of such complexity. Strategically, the program of ETC4 was focused on progress and connections regarding the classical subthemes of language of the learner, of the teacher and the classroom and of mathematics. This organisation was used as the reference point in the shaping and search of similarities, differences and future research questions in the current international domain.

Although CERME is a European conference, the ETC4 in Dresden attracted researchers from institutions all over the world who provided a crucial international dimension to the event and to the discussions. 35 researchers from 15 countries (Austria, Belgium, Canada, Denmark, France, Germany, India, Israel, Malta, Netherlands, Norway, Slovakia, South Africa, Spain/Catalonia, United Kingdom/England) and 4 different continents shared their knowledge and presented their studies (17 papers and 5 posters) to discuss and work on their research. Many interesting results and reflections were interpreted and shared. Through this work, ETC4 maintained a high scientific standard, with these conference proceedings bundling and condesing the most fruitful outcomes
of the collaborative work at the conference. These proceedings are available online on the ERME website and also on the HAL open archive. Regardless of the impact of these proceedings in the coming years and despite the relatively limited scientific impact of conference proceedings in general, this product is a milestone with a valuable role in the history of the domain of mathematics education research on language. From the beginning, ETC4 was planned to be a small conference in number of participants and, as planned, it happened to be huge in communication, collaboration and cooperation produced in line with the ERME spirit.

We were extremely happy and thankful to have Mamokgethi Phakeng, Vice Chancellor of the University of Cape Town, as our keynote speaker, who has an exceptional career in mathematics education and contributes on a very high level to mathematics education research and language diversity. Her keynote was inspiring, with ideas and arguments that do not only apply to South African schools but to all multilingual schools with children who are taught mathematics in a language that is not one of their languages at home. We would also like to thank the four panel speakers Richard Barwell (Canada), Jenni Ingram (England), Susanne Prediger (Germany) and Núria Planas (Catalonia) in the role of moderator, who participated in a lively discussion and shared their views on relevant questions and challenges regarding mathematics education and language. Conference panels are a practical way to reflect on the richness of theoretical diversity and research interests in a domain. Further, we want to give special thanks to the ERME Society and its president, Susanne Prediger, who believed in the proposal and made the arrangement of ETC4 possible. Her responses were always very encouraging. We cannot express enough our gratitude to the IPC (see members below) for their constant scientific dedication and all the helpers of the LOC (see members below) for the hard work of organization. They had thought of everything and everything worked well. The Technical University of Dresden and the Catalan Institution for Research and Advanced Studies (ICREA) were both generous in supporting a number of quality plans that contributed to a successful conference.

Last but not least, we want to thank the participants, some of which belong to the youngest generation of researchers in the domain. They all nurtured the high scientific standards of ETC4 with their outstanding presentations and their enthusiastic participation in many stimulating discussions. We were very pleased to be able to welcome the TWG09 community in Dresden, and look forward to continue the discussion in future conferences.

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Plenary sessions
ETC4 Keynote

One country, many languages: Exploring a multilingual approach to mathematics teaching and learning in South Africa

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The articulation of a multilingual language-in-education policy in South Africa meets many challenges in practice. Research shows that despite what policy says, learners, families and teachers in historically disadvantaged black African schools in the country prefer English as the language of learning and teaching, and maintain the view of African languages as the languages of the home. In this plenary lecture, I comment on a multilingual approach to mathematics teaching and learning that I see as an alternative to constraining language practices in mathematics education. The two main principles of this approach are: the deliberate, proactice and strategic use of the learners’ home language and the selection of real life, interesting and high cognitive demand mathematical tasks. The underpinnings are a holistic view of multilingual learners and a notion of language as resource for mathematics teaching and learning.

Keywords: Multilingualism, language-in-education policy, language of teaching and learning, mathematics classroom, language practices.

A context of eleven official languages

South Africa is a context of great language diversity with a progressive body of language rights in principle and on paper. Prior to 1996 and before the end of apartheid, English and Afrikaans were the two official languages. Since 1997, the language-in-education policy recognises these and nine more languages that are official. In line with the new constitution adopted in 1996, the current policy supports, encourages and values multilingualism as a resource. The principle here is that no language should be introduced at the expense of another in society and particularly at school. The policy promotes but does not mandate the use of African languages alongside English, encouraging schools to maintain the languages of the home. Families are thus allowed to choose their preferred language of learning upon admission to a school. If the school uses that language and there is a place available, then it must admit the learner. Schools have to choose a language of learning and teaching mathematics as well, and school governing bodies –comprising parents, educators and non-educator members of staff– are required to state explicitly their plan and specific measures to promote multilingualism.

The articulation of a multilingual language-in-education policy for South Africa meets many challenges in practice. Research shows that despite what policy says, teachers in historically disadvantaged black African schools in the country prefer to teach mathematics in English (Setati, 2008). Moreover, research also shows that speakers of African languages in these schools prefer to be taught mathematics in English, a language that they are still learning (Setati, 2008). Their limited fluency in English does not reduce their desire and aspirations of mobility and access to social goods such as jobs and higher education. Overall, the struggle of African-language learners
with English does not move them and their families away from choosing it as the language of learning and teaching. At the same time, debates on language and mathematics teaching and learning tend to create abstract dichotomies that are not helpful by distinguishing between the language of the school and the languages of the home. Thus, policymakers and educators seem to think that only one language can and should be used during teaching and learning. Similarly, many black African families seem to think that their languages are sufficiently learned at home and are not adequate or necessary at school, often conceived of as almost useless for their children and an obstacle to wider communication.

Why the seeming disconnection between policy and practice?

The scenario in South Africa points to the disconnection between policy and practice, with a progressive language policy in education and a monolingual orientation that values one language over others. This phenomenon is not unique to the South African context. Other mathematics education researchers have documented similar scenarios in other parts of the world (see, for instance, the early work by Barwell, 2003, with learners of English in mathematics classrooms of England, the work by Moschkovich, 2015, with Latino communities of learners in the United States, or the work by Planas, 2014, about well-established ‘minority’ languages in Catalonia). Given the hegemony of English as a world language and despite other languages may be involved in the construction of discourses of adequate language use across the world (Phakeng, Planas, Bose & Njurai, 2018), the perdurability, amplitude and pervasiveness of this phenomenon is not surprising.

There is more to it than a mere problem of disconnection. The reduced roles often attributed to language in mathematics learning and teaching remain behind and are called into question. Educational practice and research on language and mathematics learning is still framed by a cognitive perspective, which assumes the premise that language is benign and innocent, as well as the primary condition for interaction, mediation and experience (see the discussion in the introduction of the volume by Barwell et al., 2016, or the discussion in Barwell, Moschkovich & Setati Phakeng, 2017). Much have changed since the late seventies of the past century, when Austin and Howson (1979) published their survey on language and mathematics education, but the ascendency of the cognitive perspective continues. I have previously argued that, research has been dominated by cognitive and deficit approaches from the seventies and it is still marked by the idea that language issues are not important to the entire mathematics education community (Setati & Moschkovich, 2010). These are relevant considerations if we advocate for a broader understanding and conceptualization of mathematics education, and more importantly for an area that does not dismiss equity in the path towards the policy-practice nexus. By failing to view the politics of language, we do not only recreate inequality but drive mathematics education research into an illusion of precision and neutrality as well.

Language is not benign or innocent. It is not a neutral or docile tool of expression, representation and communication. It is a product and carrier of power (Bourdieu & Wacquant, 1992). Language choices of teachers and learners who prefer English are informed by a socio-political perspective, which considers the political nature of language and the power of English in particular. I have largely exposed the fact that language is political (Setati, 2005, 2008) and that not all languages are equally ‘powerful’ and do not serve the same functions. Language in the mathematics classrooms is not used for the sake of the teaching and learning of mathematics only. Language has implications for how social goods are or ought to be distributed (Gee, 2005). Social goods are anything that a group of people believes to be a source of power, status and capital. English and
school mathematics are, for example, social goods whose access implies in turn access to other social goods such as higher education, jobs and international opportunities. Nonetheless, this question of access is not easy and needs to be explained in the context of many dilemmas and contradictions. More than twenty years ago, Lodge (1997) referred to the paradox of access as a double-edged sword. Access to powerful knowledge increases and entrenches power at the same time. On the one hand, if the school system favors access to the language of power, the marginalization of home languages is perpetuated. On the other hand, if the school system favors the home African languages, learners are denied the access to social goods available in English. Moreover, enforcing purist home language or English only monolingual teaching at any level of education is not consistent with multilingual policy and can be seen as discriminatory. In the case of home language monolingual teaching (e.g., recent policy in the Gauteng Province of South Africa), it suggests that those who have capital can buy access to English. In the case of English only monolingual teaching, it suggests that learners are not allowed to be who they are.

**How the language-in-education policy plays out in multilingual classrooms**

Today the South African language-in-education policy is manifested in a diversity of classroom settings in which multilingualism is apparent. Some typified examples of these settings are:

- Township classrooms with South African learners only and a shared main language (low cognitive demands mathematics tasks are common; English accompanied by procedural discourse prevail in mathematical lessons; and there are limited occurrences of code-switching accompanied by conceptual discourse).

- Township classrooms with immigrant learners and a main language not shared (low cognitive level demand tasks are common; there is no code-switching; procedural discourse prevails in mathematical lessons; and most learners refuse to be identified as migrants).

- Urban classrooms with immigrant learners and French as a shared main language (high cognitive level demand tasks are common; teaching happens in both English and French, however, writing is in English only; conceptual discourse prevails in mathematical lessons; and learners tend to be open and proud of their migrant identity).

- Rural classroom with immigrant learners and a main language not shared (low cognitive level demand tasks are common; there is no code-switching; procedural discourse prevails in mathematical lessons; and most learners are open but not proud of their migrant identity).

Why these language practices and why embedded in these distinguishable ways? Are these practices and their variations mere responses to ‘practical’ pedagogic matters? Are they related to who the mathematics teacher is, and hence her social identity? Are they related to which the shared main language is and the socio-economic background and capital attributed to its speaking community? Are these language practices about the culture of the school and its treatment of the learners of mathematics, especially immigrant learners? If so, is the culture of the school overt and clear about what an immigrant is and which learners are called immigrants? Are participants of this culture aware of the multiple layers of language represented in multilingual classrooms
such as minority/human rights, race, socio-economic class or culture? What renders being multilingual irrelevant in mathematics teaching and learning? Level of fluency in the language of learning and teaching? Level of mathematics performance? Level of socio-economic class?

The development, by either teachers of learners, of language practices of code-switching, procedural and conceptual discourse, etc., cannot be simplified to one or two decontextualized reasons in isolation. Exploring language practices in specific multilingual mathematics classrooms of immigrant learners, for instance, provides a different gaze into teaching and learning mathematics in multilingual classrooms in South Africa. Moreover, it is different the situation in township schools, where the majority of the teachers are multilingual and many speak at least two African languages in addition to English and Afrikaans. Learners in these schools are also likely to speak more than one African language and will have ranging levels of English language proficiency. Beyond these relevant specificities and based on my research, what I claim is the existence of a network of connected reasons some of which are political. Our aim as researchers is not only to identify the practices but also to question how and why these practices have been embedded the way they have, as well as how and why their politics and implications remain under-researched. In the next section, I comment on a multilingual approach to mathematics teaching and learning that I see as an alternative to constraining language, teaching and learning practices. Such multilingual approach is in line with the argument in Planas and Setati Phakeng (2014) about “the right of using the students’ languages …. because it is itself more than an intrinsic human right; it is an option that potentially benefits the creation of mathematics learning opportunities (p. 883).

**A multilingual approach to mathematics teaching and learning**

How can we teach mathematics in multilingual classrooms to ensure that learners are challenged mathematically and interested in learning mathematics? How can we draw on the diversity of languages present in South African mathematics classrooms (English and the learners’ home languages) to provide the language support that learners need? How can we draw on the learners’ home languages to ensure a focus on developing mathematical proficiency while learners are still developing fluency in English? All these questions are very timely in a world in which the continuing domination of some language groups and their privileged speakers over others damages the identities, futures and learning opportunities of children of these other groups.

There is a holistic view of multilingual learners theoretically underpinning a multilingual approach to mathematics teaching and learning. From this view, a multilingual is not a sum of two or more complete or incomplete monolinguals, nor a bilingual with an additional language. A multilingual is like a high hurdler who blends two types of competencies, that of high jumping and that of sprinting, in her fluid use of language. When compared individually with the sprinter or the high jumper, the hurdler meets neither level of competence, and yet when taken as a whole, the hurdler is an athlete in his or her own right. No expert in track and field would ever compare a high hurdler to a sprinter or to a high jumper, even though the former blends certain characteristics of the latter two. In many ways the bilingual is like the high hurdler. The constant interaction of the many languages in the multilingual learner has produced a different but complete dynamic language system.

Importantly, a multilingual approach to mathematics teaching and learning draws on the understanding of language as resource. For a resource to be useful, it needs to be both visible and invisible in its functioning (Adler, 2001). *Visibility* is in its presence and the form of extended access to mathematics it provides, while *invisibility* or *transparency* is in the form of
unproblematic interpretation and integration of the language(s) used. This reasoning does not apply only to language in the classroom. For example, the instrumental use of technology in mathematics teaching and learning becomes effective when it is both visible and invisible or transparent. When technology becomes so visible that remains the focus, instead of the material environment for orchestration of the mathematical lesson, this occurs at the expense of the focus on the teaching and learning of mathematics. What typically happens is many multilingual mathematics classrooms is that language becomes visible to the teacher almost exclusively when being spotlighted for some learners using ‘wrong’ words and ‘wrong’ grammars. It is, however, largely invisible to the teacher and the class when conversations develop in fluid ways.

Two major principles should guide a multilingual approach to mathematics education, so that teachers take advantage of the home languages and experiences of learners rather than disregard them. These principles are as follows (Setati, 2008):

1. The deliberate, strategic and proactive use of the learners’ home languages.
   - Unlike code-switching, which is spontaneous and reactive.
   - English and the learners’ home languages operating together and not in opposition.
   - All written texts are given to learners in two languages (home language and English).
   - Learners are explicitly encouraged to interact in any language they feel comfortable with.

2. The use of real life, interesting and challenging mathematical tasks.
   - Through this, learners would develop a different orientation towards mathematics and would be more motivated to study and use it.

While learners communicate in their home language, simultaneously or not with English, they develop mathematical meanings and in this context, such meanings can be accepted, questioned and negotiated. The deliberate, proactive and strategic use of the learners’ home languages in the discussion of interesting mathematical tasks ensures that language functions both as a transparent resource and as a facilitator of mathematical learning opportunities. However, it is a challenge for some mathematics teachers to switch languages in the classroom as a flexible strategy of teaching and learning, as well as to think of posing interesting mathematical tasks to learners who are in the process of gaining fluency in English. There are practical ways and strategies of facilitating the work.

In a collaborative study with mathematics teachers in South African schools (e.g., Setati, Molefe & Langa, 2008) we grouped learners according to their home languages and gave them all written tasks, including tests and exams, in two languages: English and the learners’ home language. Learners were explicitly encouraged to communicate in any language they feel comfortable with as they were tackling the tasks. In this way, not only did the learners remain focused on the mathematics of the task but the flexible use of languages also facilitated active participation by all learners. While the home languages were visible in the sense that the learners were for the first time given written mathematical texts in their home languages, they were also invisible in that they were not distracting the learners’ attention from the tasks they were doing. The learners were not focusing on the languages but on the mathematics of the task. Not only was the use of their
home languages not perceived as a distracter or constraint, but some learners explained that having their home language versions was helpful.

Table 1. Three examples of rich mathematical tasks

<table>
<thead>
<tr>
<th>Mandla’s cinema hall can accommodate at most 150 people for one show.</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Rewrite the sentence above without using the words “at most”.</td>
</tr>
<tr>
<td>b) If there were 39 people who bought tickets for the first show, will the show go on?</td>
</tr>
<tr>
<td>c) Peter argued that if there are 39 people with tickets then Mandla should not allow the show to go on because he will make a loss. Do you agree? Why do you agree?</td>
</tr>
<tr>
<td>d) What expenses do you think Mandla incurs for one show?</td>
</tr>
<tr>
<td>e) Use restrictions to modify the statement above in order to make sure that Mandla does not make a loss.</td>
</tr>
<tr>
<td>f) If Mary was number 151 in the queue to buy a ticket for the show, will they accommodate her in the show? Explain your answer.</td>
</tr>
</tbody>
</table>

Look at the calendar with the days of rain and answer the questions.  
1. Which month has the most rain?  
2. Which month has the least rain?  
3. Which months have the same amount of rain?  
4. How many more days does it rain in February than March?  
5. How many days does it rain in September and October altogether?  
6. Which month has 20 days of rain?

The Brahm Park electricity department charges R40,00 monthly service fees then an additional 20c per kilowatt-hour (kWh). A kilowatt-hour is the amount of electricity in 1 hour at a constant power of 1 kilowatt.  
1- The estimated monthly electricity consumption of a family home is 560 kWh. Predict what the monthly account would be for electricity.  
2- Three people live in a townhouse. Their monthly electricity account is approximately R180,00. How many kilowatt-hours per month do they usually use?  
3- In winter the average electricity consumption increases by 20%, what would the monthly bills be for the family home in (1) above and for the townhouse?  
4- In your opinion, what may be the reason for the increase in the average electricity consumption in (3)?  
5- Determine a formula to assist the electricity department to calculate the monthly electricity bill for any household. State clearly what your variables represent and the units used.  
6- a) Complete the table showing the cost of electricity in Rand for differing amounts of electricity used:  

<table>
<thead>
<tr>
<th>Consumption (kWh)</th>
<th>0</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
<th>700</th>
<th>800</th>
<th>900</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost (in Rand)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

6- b) Draw a graph on the set of axes below to illustrate the cost of different units of electricity at the rate charged by the Brahm Park electricity department.
After careful consideration, the electricity department decided to alter their costing structure. They decide that there will no longer be a monthly service fee of R40.00 but now each kilowatt-hour will cost 25c.

7- What would be the new monthly electricity accounts for the family home and the townhouse?

8- a) Complete the following table showing the cost of electricity in Rand for differing amounts of electricity used using the new costing structure:

<table>
<thead>
<tr>
<th>Consumption (kWh)</th>
<th>0</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
<th>700</th>
<th>800</th>
<th>900</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost (in Rand)</td>
<td>0</td>
<td>25</td>
<td>50</td>
<td>75</td>
<td>100</td>
<td>125</td>
<td>150</td>
<td>175</td>
<td>200</td>
<td>225</td>
</tr>
</tbody>
</table>

8- b) Draw a graph on the same set of axes in question to illustrate the cost of electricity for different units of electricity using the new costing structure.

9- Do both the family home and the townhouse benefit from this new costing structure? Explain.

10- If people using the electricity had the option of choosing either of the two costing structures, which would you recommend? Clearly explain your answer using tables you have completed and graphs drawn in questions 6 a) and 6 b) and 8 a) and 8 b).

In any multilingual approach to mathematics teaching and learning, the selection of high cognitive level demand tasks rather than more routines classroom activities is key. Since mathematical tasks are central to mathematics learning, they need to be seriously considered, diversified and chosen. The tasks in which learners engage provide the contexts in which they learn. Thus, tasks must be clearly focused on what the teacher wants learners to know and do, be of varying cognitive demand, as well as allow the thinking of real world contexts that interest learners and engage them in the discussion of resolutions. Table 1 shows three examples of tasks addressed to different grade levels.

**Opportunities and concluding remarks**

The reflections presented in this report provide arguments for a multilingual approach to mathematics teaching and learning. Janks (2010) discusses the complexity of our work as she moves ‘beyond reason’ to ‘desire’. Desire for what one is excluded from, particularly mathematics and language, has material consequences. Both school mathematics and English in South Africa open and close doors to higher education and employment. Desire is thus a double-edged sword for us as teachers with a concern for the other: “As educators, changing people is our work – work that should not be done without a profound respect for the otherness of our students. Desiring what one is not should not entail giving up what one is” (Janks, 2010, p. 153)
Enabling others to access mathematics/become mathematical is our work but doing this involves more than just mathematics. The language choices of teachers and learners who prefer English are informed by the political nature of language. While language is a resource that can help advance mathematics learning, it can also be a stumbling block for successful mathematics learning. Many learners in South Africa do not have the level of fluency that enables them to engage in mathematical tasks set in English. One major challenge is bringing together the need for access to English and the need for access to mathematical knowledge. To address this, we need to be aware of the teaching and learning opportunities that a multilingual approach to mathematics education create:

- It recognizes the political role of language and thus also the inequality of languages
- More focus on mathematics rather than just ordinary language
  - Language functioning as a transparent resource (visible and invisible)
  - Engagement with high cognitive level demand mathematics tasks, which some teachers overlook because of their learners’ limited proficiency in the LoLT.
- Learner participation and interest in mathematics.

As explained in this report, the strategy for using language as a transparent resource in the teaching and learning of mathematics in multilingual classrooms is guided by two main principles: the deliberate, proactive and strategic use of the learners’ home language and the selection of real life, interesting and high cognitive demand mathematical tasks. I have alluded earlier in this report to research that argues that to facilitate multilingual learners' participation and success in mathematics, teachers should recognize home languages as legitimate languages of mathematical communication. More particularly, research shows that with this approach, language becomes a resource in the multilingual mathematics classroom. While the political nature of English is recognized, the learners’ home languages are not presented in opposition to English but as working together with English to make mathematics more accessible to the learners. Translating tasks into multiple languages is at the core of the multilingual approach. Translation is never a straight-forward enterprise, it is complex in many ways. As multilingual speakers of languages from different conceptual worlds we know from experience of living in language, what monolinguals know theoretically from training, that much loss and distortion of meaning can occur in translation. Yet translation is part of how meaning is transferred, made and re-negotiated; therefore, this aspect of linguistic activity remains an important consideration. The consequences of all this in terms of quality mathematics education and equity are far reaching.

References


ETC4 Panel

Opportunities and challenges of classroom-based research on mathematics and language

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This report is a summary of the questions, comments and issues brought up in the ETC4 panel of three expert researchers on mathematics and language. The purpose of this panel was to discuss: (1) What do we mean by the language of the learner, of the teacher and of mathematics? (2) What are today the opportunities and challenges of classroom-based research on mathematics and language? The major recommendations and position statements included: (i) Develop more nuanced theoretical frameworks for the understanding of the politics of language use in the mathematics classroom (ii) Re-evaluate conceptualizations of languages and speakers in terms of distinctions, differences, dichotomies and difficulties (iii) Conduct more language-related design research for teaching and learning of specific mathematical content areas.

Keywords: Mathematics and language, research domain, opportunities, challenges.

Introduction to the ETC4 panel

This report grows out of the contributions made during a conference panel. Three experts – Richard Barwell, Jenni Ingram and Susanne Prediger – from three parts of the world – Canada, England and Germany – were the members of the panel that took place in Dresden, March 2018, at the Fourth Topic Conference of the European Society of Research in Mathematics Education (ERME). Theoretically, Richard, Jenni and Susanne represent diverse perspectives, and some of their main statements are compiled below. In their brief statements, they complemented each other’s knowledge by sharing insights from their respective lines of expertise and by bringing attention to their ways and experiences of building research in the contemporary domain. Núria Planas in her role of moderator had posed in advance two questions for discussion to the panelists:

Question 1. What do we mean by the language of the learner, of the teacher and of mathematics?

Question 2. What are the opportunities and challenges of classroom-based research on mathematics and language, today?

Views on these questions were the context for reflection on what decades of research in mathematics and language teaches us for the present and the future, in terms of relevant issues, recommended actions, emerging directions, as well as strengths, milestones and weaknesses of the domain. This context greatly benefited from discussions with participants during the days of the conference and particularly from the interchange with the panel audience. Moreover, the exploration of ideas that follows owes much to the influence of and interaction with participants of the Thematic Working Group on Mathematics and Language (TWG09) in the more recent conferences of the ERME Society. TWG09 has been a context of opportunities for researchers in the domain to be able to work and think with the support of colleagues from different regions of the world and with different research perspectives. After the summary of comments and
recommendations by each panelist, we conclude this report with future steps for improvement of research in the domain. It is important that the next meetings of the TWG09 community provide some continuity in the discussion of these points.

Comments and recommendations by Richard Barwell

The first point I wanted to make was that language diversity is itself diverse. Language is diverse, languages are diverse, and diversity is diverse. Superdiversity refers to the increasingly diverse nature of diversity. The old, stable labels no longer work in analytic terms (even if they form part of everyday ways of talking about language). The idea of superdiversity has been accompanied by changes in how language diversity is conceptualized and examined:

Over a period of several decades—and often emerging in response to issues predating superdiversity—there has been ongoing revision of fundamental ideas (a) about languages, (b) about language groups and speakers, and (c) about communication. Rather than working with homogeneity, stability and boundedness as the starting assumptions, mobility, mixing, political dynamics and historical embedding are now central concerns in the study of languages, language groups and communication (Blommaert & Rampton, 2011, p. 3). So one challenge is to think about how we can research language diversity in mathematics classrooms from a perspective of mobility, mixing, political dynamics and historical embedding.

For an example, consider my two boys (aged 13 and 10). They are British and Canadian, go to school in Quebec in French, and discuss their mathematics homework with me in English or French. They have also spent a few months in school in the UK in English. Their experience illustrates mobility (dual nationality, time in a second country), mixing (use of two main languages), political dynamics (French is the required language of schooling for most children in Quebec, English in the UK, they are from a privileged background), and historical embedding (the use of French and English in Quebec is embedded in a long and contested colonial history).

Our ways of researching often ‘fix’ participants, classrooms, and languages. We label children as speakers of x, learners of y, etc. We treat languages as monolithic (e.g., variations in accent or pronunciation are considered bad or faulty). Moreover, we often overlook the politics and history of language in mathematics classrooms.

For a second example, consider children in an ethnographic study I conducted a few years ago, all seen as second language learners of English or French. I have many examples of students struggling to solve word problems, or struggling to explain their solutions. This situation can be examined as being strictly about how the students interpret the problems: about how, perhaps, their ‘limited’ level of English or French ‘impedes’ a ‘correct’ understanding of the problem, so that they get a ‘wrong’ answer or are unable to explain their solution. By paying attention to politics and history, however, other aspects of these situations become apparent. In one situation, the students are from an indigenous background, Cree. Their people and language have been subject to colonization and vicious oppression and the public school system is not necessarily well aligned with their lived experience. Their language is not well recognized in the school system. These dimensions are relevant in understanding their apparent struggles with solving word problems, when these problems are presented in a language that is other, based on situations that are also other.

While superdiversity represents a challenge for our research, there are opportunities to move forward. Research in sociolinguistics, sociology, anthropology, etc. has started to develop new approaches on which we can draw. There is an opportunity to develop new theoretical approaches to language diversity in mathematics classrooms that incorporate mobility, mixing, politics and history. For example, the concept of *repertoires* is now widely used in sociolinguistics, replacing
the idea of a speaker knowing a fixed number of named languages (see Barwell, 2018). Instead, speakers are thought of as drawing on repertoires made up of aspects of multiple languages, registers, genres, accents, etc. and these languages, registers, genres, accents, etc. are not seen as fixed either, but as multiple and fluid. There is not one kind of mathematical language, for example. Speakers draw on parts of their repertoires according to the situation. So one direction for our research could be to examine the nature of students’ repertoires in mathematics classrooms. This kind of approach will not result in a neat general theory, but understanding the dynamic nature of students’ use of language could be invaluable for informing mathematics teachers, and developing new pedagogical strategies.

Comments and recommendations by Jenni Ingram

Thinking about the differences, we often demark between the language of the teacher, the language of the learner, and the language of mathematics. There is an assumption here that needs to be challenged, that is that there is something that we can call the language of mathematics, the language of the learner or the language of the teacher. In reality, there are languages, or discourses. Richard talks about repertoires that individuals draw upon as a way of conceptualising these multiple and fluid discourses, but his emphasis is on the macro level. These multiple discourses are also apparent at the micro level, and at the individual level of an interaction between a teacher and a student, which are often harder to categorise as belonging to a specific register or language. Moreover, it is often us as researchers who see these categories or repertoires as being relevant to our analyses, not necessarily the teacher and the students themselves. This raises one aspect of the issue of scale that is something we need to consider seriously as an emerging field.

Making distinctions at the level of language, culture, or between the language of mathematics, the language of the learner and the language of the teacher usually results in dichotomies. The language of mathematics is contrasted with everyday language, the language of one particular type of learner is contrasted with another type of learner and so forth. This focuses our attention on differences, not similarities. This focus on differences often leads to a focus on the difficulties. The language of mathematics is hard to learn, some learners have more difficulties in contrast to others, teachers are not using, supporting, including enough mathematical language in their lessons and so forth. As Moschkovich (2018) argues we need to “move away from dichotomies that create unproductive and oversimplified approaches to research phenomena” (p. 41).

These distinctions can be helpful as they give us as researchers a focus, and it is not that we are ignoring the relationship between the three domains, but that we are highlighting or emphasising one aspect over another. They enable us to have a very specific focus on one particular aspect of teaching or learning. They have also been fundamental in examining the social and political aspects of multilingualism in the classroom (Planas & Civil, 2013). Yet too narrow a focus may tell us a great deal about what we are researching but be of little use to teachers or learners, or even other researchers. We also need a balance between more generic foci, such as particular mathematical practices that go across topics, such as argumentation, and specific foci such as what are the issues around the learning of the meaning of the word equation.

Another challenge for our domain is that we need to be more aware of our assumptions, intuitions, and beliefs and how these influence our research. Do we always articulate these or consider the impact of these in our work? In particular, the assumptions we make about what is mathematics, what are mathematical practices, what is mathematical language, what are mathematical meanings, and so on. It is often not as clear-cut as it might seem.

Something that is both a challenge and an opportunity is to bring more cohesion to our body of work. As a domain, we have expanded significantly and drawn upon a wide range of theories and methodologies and these have given us more insight into the role of languages or the role of
discourses in both the teaching and learning of mathematics, but we also need to contribute to the wider field of mathematics education research and beyond. There is a balance to be held but a key question to ask is who is engaging with our research and who do we want to engage with our research? Are we speaking to only our domain? The complexity of what we are researching can make it difficult to communicate in a meaningful way to those from outside the domain. This applies both to other researchers, but possibly more importantly mathematics teachers. It seems to me that a coherent message synthesised from the different contributions is more powerful than one single researcher or one single approach to research. For example, Susanne’s design research described below brings together theoretical frameworks for the analysis of language(s) in the mathematics classroom and the design and development of pedagogic resources for the teachers involved. This is not to say that there is just one message, more that there is strength in numbers and different ways to work together both in conducting our research but also in communicating it. At the same time, we need to recognise the complexity of languages, discourse and interaction, and avoid the risk of oversimplification which can narrow both what is taught and how it is taught.

Personally, I would like to make a positive difference to students’ experiences of learning mathematics in the classroom. Research enables us to gain further understanding of how students learn mathematics, and the practices of classroom mathematics that support students in their learning. Does this research also help to develop teachers’ understanding, meaning understanding, not knowledge? For others what matters is making structural or policy changes that benefit our students, and for others still what matters is moving the field itself forward.

Last year I was working with two groups of teachers on developing their students’ use of mathematical language during lessons, but taking the teachers’ own beliefs about what counted as mathematical language as the focus. There were noticeable differences in what the teachers’ focused on but also what they understood from what students said. In one meeting a video showed a student asking one of the teachers when does an expression become an equation. In another meeting, a teacher shared a video of a student stating that an equation included numbers but an expression included letters. Both these groups of teachers subsequently spent considerable time trying to work out what a definition of an equation would be, which examples would count as an equation, which would not, and so forth. I have subsequently also asked this question of my student teachers. On all three occasions no one definition was settled upon, and not all the teachers agreed on what counts as an equation and what does not. Yet before this discussion, we were all treating the word equation as unproblematic. We were assuming that we all had an agreed understanding of what an equation was and were thus focused on how to help students distinguish between equations, expressions, identities, functions, etc. but using prototypical examples. I use this to illustrate the complexity of what we are looking at as well as the assumptions we make about what we are researching.

However, more challenging for me was working with teachers who had different beliefs about what it meant for students to speak mathematically than I did, and to not treat or assume that these beliefs were necessarily ‘wrong’ or ‘worse’ than mine. These differences arise out of different meanings that we are all attaching to the word language alongside different values about what it meant to learn mathematics. It is not necessarily the case that one is better than the other, just that they are different. For example, one teacher equated learning mathematical language as learning vocabulary in the group discussions, but in the videos of their practice, she was doing so much more than this. To her this was not about learning language but what it meant to learn mathematics. The distinctions we were making were different, but our values about what students should be doing in the classroom were aligned.
Comments and recommendations by Susanne Prediger

All the three of us, we did not really like Question 1. What do we mean by the language of the learner, of the teacher and of mathematics? This classical distinction between the three languages stems from the earliest articles on language in mathematics (Austin & Howson, 1979), and 40 years after being posed, it seems time to overcome them. I agree to Richard and Jenni that

- “the language” does not work in singular anymore. Due to the superdiversity of modern societies and the complexities of individual language repertoires, plural, “the languages or the language repertoires” must exchange the singular;
- easy categories of students with high or low language proficiency or strong or weak mathematical achievement cannot at all take into account the complexity of superdiversity in today’s schools;
- identifying differences between the language of mathematics, the learners and the classrooms (even if posed in plural) risks to result in useless or even dangerous dichotomies;
- even if stating differences does not necessarily imply stating deficits, it is much more insightful to identify the connecting points instead of the differences: where does students’ language start from, and along which learning trajectories can we develop it further?

Instead, we are interested in language demands posed by mathematics learning and possibly mediated by the teacher as well as in students’ use of their individual language repertoires and their development for and during mathematics learning.

The research in the last forty years has contributed to identifying the diversity of students’ repertoires and substantiated the widely accepted claim that mathematics classrooms should build upon the students’ diverse language repertoires (Planas, Morgan, & Schütte, 2018; Radford & Barwell, 2016). However, so far, the research has only selectively contributed to realizing this aim in mathematics classroom practices.

This leads me to Question 2. What are the opportunities and challenges of classroom-based research on mathematics and language, today?

I agree to Richard and Jenni that classroom-based research on mathematics and language should take into account the politics of language and the complexities of multilingual superdiverse societies without deficit perspectives. Furthermore, it should stop using the dichotomy of language of the learner and of mathematics and talk more about the language demands posed during conceptually rich mathematical learning opportunities.

My personal emphasis is that our research and development activities should be extended from (very insightful!) descriptive and analytical research towards interventionist research which contributes to developing and investigating discursively and mathematically rich learning opportunities for all students. For this, language demands in learning specific mathematical contents have only selectively been specified so far, and teaching learning arrangements are to be developed more consistently. Therefore, my major claims with respect to necessary future research activities is that

- we engage in designing teaching learning arrangements which build upon students’ diverse language repertoires for engaging them in conceptually rich mathematics and develop them further;
- investigate teaching learning processes with respect to different mathematical topics;
and specify topic-specific language demands for learning specific mathematical topics in such a detail that language learning goals can be integrated in teaching learning arrangements.

Since 2009, the work of our MuM-research group in Dortmund tried to contribute to this research agenda in design research methodologies (Gravemeijer & Cobb, 2006). The design research studies have been conducted in collaboration with linguists and language education experts and their different theoretical backgrounds, including functional pragmatics and interactional discourse analysis. The investigation of the initiated teaching learning processes helped us to see sharply that we need to:

- focus on discourse practices and the syntactical and lexical means to participate in them;
- engage students in these discourse practices and give them the lexical and syntactical means to successfully participate;
- and develop a good analytical framework, and particularly suitable classroom instructional designs.

Our major goal was always to design mathematics- and language-integrated learning opportunities and to investigate the learning processes we can initiate by these learning opportunities. That means, we use design research methodologies. By these design research studies, we can specify the language demands appearing in mathematics learning processes. I absolutely agree to Jenni and Richard that there are no easy categories for the appearing complexities. These language demands are shaped by the discourse practices required for learning mathematics. According to classroom and design research studies (Erath et al., 2018; Prediger & Zindel, 2017), important discourse practices are reporting procedures, explaining meanings of mathematical concepts, arguing about the match of different representations, and describing general patterns. For these discourse practices, also lexical and syntactical means can be specified, they are partly in the students’ repertoire already, and partly need to be learnt. That is why we consider it as very important to identify thoroughly the language demands appearing in the topic-specific learning processes and to support the students to cope with them increasingly.

We have empirical evidence from a big intervention study that such kind of instructional designs can be profitable for tackle an enormous diversity of students’ learning pathways. Even those students who were labeled as language proficient profited from the instructional design. So, labelling is not necessary: when we identify the learning needs with respect to monolingual and multilingual language learners, the interventions are also strong for the students who were believed not to need a language focus. Especially, when working more consequently on the students’ language for explaining meanings of mathematical concepts, then this intensifies also the mathematics learning processes, we have found this in many of our transcripts and in the quantitative data. In contrast, emphasizing the reporting of procedures seems to hinder mathematics learning. I am grateful that the team is currently so big (including four teachers and facilitators, five postdocs and eight PhD students and me), so we can manage to handle the complexity of the projects.

Jenni has mentioned that this question must include the teachers: How can teachers develop their expertise to foster students’ mathematics and language learning? We started to work also on the second question and adopt a similar research methodology. We provide professional development courses and then investigate the teachers’ learning pathways, starting from their instructional practices, categories and orientation of what counts in a mathematics- and language integrated classroom. Overall, both Questions 1 and 2 require not treating the language of classrooms, mathematics and learners separately anymore.
Agreements on important issues for present and future research

In this report, Richard, Jenni and Susanne have made the case for a number of reflections of importance to understand the present and future of classroom-based research on mathematics and language. This is not the full text of what was spoken and discussed in the time for the panel, but four central interconnected reflections have been made clear, namely:

- to move away from using potentially unhelpful and possibly harmful dichotomies;
- to use flexible ways of conceptualising language, languages, speakers and diversity;
- and to develop rigorous ways to take into account the socio-political aspects of language, languages and diversity in our research, and make our classroom-based research accessible and useable by teachers.

There is agreement on the ambition to carry out a research that is relevant in the senses of useable and used by mathematics teachers and educators in their mathematics teaching and development. The opening to more realistic diverse views of diversity, languages and speakers has the potential to move the study of mathematics teaching, learning and thinking forward in ways that are applicable to the design and implementation of instructional practice in real classrooms where the languages of the teacher, the learner and the mathematics are diverse. In order to be successful in making our research applicable to classrooms, we must build theory that draws on the experience of teachers and learners in practice. This implies communication and reflection with teachers, and collaboration with them in carrying out design experiments for the creation of powerful learning environments in classroom cultures more inclusive for all learners.

Implicit in classroom-based research is the fact that researchers are familiar with the classrooms they are looking in. This is not always the case when, for instance, they start their interpretations with interactions converted into transcripts by means of technical and personal support. However, it is difficult if not impossible to develop a sense and appreciation of the many layers of diversity and languages of mathematics, learners and teaching without visiting schools and classrooms. In this respect, some of the benefits of the so-called naturalistic approaches to classroom-based research should not be taken for granted, at least in what refers to capturing the naturally occurring classroom as it is. Even when researchers enter the research process from its very beginning through classroom observation, they may visit lessons with closed lists of categories expected to appear in the course of mathematics teaching and learning. Thus, some relevant issues regarding the socio-political dimension of language may be either ignored or finally noted as ‘other’. Spending more time in classrooms and visiting schools can help us to understand the extent to which reality is far diverse from what some ‘naturalistic’ research has envisioned in the past and still at present. The idea that languages are not discrete, monolithic entities strongly emerges when critically observing activity in lessons. One language, one culture and ‘average’ people simply do not work in representations of real classrooms with real learners and real teachers. It is about not only verbal languages representing several cultures, several languages and singular people, but about the diversity of body and visual languages of specific learners and teachers in interaction.

In the statements of the three panellists, we find a situated empirical research root and the proposal of change in focus from conventional naturalistic views toward critically views of classroom research. Richard has mentioned one of his ethnographic studies in mathematics classroom of second language learners, Jenni her work with various groups of teachers, and Susanne her developmental projects and design research program. The emphasis on collaborative professional development, design research programs and ethnographic studies with teams of researchers and teachers is a way of promoting classroom research that realistically—and not only naturalistically—represents the complexity of languages involved in mathematics teaching and learning. In dealing with this complexity, numerous artificial dichotomies are to be overcome, particularly those
separating languages into mathematical and non-mathematical, academic and colloquial, pure and mixed, average or normal and singular or exceptional, and so forth. Immediate collaborative environments – like those mentioned by Richard, Jenni and Susanne – are a great opportunity of expanding, improving and sharing our understanding of how mathematics classroom discourses work as well as how mathematics teachers think about their work, including their language use in teaching, and their learners, including their languages.

We may enter a research process – and a mathematics classroom – by assuming that the expert teacher will share our language of mathematics – or taking the example by Jenni, the same understanding of what an equation is –. This is an assumption that prepares the identification of certain meanings and the fabrication of certain categories for the interpretation of the language of the teacher. By emphasizing the role of learners for us as researchers, other aspects and categories different to those prepared in advance for observation might appear. This lead us to end with one more commonality in the words of the three panellists: the suggestion of newer roles for researchers and less transparent ways of producing research. In the move toward more realistically representing languages and speakers in mathematics classrooms, researchers need to place themselves as learners with different skills and knowledge to offer. They need to give visibility to their roles and intentions in the research process, as well as to the assumptions made about what community is being researched, which are their languages and why, so that classroom participants can also experience their roles and communicate their perceptions in ways that contribute to uncovering more and richer languages.

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Papers
Sources of meaning and meaning-making practices in a Canadian French-immersion mathematics classroom

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Research on the learning and teaching of mathematics in the context of language diversity has highlighted how students make use of many different features of language in mathematical meaning-making, such as gestures, code-switching, genres or narratives. I refer to these features as sources of meaning, to highlight their fluid, dialogic nature. Each student has a unique repertoire of sources of meaning, on which they draw in mathematical meaning-making. I argue that to understand the mathematical meaning-making process, we need to attend to the meaning-making practices they use as they draw on these repertoires during classroom interaction. I illustrate and reflect on these ideas with an episode from an elementary school French-immersion mathematics classroom in Canada.

Keywords: Language diversity, sources of meaning, French-immersion, Bakhtin, multilingual mathematics classrooms.

Introduction

In highly multilingual classrooms, an important feature of students’ meaning-making is the use of code-switching. Setati (2005), for example, showed that students in four South African classrooms used their home languages for conceptual matters, and English for procedural matters. In many parts of the world, however, even where students are familiar with multiple languages, only one language is used in mathematics classes (e.g., Barwell, 2005). This observation implies a difference between what may be available to students for mathematical meaning-making, and what they actually use. Research has identified many other aspects of language which students may use in mathematical meaning-making, including graphs, gestures, genres, narratives, grammatical features and multiple meanings (e.g., Moschkovich, 2008; Barwell, 2005, 2014). These features of language are often referred to as resources, a term that implies a static view of language (as a fixed resource) and a monologic view of students (as users of the resource) (Barwell & Pimm, 2016). I refer instead to the sources of meaning students draw on in their mathematical meaning-making (Barwell, 2018), based on a Bakhtinian view of language use. Research on the sociolinguistics of multilingualism has adopted a similar approach, demonstrating the fluidity of language use in contexts of superdiversity (Blommaert & Rampton, 2011). Speakers are shown to use repertoires of languages, styles and genres, which they combine in particular situations to make various kinds of meaning, including the construction of identities, building relationships and identifying with particular groups or subcultures (see for example Blommaert, 2010). In this paper, I argue that to understand students’ mathematical meaning-making in contexts of language diversity, it is necessary to examine their repertoires of sources of meaning and the meaning-making practices through which these repertoires are activated.

Sources of meaning

From a Bakhtinian perspective, language is a dynamic constantly changing system of shifting relations. Although it is convenient to talk about recognisable patterns of language, such as French
or Japanese, or mathematical discourse, such labels are at best approximate. Any recognisable feature of language is in constant evolution. This evolution is driven by two opposing forces, known as centripetal and centrifugal forces (Bakhtin, 1981). Centrifugal forces are apparent in the continually changing nature of language, and its constant diversity, known as heteroglossia. Centrifugal forces are necessary, since without the possibility of change, it would be impossible to ever saying anything new, and since every utterance is in some sense says something new, all communication would be impossible. Centripetal forces arise from widespread ideologies about language as a fixed, rule-based structure, apparent in every dictionary or every time a grammar rule is cited. Again, centripetal forces are necessary, since without some degree of standardisation, communication would become a tower of Babel: many different languages, but no mutual understanding. These two forces operate in every utterance, shaping how things are said, so that every utterance to some extent follows standard forms of language, while also being a unique instance of language-in-use (Bakhtin, 1981). In previous work, I have shown how the tension between these two forces can be seen in multilingual mathematics classrooms (Barwell, 2014, 2016). Students talk about mathematics using a variety of accents, pronunciations, languages, words, grammatical forms, gestures, generic fragments and so on. Their meaning-making is also shaped by centripetal forces that imply preferred forms of mathematical discourse, classroom language, pronunciation, accent, correct spelling and so on. Students’ thinking and learning emerges through these interactions and is shaped by these two forces. The relations between a given feature of language influenced by this tension amounts to a source of meaning. For example, the relation between formal and informal geometric vocabulary results in a source of meaning, consisting of more formal and less formal terms and the various meanings that arise from the relations between these words (see Barwell, 2016).

Sources of meaning can be organised around three dimensions of heteroglossia: languages, discourses and voices (from Busch, 2014). Students can draw on a repertoire of languages, including the classroom language and any other languages they may know. Students can also draw on a repertoire of discourses, including multiple mathematical discourses, multiple educational discourses and multiple informal discourses. Mathematical discourses might include various versions of the language of algebra, statistics, geometry, etc., as well as various degrees of formality, and the language of different genres, such as textbooks, tests, classroom talk, etc. Multiple educational discourses include various institutional ways of interacting in classrooms, talking about the curriculum, discussion of assessment, etc. Multiple informal discourses refer to the many ways of talking students bring from outside of formal schooling, including discourses of the family, community, popular culture, etc. Students can draw on multiple voices, including the expressions of mathematical meaning of their teacher, their peers, their textbooks, members of their family, personalities in the media, and so on.

Although not previously organised in relation to these three sources of meaning, research has provided descriptions of language use that illustrate them. Much research, for example, has described the presence of multiple languages in mathematics classrooms. Setati’s work (e.g., 2005) has documented the use of home languages in South African classrooms in which the language of instruction is English. Moschkovich (2008) has documented the patterns of use of Spanish and English in US mathematics classrooms. Planas (2014) has examined the use of Catalan and Spanish in mathematics classrooms in Barcelona. Equally, research has documented the use of only one language, even where students may be familiar with others (e.g. Barwell, 2005). Research has also documented the different discourses used by students in multilingual mathematics classrooms. For example, in a study of the engagement of elementary school learners of English with arithmetic word problems in the UK, I showed how they drew on features of the genre of word problems, as well as narrative accounts of everyday experience (Barwell, 2005). Genres and narratives were thus sources of meanings that were part of the students’ discursive
repertoires. Similarly, in work conducted in Canada, I have shown how textbooks mediate second language learners’ meaning making in mathematics (Barwell, 2017). This analysis implies that the discourse features of mathematics textbooks are sources of meaning that form part of students’ discursive repertoires. Finally, the literature includes examples of multiple voices in mathematical meaning-making. A good example can be found in Moschkovich’s (2008) analysis of what she terms ‘multiple meanings’ for features of a graph that were a source of meaning for two students as they worked to make sense of the graph. Students and the teacher “revoiced” each other’s ideas, so that mathematical meaning arose from the interaction between their voices, even within a single utterance. In my own work, I have shown how students’ take on the words of their teacher (and vice versa) and these voices become intermingled in their mathematical meaning-making (Barwell, 2016). This interaction between a repertoire of available voices creates a relationship between students and others, including their peers, teacher, and textbook (Barwell, 2017).

Sources of meaning in use: Meaning-making practices

The concept of sources of meaning, organised into languages, discourses and voices, is a useful way to understand what students draw on in mathematical meaning-making. Comparative studies, however, have shown that there may be significant variations in how students actually conduct meaning-making in multilingual classrooms, even when their repertoires appear to be similar. In one analysis, for example, students in a classroom in Canada, two classrooms in South Africa and a classroom in Malaysia, all appear to have repertoires of multiple languages, and yet in some settings, one language predominates, while in others a mixture of languages may be used (Barwell et al., 2016). To make sense of such situations, a description of students’ repertoires of sources of meaning is not sufficient. We must also examine the meaning-making practices (what participants do) through which these sources of meaning are deployed. To illustrate this point, here are three examples of practices:

1. **Code-switching.** There is a fairly extensive literature that documents the use of code-switching in mathematics classrooms (e.g. Planas, 2014; Setati, 2005). Code-switching is a meaning-making practice that draws on multiple languages as a source of meaning. This source of meaning is available in many classrooms, but the practice of code-switching is not used in all of them (e.g., Barwell, 2005).

2. **Scale-jumping.** Although not widely reported, scale-jumping is likely to be common in mathematics classrooms: it involves indicating when mathematical formulations are too ‘local’ and prompting students to use more widely recognisable formulations. One example in Barwell (2014) is when a student is prompted to rewrite an explanation in a way that will make sense to his teacher, not just to himself. That is, he is prompted to align his writing with a more widespread discourse of explanation. Scale-jumping is thus a meaning-making practice that relates to multiple discourses as a source of meaning.

3. **Revoicing.** The revoicing of students’ contributions by the teacher is common in many classrooms, including multilingual classrooms and students may also revoice and reformulate each other’s utterances (e.g. Moschkovich, 2008; Barwell, 2016). Revoicing is a meaning-making practice that draws on multiple voices as a source of meaning.

In the following section, these ideas are illustrated with the example of a mathematics classroom in which two languages are regularly used.

An illustrative example: A French-immersion mathematics class

Canada has two official languages, English and French. Thus, schooling is available in each language throughout the country. In addition, many anglophone school boards offer French-
immersion programs, in which anglophone students follow some or all of the curriculum in French, with the goal of developing a high level of proficiency in that language. The classroom described in this paper is a Grade 3 French immersion class in Ottawa, Canada. In this immersion program, students study mathematics in French until the end of Grade 3. As part of a comparative ethnographic study (see Barwell 2014, 2016, 2017), classroom observations, fieldnotes, audio recordings and photos of classroom artefacts were collected on eleven occasions in the spring of 2012. I have selected one lesson with a focus on capacity, the second lesson in a unit on measurement that started the day before. The teacher first revises various units of measurement with the class, and also works with a poster depicting a container on which one litre is marked. The class discuss where to mark 250ml, 500ml and 750ml, as well as various connections between words for quarter, half, three-quarters, 25¢, 50¢, 75¢ and other related ideas. The students are then to complete a worksheet in pairs. The teacher introduces the worksheet in some detail, reading out each question and in some cases elaborating on what is expected. Students then work in pairs to solve the problems. In what follows, I summarise the sources of meaning and associated meaning-making practices used in the teacher-led portion of the lesson, as well as in the work of one pair of students, as identified through my analysis of the data.

**Teacher-led presentation of the worksheet**

The main sources of meaning relating to *languages* arise from a shared repertoire of English and French. The teacher also spoke Greek and several of the students spoke other languages at home. In the teacher-led portion of the lesson, the main meaning-making practice relating to multiple languages was a pattern in which students sometimes inserted English partly or entirely in their responses to the teacher’s questions, and the teacher would respond in turn in French. That is, he would focus on the students’ meaning rather than their language. On one occasion, another meaning-making practice was observed: the teacher provided a direct translation of a word arising in one of the problems:

\[d) \text{ it says “solve the problems” so the first problem is “a bucket can hold” a bucket in English is a bucket so “a bucket can hold three litres of sand how many litres of sand are needed to fill five buckets?” }\]

These meaning-making practices are similar to code-switching, although more fluid and less structured than formal code-switching.

Sources of meaning relating to *discourses* included the word problem genre, the main form of the problems on the worksheet. A key meaning-making practice related to this source involved several components to support reading and interpreting the problems: first the teacher would ask a student to read out one of the problems, which they would generally do quite disfluently; next, the teacher would read it out again himself; finally, some interpretation of the problem was usually conducted, either through direct comment from the teacher, or through interaction between the teacher and the class. In one case, the solution to the problem was actively discussed. Connections between explicitly rehearsed vocabulary relating to units of measurement, and specifically capacity, formed a second source of meaning. Several meaning-making practices involving explicit attention to vocabulary drew on this source: vocabulary was introduced in the previous lesson, reviewed at the start of the second lesson, and the teacher made explicit connections to this vocabulary when going through the worksheet. This rehearsal of mathematical vocabulary is related to scale-jumping, since it introduces more widely used forms of mathematical discourse. For example, after a student has read out the second problem, the teacher follows up:
Donc « Samuel demande chaque récipient a une capacité d’un litre utilise les mots donnés pour décrire le montant d’eau que chaque récipient contient » oh regardez ces trois moi je vois trois mots ici qui sont au tableau est-ce que tout le monde peut les voir?

So “Samuel asks each container has a capacity of one litre use the words provided to describe the amount of water each container holds” oh look these three I can see three words here that are on the board can everyone see them?

Thus, the teacher provides some interpretation of the problem and also highlights a connection with vocabulary discussed at the start of the class.

Sources of meaning relating to voices include the interactions between students’ and teacher’s voices, as well as voices carried by the word problem, both as an authored mathematics classroom text, and as texts that often feature fictional children. In the problem cited in the quotation above, for example, the problem refers to a fictional Samuel. Meaning-making practices relating to this source of meaning include the teacher’s revoicing of students’ more hesitant reading of the problem, as well as the teacher’s revoicing of his own earlier presentation of vocabulary.

Students’ work

In the interaction between Kyle and Sara, two students recorded during the work on the worksheet, the sources of meaning relating to languages were identical to those during the teacher-led part of the lesson. The meaning-making practices they used were, however, very different. The majority of the students’ interaction was in English, while their use of French was when one of the students read out the problem or referred to some element of it in their discussion of their solutions. French was generally used in direct quotation of the word problems, while English was used to interpret the meaning of the problems and discuss solutions, as in the following extract (they are working on: “Joseph drinks a half litre of milk each day. How many days are needed to finish four litres of milk?”):

Kyle : okay what the heck is this? « Joseph boit un demi-litre de lait par jour combien de jours » [Joseph drinks a half-litre of milk each day how many days]
Sara : un deux trois [one two three]
Kyle : aaahhh so four litres is like up to here (.) four litres is two sizes of this if he drinks that
Sara : (...)
Kyle : on my calcul[ulations] Sara : [but that’s a half of it already (.) two
Kyle : oh I know yeah (.) I know what this is it’s um ah grrr (3.0) I know it’s something around (.) to drink ’kay Joseph (3.0) oh demi-litre two c’est deux um litres [oh half-litre two it’s two um litres] (writing) « deux litres » (2.0) just do two and a capital L

In this extract, the problem is read out in French, while aspects are interpreted in English, such as when Kyle says “it’s something around (.) to drink”. At the point at which Kyle seems to be moving towards a solution, he code-switches back to French, coincident with the act of writing his solution, which must also be in French.

In relation to discourses, the word problem genre is again a prominent part of the students’ repertoire. Unlike with languages, however, the students adopt similar meaning-making practices to the teacher: they often read out the problem and then interpreted it with subsequent discussion, as the above extract illustrates. An additional meaning-making practice involved students making reference to their own experience of the world to interpret the word problem. For example, in the next turns following the above extract, the following exchange occurs:

Sara: combien de jours (.) [oops (how many days (.) oops)
Kyle: [no no if it’s if he does if how if he does a deux a half of a milk carton a day if he does one like you know how you have those you guys have you have mini
Sara: yeah
Kyle: [milk cartons that’s a half a demi-litre [half-litre] () if you drink four of those how much is it?

Kyle refers to a particular kind of 250ml milk carton as a point of reference to interpret “demi-litre”, although he seems to have proposed a non-standard interpretation of the problem. Finally, in relation to voices, the two students revoice parts of the word problem. In particular, Sara’s revoicing of “combien de jours” (how many days) indicates some doubt about Kyle’s proposed solution. Kyle certainly hears it in this way, since he revisits and expands his explanation.

**Discussion**

The brief example I have presented illustrates the value of the distinction between sources of meaning and meaning-making practices related to these sources of meaning. Most notably in the lesson in question, teachers and students have overlapping language repertoires of English and French, but they use quite different meaning-making practices in drawing on these sources of meaning. The teacher draws almost entirely on French, offers only occasional English glosses of key vocabulary, and generally does not comment on students’ use of English, simply focusing on the mathematical meaning of their utterance and responding in French. Kyle and Sara, in contrast, work largely in English, using French mostly to read out word problems or other instructions, or to formulate their written responses. In relation to discourses, on the other hand, and to some extent in relation to voices, teacher and students use similar meaning-making practices. For example, both teacher and students adopt the practice of reading out parts of the word problems and then interpreting them through subsequent discussion. As already noted, however, in the case of the teacher, all of these practices were conducted in French, while the students switched from French to English during this process.

The meaning-making practices observed in this class are shaped by the centripetal and centrifugal forces of language. Centripetal forces include the institutional preference for French and the broader societal dominance of English. The tussle between these languages results in a distinctive heteroglossia in which a shared repertoire of English and French is asymmetrically deployed according to the participants’ roles: French for the teacher, English for the students. This situation is relatively unusual, in that in many contexts described in the literature, English is the dominant language and is imposed in the classroom over students’ less valued local languages. The tension between the two language forces can also be seen in the realm of discourse, such as when students read out the word problems with hesitation and disfluency, while the teacher reads them clearly and offers interpretations reflecting the preferred or conventional meaning. Indeed, it is important to see the interaction between languages and discourses here: the institutional, centripetal imposition of French produces the students as disfluent. Had the problems been presented in English, it is likely that the students would be able to read them out loud with greater fluency. Sources of meaning, then, arise from the relations between various features of language. In this lesson, it is not simply the repertoire of English and French that provides a source of meaning; it is the constant interplay between utterances in these two languages. Students and the teacher make use of these relations to interpret the word problems (sometimes more successfully than others).

In conclusion, to develop a comprehensive understanding of how language is used in mathematics classrooms in contexts of language diversity, attention needs to be paid to the sources of meaning available to students and teachers, and to the meaning-making practices that bring these sources of meaning into play in different ways. This approach needs to examine languages, discourses
and voices, not least because these different dimensions interact in important ways in students’ and teachers’ repertoires and thus influence students’ learning of mathematics.

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References


Language use in different situations in the mathematics classroom: Everyday academic language and mathematical discourse

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During the last decades, the role of language in mathematics and mathematics education has increasingly caught the attention of many researchers in the field of education, teaching and linguistics as well. Within this context, a frequent use of the term “Academic Language” (AL) is noticed to label a variety of instances of language. However, today there is no universal or agreed definition for a possible exact meaning of AL in educational research. The present study in mathematics education investigates learners’ discourse competences within different situations of classroom practice, and addresses the following question: How does the use of language and discourse change during different situations in the mathematics classroom?

Keywords: Academic language, discourse, language register, interactional analysis.

The importance of language for (mathematics) learning

The role of language in the mathematics classroom is controversially discussed relative to the underlying traditions and purposes of research works in this field of research. This paper gives a short overview of the importance of language for learning in a (mathematics) classroom and shows that especially language competences in Academic Language (AL) seem to have significant influence in the learners’ mathematical achievements. But while lexical and semantical aspects of AL attract attention in many studies, discursive aspects have only received the necessary attention in more recent studies (e.g., Moschkovich, 2007/2018; Sfard, 2012; Erath, 2016; Quaschhoff & Morek, 2017; Schütte & Krummheuer, 2017; but not only). A closer look at specific discursive phenomena within the mathematics classroom could clarify how crucial the situation for the learners’ use of language might be as well as in which degree their use of language changes during different situations of interaction.

Many investigations proved the close connection of language-based and subject-based learning during different classroom activities and the concomitant educational success (e.g., Townsend et al., 2012). Three meaningful aspects of language within the classroom could be identified: It is the central medium for teaching and learning processes. To actively participate in school, pupils need to hold special language-based competences in order to, for example, read and understand mathematical texts, to follow explanations of the teacher, or to give an explanation to classmates. But not all learners bring the same language skills into the mathematics classroom, with the result that not all of them are able to equally participate. Language, according to that, is not just a medium to negotiate specialized mathematical contents, but an essential precondition for learning (often taken for granted) and a learning target as well. Indeed, several researchers in the field of mathematics education and migration research argue that worse school achievements cannot solely be explained with language-based competences in Everyday Language (e.g., Schütte, 2014). Especially these linguistic competences that play a role in educational contexts seem to be relevant – this is closely related to the term Academic Language.
**Academic language**

Closely linked to the focus on language in the context of school and mathematics education, the term *Academic Language* (AL) is frequently used. Despite many academic examinations, there exists a variety of synonymously used terms. At the moment, (in Germany) an empirically-based specification of the characteristics of AL as well as models to describe the AL competences of the learners are not available (Heppett, 2016). Additionally, normative views and considerations lead the discussion and open up dichotomous distinctions such as Everyday versus Academic Language or formal versus informal. Frequently, AL is seen as a language register that is used in the context of school and education in order to impart knowledge, and that orients itself by written language with its higher degree of complexity and explicitness (Gogolin & Lange, 2011). Moreover, in a normative way AL is seen as „that linguistic register, whose mastery is expected from a 'successful pupil’” (Gogolin & Lange, 2011, p. 111, translated by the authors). Schütte (2014) has shown that educators and teachers in kindergarten and primary school do not act as linguistic role models. AL often is not a learning target, since in his examination he could hardly find any situation in which linguistic learning was being made explicit and oral communication was mostly implemented in Everyday Language. “The children are therefore unable to learn linguistic skills [in AL] related to the mathematical concepts” (Schütte, 2014, p. 936). Nevertheless, AL remains to be a central and meaningful precondition for learning, in mathematical texts and achievement tests as well. Students are expected to communicate in an appropriate way during lessons. For example, they have to explain and justify mathematical solutions or to answer the teachers’ questions in a correct manner. “All these activities are accomplished not only by using certain syntactical constructions and academic vocabulary but within situated communicative practices …” (Heller, 2015, p. 1)

**Everyday academic language and mathematical discourse**

If we consider the fact that learners are confronted with AL in many (or even all) mathematical contents and classes, we could consider that AL in the mathematics classroom could be seen as an *Everyday Academic Language*, since students get familiar with AL and its norms when participating in lessons every day for many years. Also following Moschkovich (2018) research on language and learning mathematics needs to “move away from simplified views of language as vocabulary […] [and instead] recognize language as a complex meaning making system” (p. 38). Through the perspective that meaning is negotiated in social interactions, learning is seen as a social and co-constructive process. In this regard, language can no longer be seen solely as medium, precondition and learning target but also acquires a central significance, if not *the* central significance in the building of mathematical knowledge and the development of mathematical thought (Schütte, 2018). During classroom activities, students use multiple resources from their experiences inside and outside of school. Therefore, it is important to avoid the construction of Academic and Everyday Language as a dichotomous distinction, because it depends on how we define these two types of language, respectively discourse (Moschkovich, 2007). In this regard, the term *Everyday Academic Language* seems to underline the fact that students (and teachers as well) do not solely use “the” AL but rather a mix of multiple resources from different language registers (e.g., Schütte & Krummheuer, 2017).

“Since there are multiple mathematical Discourse practices, rather than one monolithic mathematical Discourse, we should clarify the differences among multiple ways of talking mathematically […]” (Moschkovich, 2007, p. 28). That is why this study aims to consider the use of the learners’ language during different situations within the mathematics classroom. Moving away from such dichotomies could help to suggest mathematical (classroom) discourses as a hybrid of different discourses with co-existing registers. As already mentioned, lexical and semantical characteristics of AL have been explored in many studies and research projects (e.g.,
Townsend et al., 2012; Uccelli et al., 2015) and discursive characteristics are less common at the center of attention. Some research efforts focusing on aspects of discourse are Quasthoff and Morek (2015) or Erath (2017). The former were able to show, regarding the discourse practices explanation and argumentation, that many learners do not gather enough language-based experience in family and peer groups prior to entering school. But often, such language competences in explaining and arguing are assumed (Quasthoff & Morek, 2015). Erath (2017) could show, that the discursive practice of explaining varies regarding different micro cultures, respectively classes. Depending on the class, different expressions are considered suitable. Most studies on discourse in mathematics define “discourse practices” or “discursive norms” with the focus on selected linguistic activities like explanations and arguments. However, it is differently discussed which linguistic features rank among discursive ones. Mostly it is stated that the construction and organization of such texts, which are used for the realization of specific school-based language actions (like report, presentation, discussion), has to fulfil specific conditions. Language-based actions like descriptions, explanations, comparisons, and argumentations are frequently listed (e.g., Bailey, Butler, LaFramenta & Ong, 2004; Vollmer, 2010).

The presented study takes up a broader view on mathematical discourse and the learners’ language (like Moschkovich, 2018; Sfard, 2012; and Gee, 2005), as well as on the term Everyday Academic Language, instead of opening up the dichotomy between Everyday versus Academic Language. It should not be self-evident that AL is used in all situations or by all learners in the same way to share meaning and knowledge. A more elaborate language does not necessarily have to result in a greater learning success; and even if the situation could be characterized as an educational or mathematical discourse, the learner’s language might occur as Everyday Language or less explicit, elaborate and decontextualized, but with a high impact on the learning success for the learners. In this sense, it is not enough to know what a (mathematical) word means. Learners should be able to make sense of ways in which the word is used or put together with other meaningful words and phrases to constitute a mathematical meaning and to express conceptual understanding (Moschkovich 2018). The events in the mathematics classroom present different and manifold language-based challenges to pupils and it often depends on the situation and the participants itself, if a given statement is seen as appropriate and suitable or not.

For this research project, the underlying concept of D/discourse is the one of Gee (2005; 2015). Following Gee, “language is a tool for three things: saying, doing, being. When we speak or write we simultaneously say something (inform), do something (act), and are something (be).” (2015, p. 1). In order to be recognized as a member of a community or a Discourse, it is not enough to “talk the talk” – somebody also has to “walk the walk” (Gee, 2015, p. 1). For example, being recognized as a high-achieving student in mathematics depends on the situation itself, because what works in one setting does not work in other settings. It is not enough to speak in an appropriate manner and use a prepared list of vocabulary. One also needs to behave in an adequate way. Different situations create different opportunities to behave and the classroom participants themselves could be seen as an own micro-culture that generates special ways of “doing mathematics”. Closely related to this, Gee (2005) distinguishes between discourse (with small-d) and Discourse (with capital D). The former he defines as language-in-use among people. In this sense, we are interested in “how the flow of language-in-use across time and the patterns and connections across this flow of language make sense and guide in interpretation” (Gee, 2015, p. 2) to build identities. However, such identities or activities are rarely enacted only through language as with non-language aspects. In this way, Discourse (with capital D) is defined as language plus other “stuff” (Gee, 2005, p. 7), by what he means things like gestures, bodies, interactions and beliefs. Every situation creates specific language-based requirements as well as appropriate/inappropriate and suitable/unsuitable possibilities to use language. The analysis of
Big-D Discourse embeds the former type of discourse analysis (with small d) “into the ways in which language melds with bodies and things to create society and history.” (Gee, 2015, p. 2)

Since the presented research takes up a micro-sociological perspective, I therefore use the term Discourse to signify both, language-in-use as well as some “other stuff” like gestures, actions and other context-sensitive aspects. The prime focus is on the language of students in use among different situations (discourse) but it seems also helpful to take into account other influencing factors (Discourse), like the social formation, the learning content or the existence of visual aids.

**Main goals of the study**

To clarify the importance of the situation for the learners’ language, empirical evidence would be instrumental for educators, policy makers, and funding agencies alike. To modify the existing dichotomies of Everyday versus Academic Language, respectively a “worse” and a “better” way of speaking in an academic context, a wider perspective on language is necessary. To see language as a process rather than as an inflexible object, as posed by Moschkovich (2018):

…. studies need to consider what mathematical knowledge and discourse practices learners use in different settings, what knowledge and discourse practices learners use across settings, and how to make visible the ways that learners reason mathematically across settings (p. 39).

Consequently, one main goal of this research project is to identify which mathematical Discourses exist during different situations and which special language-based discursive opportunities as well as requirements arise from this for the learners. This resembles the two sides of the same coin: on the one hand, the opportunities and possibilities to speak and behave could be very diverse, on the other hand, there could be limiting expectations as well as (implicit) rules and norms. Following Schütte (2014), demands regarding AL have seldom been made explicit by the teachers and are often being taken for granted. “It is certainly desirable for all participating children to be given an introduction to formal and mathematical linguistic aspects, and for the teacher to act as an explicit role model in this regard” (p. 936). Moreover, it is useful to bear in mind that specific classroom situations lead to the emergence of different ways of using language depending on, for example, the age of the learners or the learning content. A changed view on the use of language within the classroom would affect the production of more specific materials and methods to foster the learners’ language competences.

**Methods and research questions**

Based on a social-constructivist view on learning, mathematical and linguistic learning is seen as a collective and interactional process as it is typical for symbolic interactionism (cf. Blumer, 1969). One main assumption of this approach is that mathematical and linguistic meaning emerges and develops during communicational processes (e.g., Cobb & Bauersfeld, 1995; Krummheuer, 2011). Learners participate in different manners during different situations. To achieve successful mathematical and discursive competences, they use diverse resources from different registers (Moschkovich, 2018; Schütte & Krummheuer 2017). Despite the high theoretical and practical emphasis on improving the learners’ language skills (in AL) and despite several research efforts in this field, there are less investigations existing to describe AL, respectively discursive (Academic) Language requirements for different social formations, learning contents and class levels (Heppt, 2016).

However, opposite to previous and current research opening up dichotomous distinctions such as Everyday/Academic Language under a normative manner, I tend to describe the use of *Everyday Academic Language* during different situations in the mathematics classroom without passing judgement on “worse” and “better” ways of using language. These dichotomies are not consistent with the current assumption that “everyday and academic practices are intertwined and
dialectically connected” (Moschkovich, 2018, p. 39) and do not fit a social-constructivist view on learning. I suspect that one person uses language in different ways concerning the situation. By considering the interaction of the learners during different mathematics classroom situations, linguistic and discursive particularities should be identified. There are many influencing factors that may have an impact on the learners’ language activities, such as the following: The social formation of the learners (whole class discussions in which the teacher mostly has an outstanding position of regulating, determining and moderating the discussion in both ways, language and organization; or phases of group or pair work which seem to be more intimate). The (non-)existence of the teaching person or another audience. The (non-)existence of illustrative learning material for visualization what could affect the explicitness and clarity of the learners’ statements. To gain a broad impression of the learners’ language use during different situations of the mathematics classroom and to call into play as many situations and micro cultures as possible, it is intended to contemplate different school types and class levels. Following these fundamental theoretical concepts and ideas, this study project aims to find answers to the following questions:

- How do the language-based contributions of the learners differ in varying situations of the mathematics classroom, respectively during different mathematical discourses?
- Which scope of action and opportunities for language use can be identified during different situations of the mathematics classroom?
- Which (academic-)language-based requirements, conditions and challenges go along with that and how do the learners fulfill them?

To answer these questions, mathematics lessons of the several classes were video-recorded from 2017 to 2018. The duration of the recordings in each class varies from two to four weeks. To underline differences and similarities it is planned to observe different school types and grade levels. Afterwards, selected lessons and passages were transcribed and analyzed via a linguistic analysis on selected language-based aspects and via interactional analysis (Krummheuer, 2011) to illuminate how the situation and its opportunities and requirements in speaking and behaving could be characterized and if mathematical and linguistic meaning emerges.

**Initial results**

Until now, the database of mathematics lessons encompassed recordings from a 1st, 2nd, 3rd and 4th class in primary school, a multi-graded high school class 7 and 8 with children between 12 and 15, and one 12th class of a mathematics intensified course (between 17 and 18 years old). Many obviously interesting and meaningful scenes are already transcribed. The following excerpt is from a class discussion in grade one and the topic is about the relation terms “greater than” (>), and “less than” (<). The students already know some tasks and the teacher (T) is now initiating a discussion about the formal expression of the terms with the help of a narrative about a crocodile named “croco” and cubes on the board.

T: Our little croco always want to eat a lot. That’s why is mouth is open that wide. And now he comes and thinks about ‘Shall I eat the red ones or the blue ones?’ What do you think, Ina?

Ina: I think red.

T: You think red [turns croco with the open mouth to the two red cubes]. Why?

Ina: Red is like meat.

T: Aha. That would be a consideration. Nabil, what do you think he wants to eat?

Nabil: Blue?

T: You say blue is what he wants to eat, why? … [Nabil does not say anything for 4 seconds]. Simply because blue is beautiful. Okay. Rich, what do you think?

Rich: Ehm. He wants to eat red because it is like meat and fish.
Nagi:  Blue. Because that is more.
T:  That is our little croco who always wants to eat the most and that’s why he looks here [turns the crocodile between the towers and cubes that it looks to the four blue ones, writes a “>” between the four blue and the red cubes and again places croco between them]. Can you see this? Because he always wants to eat what is more.

First of all, we can see that the first two children, Ina and Nabil, were asked by the teacher for an explanation of their given answer. In contrast, the last two children who gave answers, Rich and Nagi, gave these explanations by themselves. We could imagine—that they recognized the specific demands and requirements of the situation, that giving an answer is not enough and instead, some remarks about the “Why” are necessary. In addition, this is typical for educational contexts.

Second, the extract shows that the teacher packs the mathematical content into a narrative, what seems to lead to student answers which are centrally oriented towards the story and less towards the mathematical content. This typical IRE-pattern (Initiation, Response, Evaluation) continues until Nagi gives a satisfactory answer with a short justification about the mathematical insight which is less oriented to the story. Ina and Rich, instead, seem to be “caught up” in the narrative and try to argue for “red” as it is similar to the color of raw meat and fish. Although it is visible that there are more blue cubes on the board, two children argue for red, what leads to the assumption that they are too much fixated to the story about croco. However, it remains unclear whether the children could see the mathematical concept behind the story, since only Nagi contributed something mathematical to the situation and the rest of the students' utterances were superficially oriented toward the story of croco and the colors of the cubes. The teacher did not guide the children in one direction and the idea of color is seen as one possible way to answer the question of what croco wants to eat. These results are comparable to Schütte and Krummheuer (2017) who found that if the (mathematical) content is “packed” in a narrative, the interpretations of the learners and their verbal utterances could be bound to the story. This could hinder them to gain mathematical-based interpretations and to understand the underlying mathematical construct.

In sum, it can be stated that in the presented transcript it was reconstructed that the learners’ language is affected by the situation, especially the existence of visual aids and the “packing” of the learning content into a narrative. In this view, interaction could be seen as a “discursive practice, primarily structured by the social action it forms, rather than by its content” (Barwell, 2003, p. 201). Telling the story about croco seems more important than the mathematical structure of “less-than” and “greater-than” that goes beyond the story. In this regard, we can identify a central challenge for early childhood educators: On the one hand, it seems necessary to “build up” a story about a mathematical content. On the other hand, this story could be an obstacle for the children, preventing them from to “seeing” the mathematical content behind.

References


Diversity of teachers’ language in mathematics classrooms about line symmetry and potential impact on students’ learning

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In the continuity of our previous research on the impact of teaching practices on the variability of learning among students, we present a first step in order to investigate teachers’ discourse more deeply. We identified crucial issues, including linguistic ones, in the conceptualization of line symmetry, related to logical aspects of the concept. This exploratory study consisting in the analysis of textbooks content and some classroom sessions suggests that these learning issues are globally hardly considered but that there are some differences in the ways teachers address them, which are likely to have differentiating impacts on students’ learning.

Keywords: Teaching practices, language of the mathematics teacher, line symmetry, logical analysis of mathematical concepts.

Introduction

Our concern about language issues arose from previous research about relations between teaching practices and social inequalities in mathematics learning (Chesnais 2012, 2014, in press). Firstly, our theoretical framework for studying teacher practice and its impact on student learning is grounded in the theory of Vygotsky (1986), which indicates the importance of language as an object of learning but also as a means for conceptualization, and finally as one of the main tools of teacher activity. Secondly, as sociological research shows, language plays a crucial role in the construction of learning inequalities and their relation with the sociocultural background of students (Rochex & Crinon, 2011).

This concern about the role of language in mathematics education meets a recent preoccupation in the French community of research in didactics of mathematics (Artigue et al., 2017). The original feature of our work in the French context lies in the use of Vygotsky to investigate these questions, altogether with our interest in analyzing “ordinary” teaching practices. It also seems to largely echo some vivid preoccupations of the international – and particularly ERME’s – community of research in mathematics education (Pimm 2004; Radford & Barwell 2016; Planas, Morgan & Schütte, 2018), questioning the three lines “classroom discourse”, “language diversity” or “conceptualization through language” (Planas, 2016). Our originality in this landscape comes from our theorizing of the process of teaching and learning mathematics and the specific way it leads us to question classroom discourse. Firstly, our use of mathematics logic to analyze mathematical discourse seems original in the field even if it may have some common ground with some other approaches (Pimm, 2004). Secondly, our investigation of “ordinary” teachers’ practices with a combined didactic and ergonomic point of view (grounded in Vygotsky, Vandebruck, 2013), leads us to consider ordinary teachers’ discourses with a “naturalistic approach” when most of the research seems to focus more on the design of experiments. Finally, our hypotheses about mathematics learning, based on a combination of Vygotskian and Piagetian theories involve a way of investigating teachers’ discourse that differs from other research
influenced by cultural-historical theories (e.g., the work of Sfard 2000, 2001 or Radford, 2013). We consider that teacher telling has a role to play (Chesnais at al., submitted), which includes offering lexical means to support student activity (Pöhler & Prediger, 2015) but not solely.

We are particularly focusing here on the “logical aspects” (the arity of properties and relations) of line symmetry and reflection and the related linguistic aspects (Chesnais, 2009; Barrier et al., 2014; Chesnais et al., 2017), as we will detail below. We previously showed that these issues generate many learning and teaching difficulties in 6th grade (first grade of secondary school, 10-11 year-olds) in France (Chesnais et al., 2013; Barrier et al., 2014; Chesnais et al., 2017). The studies we are presenting here are part of two research projects: one about the transition from primary to secondary school, funded through the “chercheurs d’avenir” campaign of the Region Languedoc-Roussillon and one about the role of language in the learning of mathematics and sciences funded by the ESPE-LR (superior school of teacher training).

The questions we tackle in this paper could then be phrased as: How do 6th grade mathematics teachers deal with linguistic issues related to logical aspects of line symmetry and reflection? Are differences among teachers’ practices likely to have a differentiating impact on students’ learning? First, we present our theoretical framework and its methodological implications. Second, we expose main findings of a study of textbooks contents and some classroom sessions and we detail a demonstrative example of variability among teaching practices.

Theoretical framework and methodological implications

Our theoretical framework is based on Activity theory adapted to mathematics teaching and learning in a school context (Robert & Rogalski, 2005; Vandebrouck, 2013). The main hypothesis is that learning results from students’ activity which results (mainly) from the tasks the teacher chooses for students and the way he/she implements them in the classroom. Learning is then characterized as conceptualization, operationalizing in a way Vergnaud’s definition of concepts, based on the theory of schemes (Vergnaud, 2009): conceptualization (as a product) of a specific piece of knowledge is characterized by its “availability” in situations in which it is relevant, with the corresponding “operatory invariants”, plus its integration in the network of previous knowledge, and the use of associated “signs” (in particular linguistic ones). Managing specific linguistic signs is then consubstantial with conceptualization. Moreover, in line with Vygotskian ideas, signs (in particular linguistic ones) play a role in the conceptualization process. Our hypotheses on the role of the teacher in this process, resulting from combining of Vygotskian and Piagetian ideas, is that she has to support the activity of students to solve mathematical tasks, but also to ensure that the solving of mathematical tasks actively supports opportunities for conceptualization. The role of teacher telling seems crucial to support the dynamics between general and contextualized knowledge (Chesnais et al., submitted), in particular when tasks are not didactically very robust, which is often a reality. Hence, what we investigate in teachers’ discourse is how the linguistic elements the teacher offers to students constitute a crucial factor but also the way they contribute to construct “proximities” (Robert & Vandebrouck, 2014) between students’ real activity and the aimed knowledge.

Our methodology starts with a preliminary study of the mathematical content at stake. We chose to particularly focus on the linguistic issues related to logical aspects of line symmetry and reflection. In this specific purpose, we use logical analysis of language (Vergnaud, 2009; Durand-Guerrier, 2013). Our study of teaching practices starts with an overview of French textbooks. We consider them as “approximations” of what might be taught in classrooms and how, since textbooks in France are often written by teams including teachers (if not exclusively composed by them) and they are an important resource for teachers, even if their use is not voluntary but compulsory. Studying textbooks then allows us to access in an economical manner a wider view
on potential teaching practices – even if it necessitates confirmation by lesson analyses. Finally, we analyze four experienced 6th grade teachers’ discourse (T1, T2, T3 and T4), during sessions devoted to working on some tasks. We took videos and compared resulting opportunities for conceptualization. Overall, our analyses show that the teachers in the study potentially support learning issues related to the logical aspects of line symmetry.

Preliminaries about logical aspects of line symmetry

As we first identified in Chesnais (2009) and formalized with the logical analysis of mathematical language in Barrier et al. (2014) and Chesnais et al. (2017), the concept is constituted of line symmetry as a property of a figure (unary predicate), but also of reflection as a relation – a binary one (two-place predicate) between two figures, a binary relation between a figure and a line and a ternary one (three-place predicate) involving two figures and a line. The property may be defined by the fact for a figure to be superimposable with itself when “flipped”. The ternary relation is defined as two figures being superimposable (one on the other) when folding along the line. The binary relation between two figures is defined by the existence of a line such that the two figures and the line satisfy the ternary relation. The binary relation between a figure and a line can be defined either as the ternary one between the two parts of the figure (situated on either side of the line) and the line, or as the case of the ternary relation where the two figures are the same one (the figure is then called globally invariant under line symmetry through the given line).

As Vergnaud (1998, 2009) already pointed out, the issues (and obstacles for learning) are both inseparably “linguistic and conceptual”. The conceptualization of line symmetry supposes to be able to consider, distinguish and articulate its different logical aspects in various situations (recognition of lines of symmetry in figures, construction of mirror images or completing a figure in order for it to be symmetric with respect to a given line) and to be able to use correctly the associated linguistic elements. Vergnaud (1998, p. 234) illustrated the polysemy of the French word “symétrique” with the following sentences: “1. The fortress is symmetrical. 2. Triangle A’B’C’ is symmetrical to triangle ABC in relation to line d.” The second sentence would probably more likely be phrased as “A’B’C’ is the mirror image of triangle ABC” in English, but we reproduced the word for word translation made by Vergnaud himself to highlight the polysemy of the word “symétrique” in French. In particular, there is no equivalent for “mirror image” or for “reflection” in French, the word “symétrique” being used in all situations – actually, the word “réflexion” is used in French to name negative isometries, but only when considering spaces of dimension greater than two. We will use word for word translations in the rest of the paper rather than a more regular English wording every time it seems useful for the reader’s comprehension.

This polysemy is particularly problematic because of some “expert” (with a sufficient level of conceptualization of line symmetry) linguistic forms, especially “F and G are symmetric”. This factorized form is ambiguous. It may either mean that the two figures share the same property (line d is a line of symmetry for each one of them) or that one is the image of the other under reflection through line d. Note that using the expanded form “F is symmetric to G” or adding “to each other” or “to itself/themselves” would eliminate the potential misunderstanding.

In our study, we chose to focus particularly on the issue of the conceptualization of line symmetry as a ternary relation: this includes distinguishing it from the property and from the binary relation, identifying the nature of the relation (as a negative isometry – materialized by folding or flipping of tracing-paper – among the isometries characterized by superimposition) and understanding the role of the third object (the line). These aspects are supposed to be tackled in France in 6th grade - not in a formal way, of course. Indeed, primary school students construct mirror images of figures using tracing-paper, folding or grids and identify lines of symmetry in elementary figures, but it is only in 6th grade that this has to be progressively formalized and unified by the
introduction of reflection as a transformation acting on points and defined by mathematical properties (perpendicularity and distance conservation). This supposes in particular to use the specific word “symétrique” with its various meanings and “par rapport à” (“with respect to”) to mention the line. However, in the learning process, some intermediate or alternative words might be used—and potentially useful.

**Empirical study**

We focused on the use of the word “symétrique” and the expression “par rapport à”. The study of six textbooks (Chesnais, 2012) revealed that in all of them, the word “symétrique” is used to refer to the property and to the relation without any comment on the polysemy of the word. Forms like “A is symmetric to B” and “A and B are symmetric” seem to be distributed randomly and sentences are sometimes difficult to interpret for students. We found in a textbook in two successive exercises these sentences: “A and B are symmetric with respect to d” and “A and B are symmetric to A’ and B’ with respect to d”. In another textbook, the definition of the ternary relation starts with “Two figures are symmetric with respect to a line d” and the definition of the property, on the next page, starts with “A figure is symmetric with respect to line d”. Students might wonder if the meaning of the term changes when it is in the singular or in the plural. Only one textbook mentions the fact that what we called the factorized form is equivalent to the expanded one. Another finding is the frequent omission of the third element, which jeopardizes the distinction between the binary and the ternary relation.

**Comparison of teachers’ language to designate the ternary relation**

The main result of our analyses of teachers’ discourse is that potentially differentiating practices are largely shared, especially the use of factorized forms, like in the textbooks, but also the use of more opaque forms, like “there is a symmetry” or “it is symmetric” (as showed the detailed analysis of sessions in T1 and T2’s classrooms in Chesnais et al., 2017). Note that it often echoes students’ language and that it results in ambiguities and misunderstandings about the subject of discourse (relation or property). The difference between textbooks and teachers’ discourse might be related to the fact that textbooks are written discourses. We also observed that uses differ when talking or writing, especially about the mention of the axis, as we will show in the example below.

Let us detail an example of such variations to highlight their difference of potential—according to our theoretical framework—for students’ learning. T3 has taught for about ten years in a socioeconomically disadvantaged school and T4 has taught for about twenty years in a mixed one. They both use the same textbook. Their students are working on the same exercise, made of five drawings for which the question is: “are the two figures symmetric with respect to line (d)?”

We focus on the third drawing.

![Figure 1. Third case of the exercise](image)

The distinction between the binary and the ternary relation is at stake together with the role of the line. The two rectangles appear to be symmetric (in the binary sense) since there exists a (vertical) line with respect to which one rectangle is the mirror image of the other, but they are not symmetric with respect to line (d): negation is about the third argument of the relation. A non-correct answer (answering “yes”) might be caused by any of the two following reasons: (1) line symmetry is considered as a binary relation between two figures, the line being obliterated: the
actual question the student answers is: “Are the two figures symmetric (to each other)?”, (2) The line is not obliterated, line symmetry being considered as a ternary relation, but the level of conceptualization of it is not sufficient. A misconception of line symmetry (Grenier, 1988) is here at stake: two figures are mirror images of each other with respect to a given line if they are globally on a horizontal direction, at the same (global) distance of the line. A right answer could also result from this misconception if the student identifies that the distance is not exactly the same (the rectangle on the left is slightly closer to the line). Answers could be based on visual perception, but could be reinforced or contradicted by the use of instruments like a ruler or tracing paper.

**T3’s way of pointing out the role of the line to students**

After the students have read the question, T3 asks them if the words “with respect to line d” are important. One student answers, “Yes, otherwise we cannot know if they are symmetric”. It is interesting that even if the students say yes, they do not mention the line in the answer. They did not say, for example, “Otherwise, we cannot know if they are symmetric with respect to this line”. After working on the task individually, numerous students answer “yes”, because they manifestly obliterated the line; some of them used tracing paper, but drew on it only the two rectangles and folded the paper to make the two rectangles match. T3 hence drives the collective discussion on the importance of the line, pointing out that it indicates where to fold the paper and that it has to be mentioned verbally using the expression “with respect to line d”, herself mentioning it systematically. T3 produces “proximities”, articulating the work on language with the solving of the task and the other dimensions – like material actions – of (real) students’ activity. At the end of the session, some students’ points of view changed. Lucien, a student with some difficulties in mathematics whose initial answer was “yes”, explained that he changed his mind and that “the two rectangles are not symmetric with respect to line d” but that if the line was vertical, then they would. This shows that the relation is now considered as a ternary one and that, simultaneously, some elements of the language of mathematics are appropriated.

**T4’s contradictions between action and discourse**

Right after reading the question, T4 asks the students to draw the figures on tracing paper and the line in red, but without mentioning any reason for it (drawing the line is not a necessity to solve the task). Later, after individual work, in the collective discussion this conversation arose:

T4: what can you say about figures in case c?"

Student: they are not symmetric.

T4 *(with an agreeing tone)*: they are not symmetric. *[And discussion goes on case d].*

Strictly speaking, this answer is incorrect but the main issue for us is that students who answered “yes” (and there were some) did not have any chance to understand why their answer is wrong since the words used do not distinguish between the binary and the ternary relation. Here, we do not suppose that mentioning the line had been sufficient. Later in the session, writing the answer concerning the cases where the answer is “yes” produces a switch in the teacher’s discourse:

T4 *(writing on the blackboard while speaking)*: the figures B, D, E are symmetric

T4 *(stopping and facing the class)*: there is something very important that you have to remember and it is the figure in the middle, figure c that allows us to say it, actually. That is, if I had drawn the line elsewhere, in figure c, don’t you think it would be symmetric? […] In other words, what is important is to say?

Student: the line.

T4: to mention the line. Then, each time, we’ll say with respect to, with respect to line d.
Ambiguity about case c is finally but partially clarified. An issue is how it may affect learning, since discourse seems contradictory with “action”: when solving case c minutes before, mentioning the line did not seem so important. Moreover, the use of the pronoun “it” at the end of the second T4’s intervention may prevent thinking in terms of relation between several objects.

In T3’s class, language supports action, and discourse about language is related to a situation where it is useful. Besides, the teacher’s discourse is adapted to students’ real activity (also because she led some space for genuine mathematical activity). In T4’s class, considerations about the importance of the line (and of the linguistic related elements) seem to be disconnected from the activity of solving the task.

Even if there is no way to assess specifically the effects of these differences on students’ mathematics learning, our theoretical background allows us to suppose that T3’s practices offer potentially better opportunities for learning (like Lucien’s answer seems to corroborate). What corroborates these assumptions is that a larger study of teaching practices of nine 6th grade teachers about line symmetry, in which we assessed the resulting learning of students by running some tests (Chesnais, 2014), attested that T3’s practices are more effective than others. Nonetheless, T4 did not participate in this larger study.

Conclusion

This exploratory study of teaching practices based on the analyses of textbooks contents and of teachers’ discourses on a sample of four teachers suggests that language issues related to logical aspects of line symmetry are far from being systematically explicitly addressed as learning issues. We hypothesize that this might limit opportunities for conceptualization potentially offered by some tasks, at least for some students. However, we suggest that some teachers’ practices offer more opportunities than others, because they address language issues more explicitly and tend to develop students’ mathematical activity simultaneously on and within language.

Obviously, these assumptions need more investigations for confirmation. First, it seems necessary to investigate more teachers’ discourse in order to get a more representative sample of teaching practices and be able to refine the characterization and variability of teachers’ ways of addressing these issues. Secondly, developing methodological tools to measure the impact on students’ learning of what our theoretical framework inclines on considering as more effective teaching practices than others constitutes a crucial issue even if we consider that it would make no sense to isolate the language issue in assessing students’ learning. One of an undoubtedly fruitful but demanding challenge remaining is also to pursue the first step that we initiated here in order to confront French didactics of mathematics research questions, frameworks and findings with international research in mathematics education on language issues in mathematics education.

References


What characterizes quality of mathematics classroom interaction for supporting language learners? Disentangling a complex phenomenon

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The crucial role of the quality of classroom interaction is highlighted in many qualitative studies, especially in connection with research on fostering (language learners’) conceptual understanding of mathematics. But what exactly characterizes quality of classroom interaction?

We want to address this question with the aim of proposing different criteria and disentangle the complex phenomenon into the talk-related, discursive, conceptual and lexical dimensions of the interaction. Therefore, we give an overview on related qualitative studies and deduce categories for the quality of classroom interaction for both, teachers’ measures for activating students and for students’ participation. This disentangling can establish a theoretical foundation for future research, as it provides directions for an operationalization for quantitative video-ratings.

Keywords: Classroom interaction, quality, activation, participation.

Why to operationalize quality of classroom interaction that can support language learners?

There is a wide consensus that language learners’ (i.e., learners still acquiring the language of instruction) access to mathematics should be supported in specific ways (Gibbons, 2002; Moschkovich, 2013; Smit, van Eerde & Bakker, 2013). However, different instructional approaches exist which vary heavily in their main principles and quality criteria. For example, some researchers mainly focus on simplifying the mathematical texts (e.g., Haag, Heppt, Stanat, Kuhl & Pant, 2013), on training vocabulary (DfEE, 2000) or general reading strategies (Hagena, Leiß & Schwippert, 2017). Although these instructional approaches have been criticized for being too reductionist (e.g., Moschkovich, 2013, 2015), they persist being empirically investigated, especially in quantitative intervention studies. One reason might be that for these kinds of approaches, the criteria for successful implementation can easily be given.

In contrast, most mathematics education researchers emphasize that classroom interaction must be taken into account with respect to students’ participation in rich discourse practices (Barwell, 2012; Erath, Prediger, Heller & Quasthoff, submitted; Moschkovich, 2013, 2015). Many qualitative studies (e.g., Erath et al., submitted; Moschkovich, 2015) highlight that the quality of communication and discourse is crucial for learning mathematics, especially for language learners. However, the quality of interaction is characterized in different ways without a systematic disentangling of the complex phenomenon. This might be a reason why so far, no quantitative studies exist which really provide quantitative evidence that high discursive quality can indeed impact on students’ learning gains.

In our mainly theoretical paper, we try to systematize existing approaches and refine the construct of quality of interaction in a way that allows a later operationalization also for quantitative purposes. For this, we pursue the following research questions: How can the quality of interaction
in mathematics classrooms be disentangled into distinguishable dimensions? How can they be operationalized for quantitative video-ratings?

The questions are posed in the context of the intervention study MESUT, where students’ conceptual understanding of fractions was fostered in a content- and language-integrated remediating intervention. Grade 7 students worked in groups of 3 to 6 with a teacher so that the interaction quality of 19 groups working with the same (conceptually focused) teaching material can be compared.

Theoretical background: Different dimensions of high quality interaction

Investigating the role of communication for learning mathematics has a long tradition in mathematics education research. Whereas large quantitative studies (e.g., Haag et al., 2013) point out that students with low language proficiency are outperformed by students with high language proficiency in mathematics tests, qualitative studies focus on analyzing learning processes and the related interplay of language and learning mathematics. In the following, we outline some of these studies with the aim of identifying different important dimensions of high quality interaction, especially with respect to supporting language learners.

The importance of giving students space for active participation in mathematics classroom discussions was already underlined twenty years ago, e.g., in Yackel and Cobb’s (1996) qualitative study on social and sociomathematical norms. They argue how students’ increased participation in the talk influences mathematical learning opportunities. This dimension related to the quantitative amount of students’ talk (in brief: talk-related dimension) of quality of classroom interaction is also highlighted by other studies, e.g. underlying the TIMSS video study (Hiebert et al., 2003).

For learning mathematics, of course not only the quantity of talk but especially its quality matters. In their research overview, Hiebert and Grouws (2007) present quantitative studies which provide evidence for the effects of high quality teaching on students' learning gains. One major feature of high quality teaching is the focus on conceptual understanding. Most qualitative studies cited see alignment of a focus on conceptual understanding and high quality of discourse. This conceptual dimension is lately further investigated by Erath (2017) who points out that on the one hand especially explaining and arguing are linguistically more demanding than reports or descriptions but on the other hand these challenging discourses are connected to talking about conceptual knowledge (Hiebert & Lefevre, 1986) and thus are important discursive practices for a meaningful learning of mathematics.

This work builds a bridge to another perspective on the quality of communication in mathematics education research. The discursive dimension emphasizes the importance of rich discourse practices for fostering (language learners’) mathematical understanding (e.g., Barwell, 2012; Erath et al., submitted; Moschkovich, 2015). It important that students participate in classroom communication processes but also that they are supported and encouraged to contribute to discursively rich communication about mathematics, which particularly means to avoid students’ answers on single word level. That is, students’ participation in discourse practices is seen as especially important for their mathematical learning. For carving out different dimensions of quality of interaction, we distinguish discursively rich discourse practices such as defining, explaining meanings, arguing, from less rich practices such as telling, reporting procedures (justified in the context of Interactional Discourse Analysis in Erath et al. submitted). Thus, the discursive dimension in this study refers to episodes of the interaction in which rich discourse practices are made relevant which especially means, that students are demanded to contribute with more than single words or half sentences.
Language learners have special needs when it comes to supporting them in participating in classroom communication: As several studies show (e.g., Gibbons, 2002; Prediger & Wessel, 2013; Smit et al., 2013), these learners need additional support on a lexical level in order to facilitate participation in discourse practices like explaining, arguing, describing etc. This lexical dimension does not imply offering lexical means for their own sake, but to offer integrative lexical support, integrated in jointly discussing mathematics. Until now, it can only be hypothesized that the lexical and discursive dimension are especially important for language learners. Since this must be investigated empirically, we hope to contribute in closing this research gap with our planned quantitative study.

First steps towards disentangling quality of classroom interaction

Systematizing the literature review leads us to distinguish four dimensions for high quality classroom interaction which can later be operationalized in codings for video-ratings:

- The talk-related dimension refers to students’ general space to talk: how much time does the teacher speak, how much is left to student talk?
- The conceptual dimension refers to the epistemic quality of the talk with respect to the forms of knowledge: how much is conceptual knowledge addressed and connected, how much procedural knowledge?
- The discursive dimension incorporates the discursive quality by valuing rich discourse practices like explanations, argumentations higher than one-word answers or simple reports of solution pathways and descriptions.
- The lexical dimension refers to the specific support which is required for language learners: How much learning opportunities are provided for lexical means which are required for the discourse practices, and how are they embedded in the discursive practices?

These dimensions are connected, but they are not the same, as a brief analysis of the following episode (cf. Erath, 2017 for closer analysis) can show. The transcript stems from a remediate intervention for weak seventh graders on conceptual understanding of fractions in the project MESUT (see above). After a rich activity of drawing fraction bars for equivalent fractions, the students are supposed to consolidate their experiences by explaining the pseudo-student’s utterance “If I’m looking up, I’m portioning more coarsely!” as printed in the task in Figure 1:

22 Dennis: I’d like to say something else
23 Teacher: W, What would you like to say?
24 Dennis: The numerator has split here, here is written eight and there four
25 Teacher: The numerator?
26 Dennis: Or the denominator, no idea, down there, I don’t know what it’s called
27 Teacher: The denominator
28 Dennis: Yes
29a Teacher: Exactly, and what the denominator um divided by two, right so in half, and what does it with the bar? [points to the bar in the task, 8 sec. break]
29b Teacher: Whereby do I see at the bar that down here, the denominator is eight and above four
30 Dennis: Because above#
31 Rahmiye: #It doubles
32 Teacher: Yes what doubles? Explain it
33 Rahmiye: The denominator
34 Teacher: The denominator from, from top to bottom it doubles, well, and how do I see it at the bar?
35 Dennis: Because the pieces are larger
In the talk-related dimension, the episode shows a high quality since the teacher initiates talk-related activation by requesting students’ active contributions (Turns 23, 29, 32, 34) and students try to fulfill the demand by talk-related participation. The episode is also rich in the conceptual dimension as the task is intended to work on students’ conceptual knowledge of connecting the new mental model for finding equal shares with the familiar representation of the fraction bar.

In the discursive dimension, the episode must be split into two parts: Until Turn 29a, there is a discursive activation by the teacher’s request for explanations, but Dennis shifts his participation into a less rich discourse activity by describing the change of the written numbers rather than explaining their meaning. Hence, his discursive participation in rich discourse practices is more limited. In Turns 25 to 29a, the discursive activation is reduced for the sake of the vocabulary work, in Turn 29a, the teacher embeds his lexical activation into the discourse activity offered by Dennis. In Turn 29b, the discursive activation shifts again to the higher level of explanations, but students’ participation stays very limited, as they mainly contribute single words. One exception is Dennis’ utterance in Turn 35 which provides a more significant contribution to the discourse practice of explanation. In the same Turn 35, Dennis also participates in the lexical learning pathway as he adopts the meaning-related lexical means of pieces which become larger. The episode closes with the teacher fulfilling his own discursive demand for explaining the meaning of equivalent fraction. So he provides a conceptual learning opportunity in which the students participate more peripherally.

Table 1. Systematizing categories for quality of classroom interaction

<table>
<thead>
<tr>
<th>Teachers’ intended activation</th>
<th>Talk-related dimension</th>
<th>Conceptual dimension</th>
<th>Discursive dimension</th>
<th>Lexical dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Space for students’ talk in collective discussions of mathematics</td>
<td>Conceptual demands and meaning-related learning opportunities</td>
<td>High discursive demands by requesting rich discourse practices</td>
<td>Integrated lexical learning opportunities</td>
<td></td>
</tr>
</tbody>
</table>

| Students’ enacted participation | Participation in collective discussions of mathematics | Participation in meaning-related activities | Participation in rich discursive practices | Taking up lexical learning opportunities |
This brief analysis suggests that the distinction of dimensions enables us to analyze their interplay. Within each dimension, it is crucial to distinguish between teachers’ intended activation and students’ enacted participation (Table 1). The quality of an interaction seems to depend on both.

**A proposal for operationalizing different criteria for video-rating**

Qualitative case studies of two data sets provide first indications that the discourse practices of explaining and arguing seem to be particularly important for working on aspects of conceptual knowledge and that only few students can participate in these sequences without the teacher’s support (Erath, 2017; Erath et al., submitted). Furthermore, the teachers’ moves seem to vary strongly in their impact on students’ participation in the different dimensions (Erath, 2018).

**Motivation for quantification**

Based on these qualitative insights, a first quantitative case study (9 groups, 5 tasks) measured the talk-related and discursive activation and participation for individuals and groups (Nienhoff, 2017). The study reveals huge differences in the two dimensions on the group level and on the individual level. In addition, the individual share of (oral and written) explaining and arguing in time on task correlates to the measured learning gains (with $r = 0.331$) as well as a group variable: all students’ share of discourse on the time on task correlated also to the learning gains (with $r = 0.318$). The latter finding suggests that listening in rich discursive learning environments can be effective for learning.

These first observations point to very interesting possibilities of quantitative analysis that motivate our attempt to operationalize the four suggested dimensions of classroom quality on an individual and a group level. Hence, based on the categories proposed in Section 3, our next step in future research is to search for operationalizing the categories for quantitatively grasping the quality of classroom interaction by video-ratings.

**Operationalizing teachers’ intended activation in four dimensions**

For operationalizing the quality criteria in a video-rating, a basic coding was conducted on the tasks and the video data that allows determining frequencies: All tasks are categorized as having a lexical, conceptual or procedural focus as well as oral or written discursive demands that are further classified as sequencing (describing, reporting,…) or integrating (explaining, arguing,…). In the video data, the time spent on the different tasks (without time for general organization) is captured and can be summed up to the total time on task. Furthermore, the students’ talking time is measured as well as more specifically students’ and teachers’ times spent on richer or less rich discourse practices as well as single-word utterances. Besides these time measurements, the teacher moves are classified as focusing discursive, lexical, conceptual or procedural aspects. For further investigating the lexical dimension, a simplified version of trace analysis (Prediger & Pöhler, 2015) is applied that relates the number of offered lexical means to those students’ take up: More precisely, we count how many of the written offered formal and meaning related expressions are taken up by students in their oral utterances. This basic analysis allows specifying criteria for the quality of classroom interaction that relate to the four introduced dimensions of talk, discourse, conceptual knowledge, and lexical means.

The operationalizations of teachers’ activation always refer to the small group, as small group characteristics determine individual learning opportunities, but not necessarily individual use:

- Criteria for the *talk-related activation* are operationalized as (CA1) the percentage of all students’ talking time related to the groups’ complete time on task and (CA2) as the percentage of time that the teacher is not speaking in the time on task, that operationalizes the
time for oral and written talk in whole group phases as well as in phases of working in pairs and individual seatwork.

- Criteria for the *discursive activation* are focusing the written or oral production of or contribution to discourse practices. On the written level, discursive activation is operationalized as (DA1) the percentage of time spent on writing tasks requesting discursive practices (including the time of orally reviewing these texts) in the time on task (for the whole lesson). On the oral level, two operationalizations are relevant: (DA2) grasps the percentage of time spent on discursive sequences of students and teacher together in the time on task, whereas (DA3) captures only the students’ percentage of time spent on discursive sequences in the time on task.

- Criteria for the *conceptual activation* are proposed on the levels of tasks, teacher moves, and oral discourse: On the level of tasks, conceptual activation is operationalized (KA1) as percentage of time spent on tasks with conceptual focus in the time on task. On the level of teacher moves, it is operationalized (KA2) as percentage of teacher moves with conceptual focus in all teacher moves. On the level of oral discourse, conceptual activation is captured (KA3) as percentage of all students’ time in sequences with integrating discourse in the time on task since qualitative studies point to the importance of these sequences (of explaining, arguing …) for the learning of conceptual knowledge whereas sequencing discourse (like describing or reporting) is more likely to be connected to working on procedural knowledge.

- Criteria for the *lexical activation* are suggested for tasks and for teacher moves: On the one hand, (LA1) operationalizes lexical activation as percentage of time spent on working on subtasks with lexical focus in the time on task. On the other hand, (LA2) operationalizes it as percentage of teacher moves with lexical focus in all teacher moves.

**Operationalizing students’ participation in four dimensions**

In each dimension, students’ participation is operationalized for each individual:

- Criteria for *talk-related participation* are operationalized (CP1) as percentage of individual talking time in the time on task and (CP2) as percentage of individual talking time in the talking time of all students.

- Criteria for *discursive participation* are operationalized (DP1) as percentage of individual time in discursive sequences in the time on task and (DP2) as percentage of individual talking time in the time spent on tasks demanding written texts.

- Criteria for *conceptual participation* are operationalized (KP1) as percentage of individual time in sequences with integrating discourse in the time on task and (KP2) as percentage of individual talking time in the time spent on tasks with conceptual focus.

- Criteria for *lexical participation* are operationalized (LP1) as percentage of the picked up terms and (LP2) as percentage of individual talking time in the time spent on tasks with lexical focus.

**Outlook on the next steps towards a quantitative video-rating study**

At the conference, we presented the suggested criteria and operationalizations for quality of interaction to the ETC participants and showed first results. Thanks to a lively discussion, we are in the process of refining our theoretical base and the operationalizations. In the future, we will be able to present empirical results based on these refinements and referring to a larger database: In a second study that is currently in the phase of coding, we refer to the video data of lesson 2 in 19 groups (one group from each teacher of the intervention; altogether 89 students). Lesson 2 was
chosen as a lesson with a lot of opportunities for communication. We are applying the operationalized criteria and are going to correlate them with the students’ learning gains as difference between scores in pre- and posttest on the conceptual understanding of fractions.

We hope to find quantitative connections in the data because this would help to overcome the qualitative / quantitative divide in which the quantitative research always limits to less complex phenomena.

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A functional linguistics analysis of a mathematics register expressed through two languages

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In Malta, Maltese and English are used for learning mathematics. In this paper, I present a starting point for investigating features of a spoken mixed Maltese/English mathematics register. Using data from an elementary classroom (two lessons), I draw on functional linguistics to describe how Maltese and English were combined to constitute the register in terms of interpersonal, textual and ideational meanings as defined by Halliday. I conclude that the translanguaging formed an integrated system due to the intricate role each language played in realizing the three elements of the mathematics register. Furthermore, diagrams and symbols were ‘read’ in English, illustrating the role of English in linking various semiotic systems. Finally, while nouns/noun phrases were embedded easily into Maltese sentence structure, I noted that some words that serve as adjectives or verbs in English were rendered nouns in Maltese. Further investigation will now require a larger scale study.

Keywords: Mathematics register, elementary mathematics education, functional grammar.

Introduction

As Lin (2017, p. 10) points out, much has already been written internationally “to uncover the good sense of rationality” of existing practices of translanguaging in various contexts. Within the area of mathematics education, two examples of research-based discussions are Halai (2009) and Norén and Andersson (2016), who focus on situations in Pakistan and Sweden respectively. However, Lin (2017) recommends that research might now take new directions. One suggestion she offers is to view the whole lesson as a curriculum genre and investigate therein the role of participants’ home language (referred to as L1). In this paper, I take up this suggestion, however focusing on both L1 and the participants’ second language (L2) as they are used in an elementary classroom in Malta. The languages in question are Maltese (L1) and English (L2). English plays a key role in Maltese education, as a result of 164 years of British colonization (1800-1964). Most notably, it continues to be the written language (textbooks, assessments, etc.) for various school subjects, including mathematics and science. This situation prompts extensive mixing of Maltese and English, especially in terms of ‘technical’ vocabulary, which tends to be stated in English. Of course, I cannot suppose that all Maltese teachers and students use language in the same way, but Camilleri Grima (2013) does note similarities in patterns of language use across Grade-levels and subjects. In her early seminal work on code-switching in Malta (Camilleri, 1995), she also comments specifically on similarities in the use of English during mathematics lessons.

Assuming the pedagogic benefit of using two languages, I wished to focus my attention on the mathematics register, as brought into existence through the practice of the bilingual participants. Halliday (1978, p. 195) defines a register as “a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings”. My research questions were: How do Maltese and English contribute to the school mathematics register? What are some features of a spoken mixed Maltese/English mathematics register? Here, I adopt the expression ‘Mixed Maltese English’ that was first used by Borg (1980). There were
two reasons for these questions. The first relates to the local debate regarding the medium of instruction for mathematics. While some practitioners and policy-makers accept the benefits of using two languages, others argue in favour of an English-only approach, generally because of the use of English for written texts and/or because they hold a deficit view of language mixing. I wished to contribute to this debate in a theoretical manner. Second, in my role as a mathematics educator involved in teacher training, I promote the explicit teaching of mathematical language. Such attention to language involves focusing on shifts from informal to more formal language. I believe that it would be beneficial for me to be aware of (possibly prevalent) ‘ways of saying’ pertaining to the informal, spoken elements of our mathematics classrooms. A full study will require observation of a large number of classrooms, and hence, the small case study presented here serves as but a starting point to indicate how a larger study might unfold in terms of research method and analysis. All this may be of relevance to other post-colonial countries with comparable educational contexts.

The mathematics register(s)

Halliday and Hasan (1985) hold a functional view of language that is based on the premise that people put language to use according to the situation in which they find themselves. A context of situation includes: the field of discourse (what the language is ‘about’), the tenor of discourse (the set of relationships between the participants) and the mode of discourse (the role or purpose of the language). These elements determine a ‘text’ within which they are realised through respective functional meanings: ideational meanings, which express categories of experience; interpersonal meanings, which express social and personal relationships; and textual meanings that help to structure the flow. Meanings are constituted through grammatical choices, and it is the particular configuration of meanings that creates a register. Hence, for Halliday and Hasan, a register is a semantic concept. I will elaborate on the three elements of the register in the course of the paper, presenting them together with the data. Schleppegrell (2004) applies the functional linguistics perspective to the language of schooling, noting that each discipline can be characterized in terms of linguistic choices that are typical and pervasive. I chose to use this perspective since I anticipated that it would help me describe the specific roles played by Maltese and English as used by the participants; furthermore, I recognised the particular relevance of the ‘ideational’ element to a discussion on mathematical terminology.

The word ‘register’ has been used in a different sense by Duval (2006), who applies the term to semiotic systems - such as graphs, geometric figures and symbolization - that are governed by rules and that allow transformations. Duval defines two types of transformations: treatments, which are transformations within the same system, and conversions, which are transformations from one system to another (e.g. algebraic notation to graphical representation). The word register in this sense has also been utilized by Prediger and Wessel (2011) in their work on ‘relating registers’. They suggest that a verbal register may be related to other registers of increasing abstractedness, namely, concrete representational, graphical, symbolic-numerical and symbolic-algebraic registers. With reference to the verbal register, Prediger and Wessel point out that this can be expressed through a student’s L1 or L2. In this paper, I will retain the term register in the sense used by Halliday due to its constituting elements, but will make reference to links between the spoken register and other semiotic systems.

Research context and method

The data for this study consists of two lessons of approximately 40 minutes each, carried out in a class of sixteen 8 to 9-year olds. The choice of class was opportunistic since I was in the school carrying out another research project. I asked the teacher to allow me to record two of her lessons, since hers was a class wherein a ‘typical’ mix of language was used, as a result of all participants
being Maltese speakers. She suggested two up-coming lessons: “Greater/less than” and “Carroll diagrams”. The lessons included long stretches of whole-class discussion, followed by individual written worksheet or textbook exercise; this lesson development is also rather typical of Maltese classrooms. The lessons were recorded using two camcorders and later transcribed in order to enable a detailed analysis in terms of the register components, and to look out for any notable features. The data consists of the whole-class interaction. Some verbatim excerpts are presented in this paper as illustrations of how Maltese and English were used to contribute to the elements of the register. Where appropriate, both Maltese and the English translation are presented; however, for the sake of conciseness, some excerpts are presented only in their translated version.

Analysis of data

Most of the spoken language observed was Maltese, with English words embedded within it, as illustrated in the excerpt below (T for teacher, P for pupils and Glen for one of the pupils). At this point of the lesson, a projected image showed some pirates and a 4-celled Carroll diagram marked with the English classifications “has 1 leg / has 2 legs” and “has a sword/does not have a sword”; the use of spoken English in relation to written text is evident in the illustration. In the transcript, Maltese speech is shown in a bold font (left-hand column), and a translation is presented alongside it (right-hand column).

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T: Mela, hawnhekk (touches Carroll diagram) qed jistaqsini “has one leg” jew “has two legs”. Ahna, kemm għandha saqajn?
P: Tnejn!
T: Tnejn. Imma hemm pirates minnhom, minflok sieq, x’jkollhom?
P: Injama!
T: Injama, stick. Bicca injama. Allura, mela dak jiġi ‘one leg’. Issa, ma’ din in-naha (touches right side of diagram), ghandi ‘has a sword’, ‘does NOT have a sword’. Kollha għandhom xabla?
P: L.e. (...)
Glen: Miss, hemm wieħed zero leg [sic] għandu!
T: Aghti ċans! Jista’ jkollu zero legs?!
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So, here (touches Carroll diagram) he’s asking us “has one leg” or “has two legs”.
How many legs do WE have?
Two!
Two. But there are pirates who, instead of a leg, what do they have?
A stick!
A stick, stick. A piece of stick. So that is ‘one leg’. Now, on this side (touches right side of diagram), I’ve got ‘has a sword’, ‘does NOT have a sword’. Do they all have a sword?
No.
Wait a bit! Can he have zero legs?!

Now I consider the elements of the register in turn: interpersonal (relationships), textual (purpose and structure) and ideational (expressing ideas) elements.

The interpersonal element: Relationships

Speakers and writers demonstrate their understanding of the role relationships inherent in a context through grammatical choices related to the interpersonal component of the register. One grammatical feature is ‘mood’, which can be, for example, declarative, interrogative or imperative (Schleppergrell, 2004). In these lessons, the declarative mood was often utilized, for example, when the teacher stated (in Maltese) “Instead of writing a sentence … you know that in maths we write only number and signs” or when a pupil said “Miss, I noticed that the ‘G’ for Gracie [the alligator] is the big one”. Written declarations were in English, as in the slide text: “You can remember when to use < or > by thinking of a crocodile.” With respect to the interrogative, Erikson (1996) notes that questioning is very commonly used in spoken whole-class interaction in the form of ‘know information questions’ (p.42). These are a type of request for information that the teacher generally already knows. The use of questioning in the class appeared to create a
relationship of the adult as the knowledgeable and guiding person; the students were placed in a role of answering questions in order to progressively achieve the lesson objectives which had been displayed at the start of each lesson. The imperative mood was evident in the spoken interaction in terms of instructional/procedural strategies stated in Maltese: “Daniela, come out”. In relation to mathematics, the imperative was noticeable mainly in written English texts. Commands imply an authority which the students are expected to follow. For example, an online game on Carroll diagrams gave the instruction “Click and drag the objects to the correct square”. Indeed, Morgan (1998) notes that the imperative (‘Draw a diagram’) is a grammatical structure very typical of a written mathematics text.

Morgan (1998) explains that the interpersonal function concerns not only the relationship between the the participants, but also the ways in which they are constructed as individuals. These relationships were expressed in Maltese. Following Morgan, I noted that the frequent use by the teacher of the third person plural (“we”) suggested that the teacher was not speaking ‘alone’ but as part of a community. At times, this community appeared to be the classroom itself: “Now, look at it [the diagram] before we do [click] ‘submit’”. This sense of community was sometimes echoed by the pupils themselves, as when Daniela asked with respect to how to proceed in a set task: “Do we keep matching them?”

I concluded that except for occasional English words, the interpersonal element of the register was constituted by the use of Maltese. Written aspects were in English.

The textual element: Purpose and structure of language

The textual element of the register relates to the purpose and structure of language. The language was spoken, and set up in a way that the teacher did a lot of the talking as she strove to introduce new ideas and to confirm that her pupils were ‘following’ the lesson. Pupils tended to give short answers, contributing to the structure of the interaction. Various language elements contributed to the structuring of the intended line of thought. At the start of each lesson, the teacher referred to previous work through (Maltese) adverbials of time “Yesterday we did…” and “Last week…”, thus creating a sense of continuity. Throughout the lessons, cohesive devices contributed to the textual element of the register, in particular the conjunctions so/or/and/because. Such ‘explicit conjunctions’ are common in school spoken interaction (Schleppergrell, 2004, p.65). For example, the word mel [so] was used extensively by the teacher at the beginning of sentences, marking a logical progression from one point to another. Another cohesive device is reference (Schleppergrell, 2004). The teacher often used the words this, that, here, there in order to take the children through a point. For example, linking a worksheet with a image projected on the whiteboard, she said: “So [now], look here. Your worksheet is like this.”

Schleppergrell (2004) also mentions other elements that enable a controlled flow of information. These include repetition, emphasis and the introduction of details and asides. The following two stretches of interaction are examples of detail and repetition respectively. The context is a story of two alligators as an introduction to the symbols > and <.

T: Gracie [female alligator], jekk ha tiekol innaha tal-left, lil fejn ha tifthu halqha? If Gracie [female alligator], eats from the left, in which direction will she open her mouth?
Rachel: Lil hemm (indicates left direction; other children do the same).
T: Lejn in-naha tal-left. Towards the left.

Heidi: (Heidi is reading English text from a projection). (Repeating written text) “They forgot to pick up the left-overs”. X’jiġiferti? What does this mean?
T: (Repeating written text) “They forgot to pick up the left-overs”. Heidi, how many tally marks are there for the month of January?

Heidi: Insew jiġbru l-fdal. They forgot to pick up the left-overs.
T: Il-fdal, prosit. U hallewhom hemm. The left-overs, well done. And they left them there.

Textual elements of the register were expressed almost exclusively through Maltese. Hence, the role of Maltese was crucial to the structuring and logical progression of the spoken interaction.

The ideational element: Expressing mathematical ideas

The field of a context is realized through ideational meanings that express circumstances, participating entities and processes (Halliday, 1976); it consists of mainly nouns, verbs (processes) and what Schleppergrell (2010) calls ‘content words’ that contribute to text in particular subject areas. It was for this element of the spoken register that English played the greatest role. Circumstantial information (e.g. time, place) relevant to mathematics was expressed through both languages with English being used for words commonly forming part of local school discourse, e.g., “In today’s lesson, we’re going to learn …. “ ‘Participants’ in mathematics include the objects being talked /written about which are expressed grammatically through nouns or noun phrases. In the observed lessons, English was used extensively for these. For example (NOTE: excerpts are not consecutive):

T: Hiedi, kemm hemm tally marks ghax-xaghar ta' January?
T: Kylie, aqraxna t-two numbers.
Dulcie: Ikun hemm zeroes, u jkun hemm lines oħrajn hdejhom.
P: Hundreds, tens u units.

Heidi, how many tally marks are there for the month of January?
Kylie, read the two numbers for us.
Pick the correct sign.
There’d be zeroes, and there’d be other lines next to them.
Hundreds, tens and units.

In the observed lessons, English nouns were always preceded by the Maltese definite article (of which there are 9, depending of the first letter of the noun). The following are various examples used by the teachers and/or children: in-number, is-sign, il-five, il-maths, iz-zero, l-objects [the number, the sign, etc.]. This is line with a linguistic feature of the Maltese language, which makes extensive use of the definite article the.

Halliday (1976) lists three types of processes: mental, action and relation. Mental processes involve consciousness, as in I liked …(p. 165); action processes indicate that something is being done, e.g. he is throwing stones (p. 161); relational processes may include attributes expressed as adjectives e.g., Mary looks happy or may involve two nouns e.g., John is the leader (p. 167). Processes are realized through verbs. Examples of the three types of processes expressed in Maltese are respectively:

T: Taqblu magha? Do you agree with her?
Sandra: Ghax iċċekkajnijhom kollha. Because we checked them all out.
T: Mhx kollha ndaq. They’re not all equal.

Mental and action verbs tended to be expressed through Maltese, with the vast majority of action verbs being everyday words such as to do, draw, check, write, choose, look at and so on. In these
two lessons, the use of ‘mathematical’ verbs was not observed, except in one case where the point at hand was multiplying by two. However, the idea was expressed as “tagħmel times two” [you do times two] by a pupil, followed by the teacher’s feedback “Brava, tagħmel id-double” [“Well done, you do the double”]. Hence, the idea was expressed through the verb ‘to do’ followed by an expression that appeared to function as a noun. This feature of expression prompted me to recall a similar phenomenon I had noted in a different study (Farrugia, 2007) wherein different 8-year-old children uttered the phrases: “tagħmel adding / plus / il-plus / multiply / division / dividing” [You do adding / plus / the plus / multiply / division / dividing]. This might be a particular feature of the local mixed register; further investigation will shed more light on this.

For relational processes regarding mathmematical ideas, integrated Maltese/English was also used. For example, “Carlton, how did you realise that they [the number of chicks] are not equal?” It was on only one occasion that the expression ‘greater than’ was used in Maltese: “Seven huwa ikbar mill-five” [Seven is bigger/greater than five]. What was striking was that at times, words that in English are attributes - and hence adjectives - were rendered nouns in the integrated language. For example: “We’ve got the odd and the even as we had in the game”. Admittedly, here the word numbers was implied, as it might be in English, however other examples of adjectives rendered nouns were: “She chose the long” and “Last week we did the greater than and the less than”.

Variation in grammatical category of words is a feature of the English mathematics register noted by Pimm (1987). To illustrate this point, Pimm mentions the number names (one, two, three, etc.) which tend to function as adjectives in everyday English, while in mathematics they often serve as nouns. For example, in the phrase “Seven birds”, the word seven is grammatically an adjective, describing the noun birds. On the other hand, in the statement “Seven is prime”, it is the word seven that functions as a noun, described by the adjective prime. Pimm notes variation from ‘ordinary’ to ‘mathematical’ English, while in my case the variation went across languages. I suggest two reasons for this grammatical shift. One reason is that the mathematical words were sometimes used as labels: ‘long’ and ‘short’ were the titles of cells in the Carroll diagram. Thus, as a child contemplated where to place an item, the teacher asked: “With the long, or with the short?” Similarly, ‘Greater/less than’ were sometimes taken as the names of the new symbols > and <=“I’m going to show you a story so that you’ll remember whether to use the greater than or the less than”; at one time the teacher asked “What’s the name of that sign, Daniela?” to which Daniela answered: “Il-le… il-greater”. [The le.. the greater”]. The teacher also refered to the = symbol as “l-equal” [the equal]. Another reason for the rendering of an adjective into a noun may have been the influence of a feature of the Maltese language, whereby an adjective can be rendered a noun phrase by attaching the definite article to it. Hence, “Għażlet il-long” [She chose the long] (‘one’ implied) may not sound so alien to a Maltese speaker, as an English translation might sound to an English speaker.

**Conclusion**

In this paper I have assumed the positive benefits of using two languages in education and, as suggested by Lin (2017), investigated the role of L1 – and L2 – in the curriculum genre that is the typical whole-class teacher-directed lesson. In the observed elementary classroom in which the teacher and children were Maltese speakers, while the academic language of mathematics was English, the translanguaging formed a truly ‘integrated system’ (Canagarajah, 2011). This was due to the role each language played in realizing the three elements of the mathematics register as defined by Halliday (1978). In particular, Maltese served a clear role in the expression of interpersonal and textual meanings (although any written form of these elements was given in English). As part of the spoken register, the greatest role played by English was as part of the expression of ideational meanings.
Another observation regarded the use of English in relation to diagrams and symbols. When the Carroll diagrams were read, this was done in English. At times the utterance may have involved a single English word, as in the teacher’s question “Issa property waħda għandha?” [Now do we have one property?] or in a rarer full sentence in English by a child, “It’s an even number”. Symbols were also read in English, resulting in number relations being expressed in English. For example, as children manipulated numbers into their correct places on a Carroll diagram with classifications <10 and >10, the teacher asked them to double check where they had placed the number: “Read for us what the sentence would say”, to which a child would answer, “Eighteen is greater than ten” or “Four is less than ten”. Thus conversions (Duval, 2006) from the graphic and symbolic semiotic systems to spoken language were realized through English or a mixed code, highlighting the role English played in relating registers in the sense of Prediger and Wessel (2011). Given the fact that, locally, educators debate the language of instruction for mathematics, the study highlights the over-simplicity of any recommendation that mathematics in Malta should be taught exclusively through English and the potential challenge of any aspirations of using Maltese alone. Rather, the study illustrates how the languages come together in a very intricate way to express mathematical meaning.

Using a functional grammar perspective allowed me to note that nouns and noun phrases were embedded easily into Maltese sentence structure; however, nouns were sometimes used where in English one might use an adjective or verb. In an English mathematics register, adjectives tend to be used for properties, and verbs for processes. I wonder whether their replacement with a noun is a widespread practice, and if so, what one might conclude about the local mixed mathematics register. An important question to address when more data is available, is the possible implications of the grammatical features for the mathematics being learnt. Halliday (1978) states that since languages differ in their structure and vocabulary, they may also differ in their paths towards mathematics. Further analysis of the local register/s can help to identify this path.

References


Pupils’ participation in collective argumentation within multi-age mathematics education at primary level

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Currently in Germany, there are increasing numbers of primary schools who install multi-age education for pedagogical reasons or due to demographic changes in rural areas. Especially for mathematics education, this often causes individualised learning to take place. However, as interaction is seen as a foundational constituent of mathematical learning (Miller, 1986), this research focuses on pupils’ interactions within multi-age mathematics education and has the objective of describing how pupils of different ages learn collaboratively. Because learning is seen as the increasingly autonomous participation in collective argumentation, we seek to identify how pupils participate in processes of collective negotiation of meaning and, thus, examines interaction by using the analysis of argumentation and the analysis of participation.

Keywords: Multi-age education, collective argumentation, participation.

Multi-age education and a sociological theory of learning

No matter which classroom one looks at, the pupils learning together are never homogeneous in their learning preconditions. Each individual is unique and therefore has differing abilities and experiences. Using age as the distinguishing factor for placing pupils in specific classes is the norm because it is seen as a possibility to make classes as homogeneous as possible. But even in learning groups which are homogeneous in age, the differences in the pupils’ preconditions for learning when they enter into school can be up to five years (Lorenz, 2000, p. 22). One concept with the aim to purposefully value and meet this existing heterogeneity is merging pupils of different ages and grades in one class deliberately (Prengel, 2007, p. 69f). In English, this concept is mainly referred to as multi-age classes. In contrast, the terms combination or multigraded classes refer to pupils of different ages and grades being combined in classes for organisational purposes (a more detailed description of the terms can be found in Wagener, 2014). For example, in rural regions of Germany, multigraded classes have been installed in order to prevent closing small schools as a result of subsiding numbers of pupils and therefore not forcing especially younger pupils to travel long distances to school (in Saxony, multigraded classed in small primary schools are being installed since 2014). In this paper, the term multi-age classes will be used, even though the schools participating in this project have different reasons for merging classes.

When making learning groups heterogeneous in age as in multi-age classes, the diversity of learning preconditions increases and with it also the necessity for differentiation. However, mathematics is often presumed to be a subject built up rectilinear whose content can only be learned following a specific order (Lorenz, 2000, p. 19). Nührenbörger and Pust (2006) found that because of this assumption pupils are rarely given the opportunity to learn from and with others in mathematics classrooms. This often causes extremely individualised and separated learning to take place as teachers either feel they need to separate the learners back into the different age groups or they need to let each pupil learn individually in their own pace. Both have its value for certain topics but can place too great a demand on the teachers trying to accompany the learners
simultaneously in their learning process as well as waste the opportunity of collaborative learning.

However in mathematics education research, collaborative learning is not only seen as the process of pupils cooperating in order to match their individual cognitive restructuring processes, but for young learners interaction is seen as foundational constituent of learning (cf. Miller, 1986, p. 10). In this context, Miller creates a sociological theory of learning of at least two individuals – the theory of collective learning processes – in order to differentiate from psychological theories of learning which predominantly focus on isolated individuals. Hereby, he does not, on principle, question that learning is also an individual process, but rather that when learning fundamentally in the early stages of a child’s development an interactive collective process precedes or determines this process of learning something new (Jung & Schütte, 2018). In this approach, the concept of argumentation is essential. Miller differentiates argumentation, and hereinafter more specifically collective argumentation, which is based on rationality according to Habermas (1985), from “communicative action” based on something uncontroversial (p. 37f). The principal specific characteristic of collective argumentations is the aspect of rationality. Mathematical learning, therefore, takes place in the negotiation of several people about what is viewed from a mathematical perspective as rational. Krummheuer and Brandt (2001) draw on this aspect of learning in collective argumentations and create a theory of interaction of mathematical learning. For them it is of great importance for mathematical learning to occur that pupils participate in collective argumentation within classroom interaction (cf. ibid., p. 20).

Analysing an increase of participation in various settings of learning

The focus of multi-age mathematics education research has, therefore, been on collaborative learning because it is seen as an important element of mathematical learning processes, however, seems to lack in teaching practices. Besides overarching concepts which have been developed for collaborative learning within multi-age mathematics education (e.g., Nührenbörger & Pust, 2006), interactions between learners who are heterogeneous in age have been examined. When analysing these interactions (e.g., with regard to the construction of knowledge), the pupils show a great variety of interaction reaching from working alongside each other to the co-construction of new knowledge by either working in the “zone of proximal development” and therefore constructing new knowledge or by reflecting on pre-existing knowledge and therefore deepening it (Nührenbörger & Steinbring, 2009, p. 118). Additionally, varying qualities within the helping interaction are identified ranging from product-oriented help to process-oriented forms of help (cf. ibid., p. 126). Another study by Matter (2017) shows that it is beneficial for learning when two pupils of different grades are collaborating when either both partners are subject-specifically balanced or when they are not that the social behaviour is important for learning to take place (cf. ibid., p. 303). When both pupils have a very low subject-specific competence, only a low gain in learning is seen (cf. ibid., p. 316).

Studies on multi-age mathematics education in primary school lay a good foundation for our project. However, they predominantly focus on dialogues between two pupils which are heterogeneous in their grade level and concentrate either on how knowledge is constructed (Nührenbörger & Steinbring, 2007) or when interactions are beneficial for learning in such settings (Gysin, 2018; Matter, 2017). In contrast, a sociological theory of learning guides our project, focusing on processes of change within the participation in the processes of negotiation of meaning and on various settings within mathematics classrooms that go beyond pair work.

Research question

One of the characteristics of multi-age education is that most of the time at the beginning of a school year older pupils leave a class and younger learners join the class. This means that those
who used to be the younger learners over time become the older learners of a class. They, thus, experience a change in their role within the class. This can provide the opportunity for learners to also change their role within subject-specific negotiation of meaning and potentially participate more autonomously in collective argumentation that will lead to a broader range of possibilities for learning mathematics.

Because learning is seen as the increasing autonomous participation of pupils in collective argumentation within classroom interaction, the overall question this research seeks to address is: How do pupils participate in collective argumentation within multi-age mathematics education and is it possible to observe a development in the participation over time? In order to answer this, firstly one has to identify how this participation in collective argumentation takes place in general and if patterns of interaction can be identified. Secondly, the analyses of the interactions over a period are then compared to each other in order to answer the question whether there is a change in the degree of autonomy. As various settings (e.g., pair work, group work and whole class discussion) are seen as beneficial for multi-age education because they can all lead to bringing forth-collective argumentation, the third sub-question is raised whether there are differences in the interaction of the learners between different settings of learning within multi-age education. In this paper, only the first sub-question of how learners participate in collective argumentation with multi-age education is addressed further, as this is the basis for the other research questions.

Interactionist approaches of classroom research

Methodologically, this work can be located within qualitative methods of social research which follows a reconstructive-interpretative approach and have the aim to ‘understand’ the actions of the individuals participating in class and to develop local theories (Schütte, 2011, p. 776). More specifically, this work is located within interactionist approaches of mathematics educational classroom research which have their origin in Germany in the 1980s (Krummheuer, 2011; Krummheuer & Brandt, 2001). For this study, the classes are filmed on several occasions over a period of one to two years in order to reconstruct possible age-specific changes concerning the participation in collective argumentation of the children. Through this, a potential change in the pupil’s autonomous participation in collective argumentations is to be identified and described.

The interactions of the pupils are analysed using Krummheuer and Brandt (2001), who also filmed in multi-age schools and subsequently analysed the scenes by using 1) the analysis of interaction, 2) the analysis of argumentation, and 3) the analysis of participation. Step one is the basis for the following analyses by reconstructing the processes of interactive negotiation of meaning. The subsequent analyses of argumentation and of participation are done on the basis of the summaries of the interpretations (Krummheuer, 2015, p. 53). Through the analysis of argumentation, which is based on Toulmin (1969), one can identify which children contribute to which of the four possible functional categories of an argumentation: data (undoubted statements), conclusion (inference together with the data), warrant (contribution to the legitimation of the inference) or backing (undoubtable basic convictions which refer to the permissibility of the warrant). In this research project, the analysis of argumentation is used to show potential differences in the functional categories of an argumentation when looking at the different children participating in multi-age mathematics education depending on their age or the setting they are in.

In order to identify differences in the participation of the pupils in collective argumentation the analysis of participation is used. This analysis includes the production and the recipient design and is able to show which pupils participate actively or receptively in a polyadic interaction (Krummheuer, 2011). For this paper only the production design is relevant which analyses the utterances in order to identify which individual speaker is responsible for a functional category of an argument on a syntactic and/or on a semantic level. Overall, there are four roles that pupils
can have according to this analysis: the author (who is responsible both for the content and the formulation of the utterance), the relayer (who is responsible for neither the content nor the formulation), the ghostee (who is responsible only for the content of an utterance) and the spokesman (who is responsible only for the formulation of an utterance) (Krummheuer, 2015, p. 58). By analysing these roles, the question is answered in what way there are differences in the participation of the pupils in collective argumentation regarding them taking syntactic or/and semantic responsibility within multi-age education.

Overall, this research seeks to describe how mathematical learning takes place collaboratively in multi-age education. According to the theory of interaction of mathematical learning by Krummheuer and Brandt (2001), the possibility of mathematical learning in a school context is seen within an increase of autonomy in formats of argumentation when spokesman and ghostees increasingly occur within production-designs, and when processes of argumentation with a complete “core” of an argumentation - meaning data, conclusion and warrant - are produced (p. 59f). These are seen as possible indicators of successful learning processes because e.g., spokesmen or ghostees take up previous utterances but also add new aspects to the interaction and therefore add to or deepen the thematic development of the interaction. This is seen as an advanced form of autonomy, whereas in contrast to these, being an author points to pupils already having learned something and being a relayer can be an indication for someone being at the beginning of a learning process (Brandt, 2004, p. 37f).

Autonomous participation of all pupils in multi-age education

The group work analysed in this paper is from a mathematics lesson in a multigrade class including 24 children from first, second and third grade. At the beginning of the lesson the students are divided into groups of three which mostly contain one pupil of every grade level. The task given is: “Tina and her family want to decorate their Christmas tree with hand painted Christmas ball ornaments. They want every ornament to look different. To paint the ornaments they have the colours red, green and blue. With these, they can paint dots or stripes. For each ornament they want to use two colours and one pattern at the most. Which possibilities do they have when painting their ornaments?” To solve this task the pupils have material on their tables: bigger circles, stripes and dots of each colour. The group, whose interaction is analysed, includes Isabella (1st grade), Hans (2nd grade) and Elias (3rd grade). The recording starts after they read the task aloud and created two possibilities (two green circles one with three blue dots and one with two red stripes). Only a summary of the analysis of interaction is included, because the focus is here on the analysis of argumentation and of participation.

During the first part of this group work, Isabella argues that they should not only use green circles but also the two other colours because they should not only use the same colour. Elias then picks up this idea and starts suggesting to put blue dots on a red circle. Hans also follows up on Isabella’s idea and suggests to put green dots on a red circle. Elias seems to misunderstand Hans’ utterance and reiterates Isabella’s idea. Then, Hans repeats his idea to clarify it and Hans and Elias agree on creating a red circle with green dots. Hans and Elias briefly disagree on the number of dots they should use but both seem to agree implicitly that for the differentiation of the possibilities the number of dots is not relevant. After settling for three, they also disagree on which background colour they should continue with. Elias now wants to use a blue circle and Hans wants to create a second red one using the other colour left for the pattern. They then agree on using a red circle and add two blue stripes. Isabella says that she has found another possibility and points to a blue circle with one green stripe. Contrary to Elias who then wants to create another red circle with a pattern, Hans explains that they have created two blue and two red ones so far and suggests to now create blue circles. All three quickly agree that they will first use a blue circle with red dots.
We see that all three children participate in the creation of possibilities. To show how the pupils participate more specifically, the analyses of three exemplary arguments are presented below.

As shown in Table 1 (with all transcripts translated from German), the first argument is brought forth by Isabella as the author, since she has the responsibility for both the content and the formulation of warrant 1 and conclusion 1. Isabella’s conclusion is then picked up by Elias as a spokesman and used as data 2 to draw the new conclusion 2a, which he is responsible for both syntactically and semantically. Hans then also picks up conclusion 1 as data and draws his own conclusion 2b as an author. Because of a misunderstanding, Elias reformulates the conclusion 1 as a spokesman by taking responsibility of the formulation of Isabella’s content but goes further and is the first one to explicitly state data 1 for the first argument. He can, thus, be seen as author of data 1. Hans then acts as a relayer of his conclusion 2b by repeating it. Hans and Elias never explicitly state the warrants for their inferences, so the warrants were deducted by the analysis of interaction. The backing of the first argument was deducted from the interaction and is based on the conviction the children seem to have that the task should be followed.

Table 1. Analysis of participation for argument 1 and 2 (format based on Krummheuer, 2015, p. 60)
Later in the interaction another argument is brought forth which consists of these components:

Figure 2. Analysis of argumentation for argument 3

It is similar to the first argument and the content is thus not treated as new but with reference to the prior speakers. Specifically, Hans takes up the content of Isabella’s conclusion 1 and the data 1 Elias previously stated and rephrases these. Since he takes responsibility only of the formulation of these functional categories, he acts as a spokesman. Isabella then repeats conclusion 3 to use two blue ones next and becomes the relayer of Hans’ and her own idea and formulation.

Table 2. Analysis of participation for argument 3
Overall, these analyses show that within this group work in multi-age mathematics education at primary level all three pupils contribute in some way autonomously to the argumentation. Specifically, Elias is the author of data 1 and conclusion 2a and is a spokesman of the conclusion 1. Hans is the author of conclusion 2b and a spokesman of conclusion 3. Isabella is the author for the conclusion and warrant of argument 1 and a relayer of conclusion 3. In summary, possibilities of mathematical learning (according to Krummheuer and Brandt, 2001) can be found in this interaction because Hans and Elias are predominantly spokesman of utterances and by this pick up utterances of others and change them structurally. Isabella on the other hand never acts as a spokesman, however, she is the only one contributing a warrant to an argument and therefore is the only one who completes a “core” of an argument. By this she, as the youngest participant in this interaction, not only has a receptive role but also an active yet, compared to the two boys, unique role. In the future, these analyses will be compared with analyses of further group work but also of pair work and of whole class discussions. Furthermore, they will have to be compared to interactions brought forth later in time in order to identify possible changes in the participation of these pupils over time.

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References


Fostering a conceptual understanding of division: A language- and mathematics-integrated project in primary school

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About 20% of the 15-year-old pupils in Germany fail to develop an understanding of the four basic arithmetical operations in the course of their schooling, and of division least of all. The study presented in this paper affords an insight into the conception and evaluation of a language-sensitive intervention project involving 45 third- and fourth-graders from schools whose catchment areas have low sociographic status. Individual misconceptions of division serve as a basis for initiating a sustainable understanding amongst the children in a language-sensitive manner. The analysis shows that the development of a conceptual understanding of division depends on language structures for expressing the connection between division and multiplication and for verbalizing division concepts.

Keywords: Division, divisibility, multiplication, language-based research, primary school.

Introduction

Elementary school children are meant to acquire a confident and workable idea of the four basic arithmetical operations in their first years at school. The arithmetical operation that seems to be the most difficult to learn is demonstrably division, with children especially experiencing mathematical difficulties, and even not acquiring any concept of this arithmetical operation at all (Cawley et al., 2001; Ehlert et al., 2013; Moser Opitz, 2013; Robinson et al., 2006). While there is a relatively large number of studies delving into the operational ideas of addition, subtraction and also multiplication amongst children who are underachieving in arithmetic, division-related studies in mathematics education research tend to be rare. Some exceptions are Ehlert et al. (2013) and Robinson et al. (2006). The research findings currently at hand largely focus on two central aspects: the specific strategies children of different ages use to solve division problems, or the frequency of solutions amongst children who are perceived to have learning difficulties in mathematics in particular. And even though it is well-known that division is distinguished by specific language structures (Anghileri, 1995), owing to the two basic models of division (partitive and quotative), less is known about how to foster a conceptual understanding of division and divisors in a language- and mathematics-integrated way. The study presented in this paper focusses on these central ideas. Therefore, it is necessary to analyze the mathematical and language-based requirements for developing a division concept.

Research findings about division concepts in primary school

According to Robinson et al. (2006), typical strategies pursued by fourth- to seventh-graders to solve division problems include: factual knowledge, recourse to multiplication, partitive division, recourse to repeated addition and/or subtraction in the sense of quotative division, and derived fact strategies. In addition to this, solutions are also guessed, inappropriate strategies are pursued, or the children do not name strategies. Only the strategies of factual knowledge, recourse to multiplication and recourse to repeated addition are found on a regular basis in all grades. The strategy of repeated addition appears particularly dominant in the lower grades. Downton (2008)
resorts to a similar strategy system. She uses the additional strategies of building up as a recital of multiplication tables, as well as doubling and halving. But many research projects do not privilege or pay attention to whether the children have a conceptual understanding of division or of the inverse operation of multiplication. In fact, children can solve division tasks without any idea of, for example, grouping or splitting in equal parts. Especially, children who focus on strategies like knowing, recourse to multiplication or to repeated addition indeed know what to do technically and how to divide but they have no idea what is underneath and what an appropriate mental picture could look like (for representations of multiplication, e.g., see Kuhnke, 2013).

Moreover, it is well-known that correct solutions of division problems are scarcer, particularly amongst children who have difficulties in mathematics. Robinson and LeFevre (2012) show, for example, that sixth- to eighth-graders who are weak in arithmetic fail to understand the connection between multiplication and division as reverse operations. Cawley et al. (2001) furthermore show that the understanding of division amongst eighth-graders fail to understand the connection between multiplication and division as reverse operations. Cawley et al. (2001) therefore refer to the introduction of division as a ‘cut-off’ point in mathematics teaching for many children. Without a confident understanding of division, all children lack central foundations for an understanding of divisors, and hence for elementary number theory, but also arithmetic learning contents (Feldman, 2012). Therefore, a conceptual understanding of divisibility is a central learning target for all children (Moser Optiz, 2013). A stronger networking of multiplication and division is therefore unanimously demanded to advance the understanding of division (Downton, 2008; Moser Optiz, 2013; Robinson & LeFevre, 2012).

Although a subject of little scientific attention yet, is the linguistic sensibility. In learning division, the close association between words describing a real context and the mathematical procedure for solving the related problems (Anghileri, 1995; Downton, 2008) –in the early grades– characterizes a conceptual understanding. At this stage, the interpretation of division is restricted by a somehow ‘simple’ but meaningful language. It is important that children have a conceptual understanding of dividing quantities into equal parts for example. However, many children after the third grade use these ‘simple’ words without recourse to any contextual meaning. Phrases like ‘divided by’, ‘divided into’ or ‘…time…makes’ are often used without any idea of the encoded meaning. These phrases seem to be something like a secret language in the classroom discourse: everybody uses them but only few children understand them. We have to clarify and to connect the meaning of these phrases with the conceptual idea of division. Otherwise, especially children with limited language proficiency in the language of teaching might be overtaxed by the verbal information, by differentiating between everyday words and technical terms, and by recognizing the connection between mathematical symbols and language on their own. Children with mild disabilities particularly face such situation as their language competences are still less differentiated than those of mainstream school children (Cawley et al., 2001).

What can be said in summary is that we need to know more about whether and how the children’s arithmetical skills in this regard, after the introduction of division, can be expanded or misconceptions dismantled, respectively. The support provided for this should meanwhile focus on the language- and mathematics-integrated connections between multiplication and division and on deepening the conceptual understanding of division, but also on a conceptual understanding of divisors as a preparation for future contents of mathematics learning and teaching. This is exactly the approach taken by the research project described in the next section.
Methodology: Design experiments for generating qualitative data

As a basis for this support, a design experiment was developed for the ‘factor tree’ (Figure 1) for the third and fourth grade. In a factor tree the starting number $a \in \mathbb{N}$ is progressively factorised into its natural divisors. The trivial factors one and the original number itself are left out as the factor tree would never come to an end otherwise. This task permits the connections between multiplication and division to be worked out by way of discovery: calculating from the top to the bottom shows the division, and from the bottom to the top multiplication.

![Factor tree](image)

**Figure 1. Two factor trees for 24**

<table>
<thead>
<tr>
<th>Factor tree</th>
<th>starting number</th>
<th>first level</th>
<th>end numbers</th>
<th>prime numbers</th>
<th>12 is factorized into 3 and 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>24</td>
<td>6</td>
<td>3 2 2 2</td>
<td>3 4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>3</td>
<td>3 4</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

12 divided by 4 makes $12:4=3$
12 divided in 3 group
12 divided in fours, 3
3 times 4 equals 12. $3 \times 4 = 12$
3 groups of 4
4 fits into 12, 3 times
3 and 2 cannot be divided any further.
12 has the divisors...
3 is a divisor of 12.
4 is the co-divisor of 3, cause 4 times 3

There was support at four primary schools in major cities of the Ruhr area to 45 third- and fourth-graders in assignments for pairs or small teams (40 or 45 minutes per session). All the schools are located in districts with low socioeconomic status and each support lesson was videotaped. Four pre-service elementary teachers were trained and intensively consulted to serve as resource teachers. The language-sensitive provision of the support was performed in keeping with the scaffolding approach, as posed by Hammond and Gibbons (2005):

[T]his functional theory provided a strong framework for the deliberate and explicit focus on teaching language, teaching through language, and teaching about language. (p. 9)

This differentiates between two levels of language support for technical learning processes: macro-(designed-in) and micro-(interactional) scaffolding. Both language support levels were relied upon in the design experiment with the factor trees. On micro-level, the pre-service teachers were trained to accompany the support in a language-sensitive manner, i.e., by inviting the
children to use more precise language, by introducing technical terms and making consistent use of them but especially by helping the children expressing connections. Thus, the shared language basis should not only provide the children with access to the language of teaching and technical vocabulary, but demonstrably also support content learning (Prediger & Wessel, 2013). On macro-level, word lists that record and visualize technical terms and sentence phrases were drawn up in cooperation with the children (Figure 2). If the children wanted to verbalize their deliberations and lacked the words to do so, this word list provided them with various forms of language support. If it was necessary the children could use mathematical manipulatives to show their thinking with the help of concrete material.

The following provides insight into the individual learning pathways of three primary school children by contrasting documents and short transcripts of the videotaped episodes from the beginning of the support and from the end. The analysis in this paper could only take a local perspective asking whether the support based on the factor trees contributes the required concepts and follows the research question: How can a language- and mathematics-integrated intervention with factor trees support students’ conceptual understanding of division locally, contrasting specific moments at the beginning and at the end of the learning process?

**Empirical snapshots from the design experiments**

The analysis focusses—in a local perspective—the individual learning pathways of the third graders Abbas, Hamit and Xara. Abbas’ family is originally from Iraq. The family has lived in Germany for a number of years already. His performance in arithmetic is on a medium level. Hamit is from Syria. He has only been in Germany for nine months at the time of the support. The school advises him to repeat the fourth grade because his language and mathematics skills are too weak. Xara comes from a family home with little access to education. She receives additional support in mathematics. She is a very bright child, however, who loves playing an active part in discussions, as the analyses will show. In Germany, the children learn the mathematical operation of division in the second grade. Hence, these three children should be expected to be familiar with the division concept.

**Snapshots from the beginning**

In the first support lesson, the children initially worked out the assignment format of the factor trees for themselves. They had to analyse six completely filled factor trees. Then they had to find factor trees with self-selected numbers and describe their calculations in written form. Figure 3 shows the results of this individual work.

Abbas writes that he is calculating ‘multiplication tasks’ and appears to have doubled the ten. He notes this doubling in an additive manner, but still describes it as ‘multiplication tasks’. He seems to connect multiplication with doubling. He moreover fails to divide the ten any further, buttressing the impression that he is solving the factor tree from the bottom up. His written text offers no valuable clues if he has understood the connection between division, multiplication, factors and divisors.

Hamit’s factor trees are correct. Any further diagnosis of his mathematical skills is elusive because he has selected easily divisible numbers (21 and ten) and not made any notes, probably because of his poor German. Xara appears to develop the factor tree multiplicatively from the bottom up because she writes ‘2 times 6 is 12’ and ‘9 times 2 is 18’. Due to this perspective, it makes it difficult to see that she could have divided the six and nine further. What is interesting is that she is adding the numbers twelve and 18, not multiplying them. Perhaps she does not know how two-digit numbers are multiplied. Anyhow, she only describes how she has calculated. But nothing is known about if she has an idea of what ‘2 times 6’ really means. None of the three
children’s documents provide any clues as to whether they have understood the conceptual meaning of division, divided by, multiplying or …times… and whether they have perceived the inversion of division and multiplication in the factor tree.

Figure 2. Written work by Abbas (a), Hamit (b) and Xara (c) in the first remedial lesson

Once the assignment format had been clarified in the first remedial lesson and the word list elaborated (see Figure 2), the game ‘Who divides last?’ was introduced in the second remedial session. In this game, the game master selects a starting number, the opponent divides it once, then the game master divides it again, etc. Whoever performs the last possible division is the winner. After a few rounds of the game, Xara suggests the use of 100 as the starting number.

313 Xara: With the 100 one can calculate as long as one wants. Do you want me to divide now directly?
314 Teacher: No, It’s Hamit’s turn now.
315 Hamit: (H notes 10 and 10. He passes the piece of paper to A)
316 Abbas: (A divides 10 into 5 and 2, points to the 2) Primes.
317 Teacher: Mhm, and what about the 5?
318 Abbas: Do you want me to do that as well? Oh, I know, that is also a prime just like the 2.
319 Xara: Can I now also divide the 10 into something else?
320 Teacher: Yes. You are doing a new division now.
321 Xara: Yippee! (…) er. OK. Let me see first what’s still possible! Ah! (X takes her fingers and counts loudly) 2, 4 (…) 2, 4, 6, 8, 10.5, um, but that is the same as just now. (…) Fiver, tooer ahhh, the 2 and the 5 are the main numbers somehow. 2 times 5 makes 10.
322 Teacher: Are there any other ways then?
323 Xara: No, with the 3 I’d be at 9 then. The 1 would work but that isn’t supposed to. The 6 would be 2 too much. The 8 would be much too much in any case. No, there are no other ways, actually.

Xara appears to assume at the beginning (l. 313) that the number 100, as a particularly large number, also has more partitions and hence more divisors. From line 319 onward she starts thinking about whether the ten can possibly also be divided in any other ways than into the primes of two and five. She is taking a consistently additive approach to this, but also considers numbers such as six and eight that are completely out of the question as divisors. She seems to have a vague idea of the connection between divisors, primes, division and multiplication, but these ideas are still marked by precursor concepts of division and divisibility.
In the third support lesson, the children were invited to find all the factor trees of the numbers two to 25. The finding of all factor trees served to highlight further individual (mis-)conceptions but also problems with the specific language structures of division tasks.

451 Xara: Yes. I think the 9 doesn’t work either.
452 Abbas: The 9? The 4 can’t be divided any further and the 5 neither. Well, then the 9 is out.
453 Xara: Yes, the 9 is out.

Abbas and Xara seem to equate division with halving. This may explain why Abbas takes four and five as divisors of nine into account (4+5=9). Interestingly he mentions that four cannot be divided any further (l. 452) but shortly before they have noted the factor tree of four. Maybe he struggles with the expression ‘divided by’ and wants to express ‘four and five cannot be divisors of nine’. The children seem to have individual ideas about how division works but they do not express a conceptual understanding of ‘sharing equally’, ‘fitting in’ or ‘partitioning in groups’ in fact and even seem to struggle with the specific language structures of division.

Snapshots from the end of the support

The finding and sorting of the factor trees and look at their commonalities combined with the need to express connections have turned out to be very fruitful activities for eroding misconceptions and for fostering expressing connections. The children’s concept of division and divisors is expanding in this lesson in the sense of them taking all the numbers in all the factor trees into consideration as divisors of the starting number and especially in expressing the connections between divisors and division:

535 Abbas: Er, 6 and 4 are divisors of 24.
536 Teacher: Okay. But how can you explain that to me?
537 Abbas: Cause in 24 the 6 fits 4 times. 6 and 4 are divisors of 24.
538 Xara: (points to the factor tree with the first partition into 12 and 2) The same as with the 12, the 4 and the 3. 4 fits 3 times into 12.
539 Teacher: And what about the 2 and the 3, then? (points to the divisors 2 and 3 in the factor tree with the first partition into 6 and 4).
540 Abbas: 2 and 3?
541 Xara: Yes, look, 2 is a divisor cause, 24 divided into twos you get 12, 12 twos into 24. And that is the same with the 3. 3 times 8 is 24, three groups of eight into 24. I think they are divisors, too, because those further down there are also divisors of 24.

While in turn 535 Abbas considers the numbers on the first level as divisors, an enquiry by the teacher (turn 536) forces Abbas and Xara to explain the mathematical meaning of the sentence ‘8 and 3 are divisors of 24’. Abbas defines divisors by using a quotative strategy (turn 537): a divisor fits in an integer without remainder. He expresses a conceptual understanding of division as a fitting in’ concept. Xara seems to share this idea of division with Abbas (538). And in fact, it is Xara who presents another idea of divisors: an idea of partitive division. Three is a divisor of 24 because you can build three groups of eight.

16 is factorised into 4 and 4 because four times four is 16. 4 is factorised into 2 and 2 because 2 times 2 is 4.
2 and 4 are divisors of 16.
Fours and twos fit in 16.
And refugee child Hamit, who has hardly actively participated in the discussions, also appears to have realized the mathematical core of division, as is evidenced by a written document at the end of the support lessons (Figure 4). He describes accurately how a factor tree needs to be calculated, using the language of teaching and technical phrases. His text reflects the connection between multiplication and division as an inversion as he argues within the factor tree from the bottom up as well as top down. And he also expresses a conceptual idea of quotative division as a ‘fitting in’ concept. He furthermore correctly describes all the numbers in the factor tree as divisors, but forgets that 16 can be factored in different ways, and hence at least the additional divisor of 8.

Discussion and conclusions

This short insight into the empirical videotaped data has shown that the deliberate support of the connection between multiplication and division and of expressing the relationship of multiplication and division and of divisors and factors appears to be a fruitful approach for developing a conceptual understanding of division (and for multiplication as well). It underpins the demand that “[p]lacing emphasis on the relationship between multiplication and division and the language associated with both operations before any use of symbols or formal recording needs to be a priority” (Downton, 2008, p. 177). Thus, initial misconceptions could be supplanted by first basic conceptual ideas. Especially the math underachievers Xara and Hamit impress in their individual development. With how much stability these new concepts are invested remains unanswered at this juncture. Moreover, little is known whether these children have really got the core of the relationship between multiplication and division. Much more long-term research is needed to analyse how a conceptual understanding of division and multiplication and the connection of these two operations develops.

But the phrases and technical vocabulary supported in mathematical discourses connected with the necessity of expressing mathematical understanding appears to be a central requirement for success because with the help of these terms and phrases the children have the chance to develop a language-based understanding. The phrases ‘divided into’, ‘divided by’ or ‘…times…makes’ are often used as meaningless phrases. Using phrases like ‘20 divided into two groups of ten each’, ‘four fives in 20’ or ‘a divisor fits in a number without remainder’ shows the thinking practice of the children – a thinking practice that is deeply connected to the ‘language of schooling’ (Feilke, 2012). By the way, misconceptions can be discursively discussed by the use of the technical terms and phrases in the first place. It seems to be possible to foster an understanding of division and divisors (and also of multiplication and factors) simultaneously because a potential language barrier could be deliberately avoided right from the start.

References


Use and meaning: What students are doing with specialised vocabulary

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Words are powerful and give students the ability to communicate mathematical ideas and relationships precisely. Yet not all students are able to access the technical vocabulary encountered in mathematics lessons. Students use mathematical words in various ways and teachers give students different opportunities to both use mathematical language and to develop meanings associated with this language. In this paper we explore how previously low attaining students use mathematical vocabulary during their lessons and how their teachers offered different opportunities for language use. The analysis reveals that there are shifts between developing language use and developing language meaning that depend upon the meaning demonstrated by students in their interactions.

Keywords: Specialised vocabulary, word meaning, classroom interaction.

Introduction

Being able to communicate about mathematics is a key part of the process of learning mathematics, yet many students face difficulties and barriers when learning to use the language of mathematics. Learning mathematics is far more than coming to know specialised vocabulary (Morgan, 2005) but this vocabulary can support and enable students to develop their understanding of mathematics. In this paper, we examine students’ use of mathematics specific vocabulary using an ethnomethodological approach, and explore the opportunities offered by the teacher to use this vocabulary in different ways within classroom interactions. In particular, we offer examples of how these opportunities differ depending on the meaning the students appear to have for these specialist terms as they use them during whole class interactions. These opportunities include the different prompts that the teacher used to generate a need for the specialist word to be used in the interaction.

Learning mathematical words

What it means to know a word is multifaceted and complex, and different researchers have attempted to categorise this complexity in different ways. We take a situated perspective on learning mathematical words, focusing on their use within what Moschkovich (2012) calls the mathematical Discourse practices of the classroom. One simple way of categorising word knowledge is to make a distinction between receptive or passive knowledge and productive or active knowledge, which distinguishes between students recognising and understanding words that they hear or read and students using words for themselves either in speech or writing (Bravo, Cervetti, Hiebert, & Pearson, 2008). Sfard (2008) makes a further distinction between three types of active use, routine-driven, phase-driven and object-driven use. However, students use words in different ways and within these distinctions there is more of a continuum of what it means to know and use a word.

Within mathematics there is a specific need to use words very precisely in contexts that are often unique to mathematics (Bauersfeld, 1995). Students’ understanding of definitions, informal or formal, also needs to enable them to make distinctions between what is or is not an example or specific use. Furthermore, the contexts in which students need to use words are often highly
abstract and connections with existing knowledge or often only possible within mathematical contexts too.

Many researchers have emphasised the role of contexts and connections to other words and ideas in the learning of new words. Words need to be studied in their natural habitat; within sentences with a communicative purpose or within contexts where the need for the word arises (Moschkovich, 2015). Students need to be able to make connections with what they already know (Bravo, Cervetti, Hiebert, & Pearson, 2008). In the case of words that students are meeting for the first time in the mathematics classroom, these connections may need to be with existing mathematical words or concepts with which the students are more familiar.

Students also need to experience the word in a variety of contexts in which it may arise (Bolger, Balass, Landen, & Perfetti, 2008). Yet in mathematics, students may only experience a particular word in one lesson or across a short sequence of lessons. Some words rarely arise within new topics and some words may only be met when a topic is revisited months or perhaps years later. In mathematics, we also tend to use language more precisely than we do in everyday contexts, and use particular technical terms in a narrow range of contexts. This can make it easier to misuse words or lead to broader concept definitions than those generally accepted by mathematicians.

Methods
The data for this paper comes from a larger project where two groups of mathematics teachers worked with the authors to develop their students’ use of mathematical language. This involved the teachers regularly videoing themselves teaching and sharing clips of this teaching in meetings for discussion. The project itself did not focus on lower attaining students, but for the analysis discussed below we consider videos from classes identified as having low prior attainment as the teachers sharing the clip focused their discussion on their students’ use (or lack of use) of technical vocabulary. These classes were described by the teachers as including many students with identified additional literacy needs, such as a learning disability or very low reading age. In the UK classes are usually set by prior attainment, particularly in secondary school (age 11-18), and the class were the lowest attaining classes within each school. The extracts included in this paper come from one such class and are used to illustrate the complex relationships between developing word meaning and developing word use. The students are around 12 years old. The video clip that the extracts are from were chosen by the teacher herself to share with her department in order to explore with them and the authors how her students were using mathematical language and the difficulties they faced when doing this.

Within the videos that the teachers shared with us, all the whole class interactions were transcribed and for the analysis below, all student turns that included a mathematics specialist word or phrase were identified. These turns were then analysed within the wider sequence of interaction looking specifically at what the students were doing with these word or phrases and how this use was reflexively related to the teacher and student turns that occurred before and after. An ethnomethodological approach was used for the analysis which is an inductive process focusing on how the participants interpret what is going on through how they interact in the classroom. Specifically, the analysis presented in this paper uses Conversation Analysis to focus on the actions of the teacher and her students when using technical mathematical vocabulary to describe properties of numbers. The analysis focuses on vocabulary use as this arose explicitly in all the classes considered and was a focus for the teachers sharing the videos in the team meetings.

Results
In this section, we explore the relationships between students developing meaning for particular mathematics specialist vocabulary and students using this specialist vocabulary. Students use
mathematical specialist vocabulary in different ways during classroom interaction and we illustrate how teachers offer different opportunities for students to use these words and/or develop their meaning in relation to these words. The opportunities offered for students to use specialist vocabulary themselves is influenced by the previous meanings students display in the interaction.

The extracts used below to illustrate the different opportunities come from Emma’s lesson on properties of numbers. The particular lesson was towards the end of a sequence of lessons looking at factors, multiples, and prime, square, cube and triangle numbers. The video focused on a task where the students needed to ask the teacher if she ‘liked’ different numbers between 1 and 25 (see I like… http://nrich.maths.org/6962 for the task used). Emma used a rule related to number properties to classify numbers as ones that she liked, e.g. those that were a multiple of 3, and those that she did not like, e.g. those that were not a multiple of 3. The students needed to work out the criteria that Emma was using to make the decision of whether she liked a number or not. The first extract below focuses on an example where the numbers were all multiples of three and the second and third extracts focus on an example where the numbers were factors of twenty.

After around 9 minutes of working on the task as a whole-class, the students have identified several numbers that Emma likes and a few that she does not like and have begun talking about whether the numbers belong to particular times tables. The students have offered suggestions that the numbers belong to the three, six, twelve and twenty-four times table (though they have not said specifically that they all belong to the three times table or six times table). In turn 145 Emma introduces the word multiple for the first time in this activity and models its use in turn 147. The students continue to use the phrase times table, such as in turn 151 until several turns later where in turn 193 a student offers the rule that they are all multiples of three. From this point onwards within this task different students use both times table and multiple fluently and interchangeably.

The students use the word multiple in both semantically and lexically appropriate ways. Emma repeats both answers and rewards the students for ‘a good maths reason’. The need to use the word multiple arises from Emma’s prompts to use the word specifically, and her general prompt to use mathematical reasons and mathematical talk. The need does not arise naturally in this context as the students are able to articulate the meanings they are attempting to express using the more informal language of ‘belonging to the times table of’.

**Extract 1**

145 Teacher: you are giving me (.) such good reasons. as well as thinking about times tables, can you think of the word multiples

146 Student: yeah.

147 Teacher: so, could we say it’s a mult- those- they are multiples of three, they are multiples of six (.) some of them are multiples of twelve, number twenty-four. you are so close to getting the answer.

148 Student: it's that kind of right, because like basically they goes in order, like six, like because they're going down [by ]

149 Student: [hey] we just said that (.) hhh

Transcript Omitted

190 Teacher: what was my reason for liking those numbers?

191 Student: because (.) I don't know, they're special.

192 Student: because, because they're all -

193 Student: they're multiples of three
Extract 2 comes from the second scenario where the numbers Emma likes are all factors of twenty. Before the extract begins the students have discovered that Emma likes the numbers 5 and 10 but not the number 12. The next student asks if Emma likes the number 20 and then offers a rule that ‘ten times two is twenty’ in turn 242. The rule focuses on the relationships of twenty being a multiple of 2 and/or 10. This response is repeated before the student also offers that it’s ‘in the five and ten multiples’. Whilst in what the students are saying they are focusing on twenty being a multiple of 2, 5 and 10, the connection they are making also refers to the relationship that 2, 5 and 10 are all factors of twenty. Additionally, at the end of turn 250 the student is using the mathematical word in a lexically incorrect way but semantically the word is used appropriately. The way that the word is used would be lexically correct if the word multiple was swapped for times tables. This is rephrased by another student into its usual form in turn 256 and this response is repeated by the teacher. Each of these reasons, and other reasons such as the numbers are all even, are accepted by Emma as being ‘good maths talk’ (turn 259) even though none of them describe all of the numbers that are on the board (factors of twenty). The students are continuing to use the mathematical language in semantically and lexically appropriate ways in that they are able to use the word multiple to express the meanings they want to convey, and the teacher is rewarding and reinforcing this through how she responds to their suggestions for rules.

Extract 2

237 Student because it's (in the five times table).
238 Teacher or what else could you say?
239 Student it is a multiple -
240 Student it's a multiple -
241 Teacher it's a multiple of five. it’s in the five times table, I think I can give you a stick for a good maths reason.
242 Student1 twenty, because ten times two equals twenty.
243 Student oh, no
244 Student oh, yeah.
245 Teacher I do, I do like twenty and your reason was? what was your reason for choosing twenty if [I'm going]
246 Student [ oh um ]
247 Teacher to give you a stick?
248 Student1 ten times two is twenty.
249 Teacher because ten times two is twenty.
250 Student2 and it's in the five and the ten multiples yeah so
251 Teacher now,
252 Student so
253 Teacher you're right that twenty is ten times two. but I like sname's reason. he said it was
254 Student2 it's a multiple of five and ten.
255 Teacher it's a multiple of five and a multiple of ten.
256 Student3 and it’s even
257 Student yeah. and-
258 Teacher and it’s even. okay, um, there's definitely some good maths talk in there.
259 Student yeah!
Teacher a lollypop stick. you haven't discovered my reason for my choice yet

In Extract 3 the focus of the discussion then shifts closer to ideas around factors following the inclusion of the number 4. In turn 298 Emma indicates that the rule is not that all the numbers are multiples of 5 now that 4 has been included, but the students are still focusing on a multiple relationship as is shown in turn 329 where a student suggests that all the numbers are a multiple of 2 except 5. The interaction continues with the students now referring to a relationship between two specific numbers on the board, such as two goes into four, rather than a general property that is true of all the numbers. After several students use the phrase ‘goes into’ to describe these relationships, Emma prompts for the mathematical word in turn 344. This relationship of ‘goes into’ is generalised to a common relationship by a student in turn 374 but the use of goes into continues. Emma asks again for the maths word that describes this relationship in turn 375. The responses that follow over several turns are all one or two-word answers and include add, divide, multiple, integer, cube number and triangle number. The interaction becomes a game of guess the word the teacher is thinking of, and the students do not give the word factor until turn 461. The relationship being described by the students is bidirectional in that 2 goes into 4 means both that 4 is a multiple of 2 and that 2 is a factor of 4. The distinction between these two meanings is not clear from the students’ contributions and their responses to Emma’s request to name the relationship indicate that factor is not yet a word that they have active control over (and presumably integer too).

Extract 3

297   Student number four? that's not in the five times table.
298   Teacher doesn't have to be, does it, or even a multiple of five.
Transcript omitted
329   Student so basically, every number except five is a multiple of two.
330   Student yeah, and ((inaudible)).
331   Teacher that's true, but I have to have one reason for all my numbers.
Transcript omitted
342   Teacher That was really good thinking, you are nearly there, but I didn't hear many maths words. ’cause, you were thinking, you said two goes into four.
343   Student yeah.
344   Teacher what's the posh maths word for goes into?
345   Student oh, multiple. uh -
346   Student add.
Transcript omitted
366   Teacher you can ask me about another number in a minute, see if that changes anything. sname?
367   Student um, two times five is ten.
368   Teacher two times five is ten.
369   Student and then two, two times ten is twenty.
370   Teacher two times ten is twenty.
371   Student so, like they ((inaudible)) and then four times five is, um, 25. so like -
372   Teacher think that one through again. four times five is…?
373   Student twenty.
374   Student so they like all go into each other.
Teacher: oh they’re all going into. what is the maths word for the going into thing?

Student: divide.

Teacher: yes, but there's another maths word that was on your sheet this morning. I'm going to give you the stick.

Student: uh, can we get another one if we get the word?

Teacher: [laughs] um -

Student: integer

Teacher: you are talking about [maths aren’t you good good]

Student: [integer, integer!]

Teacher: that wasn't the maths word, not the one that we're thinking of here. it's all about [things go]ing into.

Student: [subtract ]

Student: multiple.

Teacher: [laughs] and it is to do with dividing, but it's not the word that I'm looking for.

Student: [((inaudible))]}

Student: cubed num[ber ]

Teacher: [what]’s the connection between all these?

Student: cube number.

Teacher: there's one missing. shall I put the one missing?

Student: triangle number.

Teacher: there's one more number here that I like.

Student: squared.

Emma’s prompts and questions have shifted from asking students to describe the relationship where students’ responses include them trying to express the meaning of the relationship, to asking them identify the ‘maths word’ for goes into. Her prompts attempt to give the students the language to describe the general relationship that their specific examples are beginning to point to. The students are able to clearly explain a relationship of multiples using both formal and informal language, but this is not the case for factors. Here the students are not able to communicate the relationship that all the numbers are factors of 20 either formally or informally. This prompts a shift in the nature of the interaction to the students attempting to name or label the relationship. However, this is unsuccessful and the students are instead blurtling out all the other technical words they have encountered this lesson that appear on the worksheet on their desks.

**Conclusion**

In this paper, we have examined students’ use of mathematics specific vocabulary and the opportunities offered by the teacher to use this vocabulary in different ways. Students need opportunities to construct meaningful discourse about mathematics in order to develop their use and understanding of the language of mathematics (Schleppegrell, 2007). These opportunities were adapted throughout the lesson depending on the meanings the students appear to have for these specialist terms as they use them during whole class interactions. When students demonstrated that they had some appropriate meanings and other ways of expressing mathematical ideas using informal language the opportunities meant students were able to use the
technical vocabulary in meaningful ways. That is, they were using these technical words to communicate mathematical ideas and relationships. When the meanings of the words were less clear or where the concept that the words label were less fully formed, these opportunities focused on using words to label or name.

These opportunities were also created through the prompts that the teacher used to generate a need for the specialist word to be used in the interaction. The need for the word multiple does not arise from the mathematics that the students are working on as they already have words they can use to describe the relationship and they are able to express this relationship clearly. Instead, the need arises from the task and the way the teacher enacts the task. The teacher prompts students to use the word multiple in their descriptions and offers them opportunities to describe the relationship between the numbers on the board. The need for the word factor arises from a need to generalise a relationship and to name this generalisation. The teacher uses prompts that offer students the opportunity to name the relationship rather than to communicate the relationship they are attempting to describe.

The use of the technical language involved in communicating mathematics can be a particular challenge for students whose difficulties in learning mathematics are confounded by broader literacy or language issues. In the case of factors, the students were able to identify the individual relationship of one number being a factor of another, possibly as this is the same as the relationship of one number being a multiple of another. However, generalising this to a relationship between all the numbers on the board took considerable prompting from the teacher and this was accompanied by the challenge of identifying the word used to describe this relationship. Further research is needed to investigate this relationship between word use and word meaning the interdependence of these two aspects of technical language.

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Transcription Conventions (Jefferson, 1984)

[ text ] Brackets Indicates the start and end points of overlapping speech.
( ) Pause A noticeable pause.
. Period Indicates falling pitch or intonation.
? Question Mark Indicates rising pitch or intonation.
, Comma Indicates a temporary rise or fall in intonation.
( text ) Parentheses Speech which is unclear or in doubt in the transcript.

References


Language hurdles on the way to an understanding of length in early mathematics education
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Early mathematics education has been gaining more and more attention and importance in the domain of mathematics education research on language. The role of language, especially in early mathematical learning processes, is repeatedly emphasized. In this paper we present the complexity of linguistic difficulties for mathematical learning processes using the example of measurement and length, and illustrate aspects of such complexity with empirical data. In particular, we examine lexical, grammatical and semantical aspects.

Keywords: Early mathematics education, measurement, linguistic competence, oral language.

Introduction
Research in early mathematics education has shown the relevance of language for cognitive learning processes. Academic language proficiency is widely acknowledged as an important factor for successful education and schooling. Experts agree that academic language education processes should start as early as possible, be designed age-appropriately and be oriented to a specific content (Prediger, 2015; Rudd, Satterwhite, & Lambert, 2010). Unfortunately, the German school system is still in need for effective concepts to support children with disadvantageous starting conditions like migration, low socio-economic status or developmental speech disorder, in order to provide them with an equal chance to participate in (mathematics) education processes (Gogolin & Lange, 2010; Prediger, Renk, Büchter, Gursoy & Benholz, 2013). While most German preschool teachers seem to be aware of their function as language role models, only few have acquired a professional background that enables them to support interactive language learning processes (Ritterfeld, 2000). Michel, Oflner and Thoma (2014) examined German kindergarten teachers concerning their linguistic knowledge, their knowledge about children’s language development and their ability to choose effective interventions. In this study only half of the questions, which experts see as relevant to foster language development in young children, were correctly answered. Isler, Künzli and Wiesner (2014) analysed conversations between Swiss kindergarten teachers and children in order to investigate the potential for the acquisition and fostering of academic language skills. Their results show that kindergarten teachers have to be made more aware of the central meaning of their language acts and to support a setup of practical action patterns for the fostering of academic language skills. Our research supports their findings since we find only few approaches for supporting children’s language development in mathematic learning opportunities (Brandt & Keuch, 2017, in press). Based on these results, our aim is to raise preschool teachers’ awareness for possible language hurdles, so that they are able to pay special attention to them in connection with supporting mathematical learning. Our objective is not to avoid these challenging language structures, but to use them in a way that fosters the children’s language as well as mathematic development.

What we call language hurdles are first of all special features (lexical, syntactic or pragmatic) of academic German, a register often expected to be spoken but not explicitly taught at school (see Gogolin & Lange (2010) and Leisen (2013) for first approaches to integrate Academic German
language teaching in (mathematics) content teaching). Especially in pre-primary school settings, this means more than the usage of technical terms but also grammatical structures that enable children to grasp and express concepts. Furthermore, we look at characteristics that are hard to acquire for children with German as a second language or even first language. Finally, we are interested in words and expressions typical of a specific content. These features are analyzed with respect to the question whether they may impact on the understanding of a specific mathematical topic. For this specific mathematical topic, we choose measuring as one of Bishop’s (1988) six basic activities and length as a magnitude that young children can directly perceive. Our aim is to figure out special features of the German language that preschool teachers can use as an opportunity to foster children’s linguistic as well as mathematical development. Hence, we will first try to answer the following research question:

- Which linguistic structures (lexical, syntactic and semantic) can be problematic concerning the establishment of early concepts of length and how do preschool teachers deal with these problematic features?

**Measurement and length**

In order to answer the question above, this paper addresses measurement and the central magnitude length. Bishop (1988) claims that “measuring (...) is concerned with comparing, ordering, and with quantifying qualities” (p. 34). For the activities concerning measuring, an abstraction process is key, which results from a concentration on a quantifying characteristic. Real objects are compared regarding for example their length or weight, independent from their form, colour or other characteristics. The quantification of quality results from comparing with a unit, which is seen as a fundamental idea of all measuring activities independent from the magnitude. Here it becomes clear that talking about comparing, ordering and quantifying quality asks for a differentiated language usage, including technical terms and specific grammatical structures that are needed for example to describe a comparison or a quantification.

Many curricula for early mathematical education in Germany put emphasis on measurement. It does not only represent a link between mathematically abstract concepts and everyday life, but also comprises multiple inner-mathematical relations, especially with numbers and geometry (Barrett et al., 2011; Sarama, Clements, Barrett, van Dine & McDonel, 2011). Beyond, the concept of measurement can be seen as a basis for further concepts, for example fractions and rational numbers (Barrett et al., 2011). While we take research on the question how children acquire a (geometric) concept of magnitudes, which milestones children have to master and where they face mathematical difficulties (e.g., Sarama et al., 2011) as a background for our linguistic analysis, we will not discuss it in detail. In general, magnitudes like length and area are directly perceivable and accessible for young children. However, they are not easily to grasp because of their relations between each other and because children have difficulties to distinguish between them (Barrett et al., 2011; Skoumpourdi, 2015; Castle & Needham, 2007). Although an integrated approach for different spatial magnitudes, especially in early education, is seen as reasonable (Barrett et al., 2011) in order to understand the differences and the fundamental idea of measuring as comparison with a unit, here we only concentrate our linguistic analysis on length. Length belongs to spatial measurement. Piaget, Inhelder and Szeminska (1960) define the idea of special measurement in this way: "To measure (in Euclidean metrics) is to take out of a whole one element, taken as a unit, and to transpose this unit on the remainder of a whole: measurement is therefore a synthesis of sub-division and change of position" (p. 3). This change of position requires the understanding that (a) the size of the unit is conserved and (b) that the unit can be used iteratively. In doing so, the unit must be copied and repeated without a gap and without overlapping. Concrete objects become representations of length and their mutual characteristic is constituted in their one-dimensional linearity. The activity of measuring length concentrates on the determination of the
linear expansion. Therefore, you have to distinguish between objects with a rather clear linear characteristic, for example sticks or distances, and objects with more than one dimension that can be measured (width, height, depth) (Nührenbörger, 2002; Skoumpourdi, 2015). Consequently, it becomes obvious that speaking about length comes along with specific linguistic challenges, for example concerning the characteristic of linearity and the differentiation from area.

**Language hurdles – Empirical examples**

Our data is taken from Project erStMaL (early Steps in Mathematical Learning) (Acar Bayraktar, Hümer, Huth, & Münz, 2011). The examples stem from seventeen group interactions with a preschool teacher and one to four children prepared and realized by the preschool teachers themselves. The videos were transcribed and annotated with EXMARaLDA (see http://exmaralda.org/en). Initially, our categories were deductive from research on problems in first and second language acquisition. In addition, we generated further categories inductively. By looking for signs of language awareness in preschool teachers, we detected a few situations where the teachers rather inhibit than facilitate mathematical as well as language learning. The wrong handling with some hurdles might lead to (partially) wrong concepts that could inhibit further mathematical learning. Other seem to be of importance only concerning language learning at first sight. Participating in mathematical negotiation processes, however, is seen as a main condition for mathematical learning.

**Lexical aspects: Interferences**

Research shows that when it comes to technical terms in mathematics, interferences are a specific difficulty, or rather learning opportunity (e.g., Abshagen, 2015; Lorenz, 2012). Interferences are a result of cross-linguistic influence (Lightbown & Spada, 2013), which means that two or more languages or two or more registers, respectively, interfere with each other. Depending on the languages or registers you look at, the amount and kind of interferences can vary. Mathematical language contains many words that exist in everyday language with a different meaning. This might lead to conflicts within the learner’s mental lexicon. Maier and Schweiger (2008) call attention to the fact that learners might not consider words that they think they are already familiar with, and therefore miss the meaning of the word as a technical term. In the following example, the preschool teacher Barbara uses the word “point” (“Punkt”), which in German can be used in six different contexts (see https://www.duden.de/rechtschreibung/Punkt).

**Barbara:** From THIS point here to the finger where I hold it, we had ONE meter [Von DEM Punkt hier bis zu dem Finger wo ich ihn festhalte, hatten wir ja EIN Meter]

A Punkt on the one hand is a small round spot that you might find in a polka dot dress. On the other hand, it can be not a concrete point but something abstract like a geographic point (meeting point) or a point in time. You can also make a point in an argumentation or reach a certain number of points in an exam. In German you say einen Punkt erzielen when you score a goal. While point in the sense of scoring a goal in sports is rather unlikely in this situation, it might not be obvious if Barbara is indicating an existing graphic dot or rather an imagined point in the sense of place or time. Since she does not seem to be aware of the ambiguity of her utterance, she misses a possible linguistic learning opportunity and might also aggravate the children’s understanding of what one meter actually is. On another occasion, Barbara tries to explain the meaning of centimeter, while indicating the distance of one centimeter on a measuring tape with her fingers:

**Barbara:** From one long line to the next, so just this little box yes? That’s a … that’s a centimeter there [Von einem langen Strich bis zu dem nächsten, also nur dieses Kästchen ja? Das ist ein … das ist ein Zentimeter da]
To explain the concept of centimeter with the word little box can be difficult for various reasons, as Kästchen (little box) can have various meanings in German (see https://www.duden.de/rechtschreibung/Kaestchen). First, it is the diminutive form for Kasten (box). However, Kasten again has several meanings. Most importantly, it can be either a three-dimensional object (a box to put things, like jewelry) or a two-dimensional square on a sheet of paper (Tick a box). Second and very prominent in school contexts, we have the meaning of Kästchen as a name for each single square on graph paper. Barbara might perceive the two lines that indicate where one centimeter starts or ends, and the boundaries of the tape as a little box. However, in this situation it is not clear if the children achieve a similar degree of abstraction.

Lexical aspects: Word formation

Abshagen (2015) mentions compounds and nominalizations as being difficult to understand for learners. There is a distinction between endocentric (the meaning of a compound word can be guessed by combining the meanings of its components) and exocentric compounds which obtain meanings that cannot be guessed by the combination of their components (Bieswanger & Becker, 2017). While the latter has to be learned as individual vocabulary, the former seems to be a plausible principle, even for young children. When one preschool teacher holds up a ruler (Lineal) and asks the children for its name, one boy calls it a Maßbrett (Measureboard) and intuitively constructs an endocentric compound. It is not the correct word in this context but resembles the names for other measuring devices like Maßband (measuring tape). Instead of using it as a learning opportunity, the preschool teacher ignores the word building process and says “That’s a ruler”. Nominalizations, however, are a kind of derivation and often emerge by adding a suffix (for example -er) to the stem of a verb (Bieswanger & Becker, 2017). Again, children implicitly seem to know this process, as the following example shows. A preschool teacher asks for the name for ruler (Lineal). When a girl with German as a second language answers Messer (knife, but also measure-er), the preschool teacher praises her answer with the words “good name”. Although it seems to be the wrong word at first sight, there probably lies a very interesting word building process behind it, which the preschool teacher at least subconsciously seems to notice and even appreciates it (for this example also see Brandt & Keuch, in press).

Lexical aspects: Measuring devices, measuring units and indication of size

In most situations, the preschool teachers measure the children’s body length, name and record them in different ways. Some write them down, others document them with woolen strings (Brandt & Keuch, in press). When you capture body length with standardized measuring tools, you read the numbers on the measuring tools as a scale value. With measuring tools, the scale value indicates the corresponding measuring value based on a certain scale unit; for ordinary levelling boards or folding rules, that is centimeter. When using measuring sticks and folding rules, the kindergartners on the one hand are confronted with measuring units whose meaning they rarely comprehend and only hesitantly take over into their active vocabulary (Brandt & Keuch, in press). Moreover, they also have to deal with numbers that exceed their actively mastered range of numbers. Some preschool teachers become very creative when trying to make these numbers more accessible for young children, like Sabine in the following example:

Sabine: And you are exactly as big as this red number [Und du bist genau so groß wie diese rote Zahl ist]

On the folding rule used in this situation, the scale values are marked in red for every ten centimeters, while all other numbers are black. The red number therefore references the measured body length. Sabine syntactically uses the red number as a representation for the measured size value 110 centimeters. In the passage before, we could show that children become very inventive when it comes to naming standardized normed measuring devices. In Brandt and Keuch (2017) we show how children particularly struggle with objective non-normed measuring devices. After
having used building blocks to measure a child’s body length, one of the children takes a piece of chalk and says: “I measure it with the chalk” [“ich messe mal mit der Kreide”]. What he actually does, however, is drawing a line on the floor between two marks that the child’s head and feet. In this situation, the preschool teacher could have taken up this utterance by showing that you can measure a distance with a piece of chalk, but just by drawing a line on the floor. Unfortunately, the preschool teacher does not comment on Can’s utterance or actions.

**Grammatical aspects: Valence**

In Brandt and Keuch (in press) we analyze the grammatical valence of the verb measure in oral language. The term valence derives from chemistry, where it describes the ability of an atom to link with other atoms. Linguistics takes this model to explain the fact that in valence theory, a verb asks for a certain number and kind of sentence constituents (like subject, different objects or adverbial phrases) in order to form a correct sentence (Herbst & Götz-Votteler, 2008). The addition as well as the omission of constituents might lead to different meanings or even incorrect utterances. The verb “measure” asks for a subject (someone who measures) and an object (something or someone that is measured). Often, you also have an adverbial phrase that tells you the measuring device used (with what you measure). When one preschool teacher asks a child to stand back-to-back with another girl and compare their sizes, she accompanies her request with the words “Do you want to measure yourself with Sadira?” [“Willst du dich jetzt mit der Sadira messen?”]. By adding the reflexive pronoun yourself [dich], the teacher (probably involuntarily) changes the meaning of the verb and literally asks for a competition, but not necessarily a measurement (Brandt & Keuch, in press).

**Semantic aspects: Conventionalized expressions (Phraseologisms)**

Another aspect that has been neglected for a long time are so called phraseologisms, in this case a combination of words or a functional unit whose meaning cannot be solely deduced from the combination of the single words (Cowie, 2001). Experts agree that the acquisition of phraseologisms demands special strategies that only develop during primary school (Buhofer, Burger & Sialm, 2012) and that it constitutes as a hurdle especially in a second language (Granger & Meunier, 2008). In the following example, a preschool teacher uses the formulaic expression “back to back” to ask the children Deny and Can to compare their height:

Barbara: Get up both, back to back, back to back, BACK to back, so [Steht mal beide auf, Rücken zu Rücken, Rücken zu Rücken, RÜcken zu RÜcken]

After being asked to stand up and to stand back to back for the first time, Deny and Can stand behind each other, Deny’s face is facing Can’s back. Barbara stands up while repeating the phrase repeatedly without any reaction from the boy. Studies about the acquisition of phrases have shown that these idiomatic expressions are not solely conceived by steady repetition. Learners rather have to deduce the meaning of phrases from the context in the particular situation (Häcki Buhofer, 1997). In this short extract, it becomes obvious that the children do not understand the utterance despite Barbara’s repeated articulation. The action she asks for becomes only clear when she touches Deny’s shoulders and turns him around so they can compare their height.

**Conclusion**

The literature concerning first and second language acquisition shows many starting points for possible linguistic barriers that can used as learning opportunities. In this paper, we have first tried to choose the ones that might be important concerning the development of an understanding of length from a theoretical point of view. Examples from our empirical data supported our categories. It became obvious that some preschool teachers are able to use these difficulties productively to create possible learning opportunities. On the other hand, there are also preschool
teachers who, with their situative language usage, reinforce difficulties concerning language acquisition as well as the understanding of length. Further desiderata of our study are to systematize these difficulties and to apply them to other magnitudes in order to raise awareness for difficulties and hurdles when dealing with language in mathematical learning situations.

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Using multiple representations as part of the mathematical language in classrooms: Investigating teachers’ support in a video analysis

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When making sense of mathematical objects in classroom interaction, the use of representations is necessary. Mathematical objects can be represented by the means of language, and even if they are represented in ways that do not use words, language is mostly used for explaining the meaning and connectedness of different representations of mathematical objects. Hence, it makes sense to consider any form of dealing with representations as being part of the mathematical language used in a classroom. For the sake of research on how representations are dealt with in learning support interactions, this video study examines how teachers supported students in using representations. Results show that teachers’ support in using multiple representations was rare, indicating that teachers’ mathematical language in these interactions could be improved.

Keywords: Video analysis, mathematical language, multiple representations, learning support situations.

Introduction

Supporting students in building up mathematical competence is a key feature of the teachers’ role in classroom interaction. This can be expected to be the case particularly in so-called learning support situations, in which the teacher has the possibility to react to students’ individual questions or to interact with them individually. Such interactions do not only consist of spoken words, but sense-making may also be based on multiple other representations of mathematical objects. Since being able to represent mathematical objects in multiple ways is a core element of mathematical competence and at the same time of communicating mathematically, any form of using representations of mathematical objects or dealing with them should be regarded as an integral part of the mathematical language which can be observed in learning support situations. Studies which explore the teachers’ support of the students with respect to the quality of the mathematical language in this sense are, however, scarce. Consequently, an analysis of video data from 30 classrooms was carried out in order to describe whether and how teachers supported their students in using multiple representations during learning support situations.

We will then first introduce the theoretical background and the research aims of this study, then describe design and methods, present results, and discuss these in a concluding section.

Theoretical background

The way representations are dealt with is a key quality aspect of interaction processes in the mathematics classroom (e.g., Duval, 2006; Ainsworth, 2006; Kuntze, 2013). As mathematical objects can only be accessed through representations and being able to handle them as well as to change between representations is central for students’ mathematical competency (Ainsworth,
students should be encouraged and supported to use flexibly multiple representations. Language can play a role for such support in two ways. Firstly, mathematical objects often can be represented by means of words, for example, descriptions of situation contexts that can stand for a mathematical object (Goldin & Shteingold, 2001). Secondly, such support can also consist in explanations of how different representations, for example by symbols or in graphical representation registers (Duval, 2006), stand for a mathematical object.

Indeed, teachers’ support for learners is important when dealing with multiple representations, as conversions between representation registers are complex for students and have shown to be potential obstacles for understanding (Ainsworth, 2006; Dreher & Kuntze, 2015b). Thus, teacher’s help for students to connect representations and to translate between them is needed.

We may conclude that representations play a key role in the mathematics-related interaction between teacher and students in the classroom, and that they are agents for mathematical sense-making, communication and reasoning, as it is the case for language. Moreover, the use of language can hardly be separated theoretically from the use of representations in class interaction. Thus, we define the mathematical language appearing in class interaction as encompassing both the use of words and any form of dealing with representations of mathematical objects.

According to relevant specific literature (e.g., Duval, 2006; Ainsworth, 2006), core aspects of the mathematical language can be considered as quality factors of classrooms:

- The mathematical language of the teacher should be rich in the sense of using multiple representations in order to avoid confusion between a mathematical object and one of its representations (Duval, 2006) and to encourage students to build up a rich concept image (Ainsworth, 2006). Students should thus be encouraged to use multiple representations.

- As conversions between representation registers can be obstacles for understanding, the teacher’s mathematical language should also provide support for the learners. This implies that connections between different representation registers need to be explained and examined together with the learners, that the teacher connects to the students’ representations when introducing a further representation register, and that the students are encouraged to reflect on conversions of representation registers.

These quality factors of learning support in classroom interaction merit closer examination in corresponding video studies. In a more general sense, the discussion about quality factors of learning support in the mathematics classroom has been advanced significantly from the 1990ties through large-scale international video studies (Hiebert et al., 2003; Stigler et al., 1999). In particular, the results of the TIMSS 1999 Video Study (Hiebert et al., 2003) pointed to a need for identifying specific quality factors related to mathematics education. This is important since there was not a simple observation pattern that allowed for distinguishing ‘the’ successful mathematics classroom from less successful teaching and classrooms. Investigating quality factors associated with the mathematical language in classrooms is hence highly relevant with this respect.

The mathematical language including interaction processes around representations can be expected to play a central role particularly in learning support situations, in which the teacher has the possibility to react to students’ individual questions or to interact with them individually (Krammer, 2009; Schnebel, 2013). By learning support situations, we understand classroom situations in which there is not a whole-class dialogue, but interactions between teacher and students during seatwork phases of the students, who are working on tasks on their own, in pairs or in small groups. Students can initiate these learning support situations, for example, when asking a question. Alternatively, the teacher can initiate them, for example, when asking the student(s) or giving a hint or feedback related to the working process. Such interaction situations are a key opportunity for helping the students with conversions between representational registers.
(as they are often presented or required by mathematical tasks) or to encourage them to change between representations (in order to support their ability to change flexibly between representations of mathematical objects). Hence, the quality factors for the teacher’s mathematical language as introduced above apply in particular to learning support situations.

However, so far and despite the international growing relevance of these quality factors, there are to our knowledge hardly any quantitative studies about whether and how mathematics teachers encourage their students to use multiple representations and provide them with help in learning support situations.

**Research interest and questions**

The study reported here focuses on the research need mentioned above. The key research interest consists in exploring the quality of teachers’ mathematical language used in learning support situations. Accordingly, our study concentrates on the following grouped research questions:

(a) What role does the use of multiple representations play in the teachers’ interaction with students in learning support situations? Do the secondary school mathematics teachers actively promote this use? Do these teachers support the change between representational registers of their students by help focused on connecting such registers?

(b) Which form does such help focused on connecting the different representational registers take in cases of learning support situations?

**Design and methods**

For answering these research questions, it is advantageous to investigate mathematics lessons that concentrate on the same topic area. In this case, expectations related to usable representations can be stated in a more content-valid way and comparisons between classrooms and results of analyses are supported. For this reason, we analysed 8th-grade videotaped lessons on the topic of “increased and decreased basic value” from a learning unit on percentage calculation from classrooms of 30 different German secondary mathematics teachers. The video sample stems from a data set collected by the research group around Thorsten Bohl from the University of Tuebingen (Batzel-Kremer et al., 2013). In this video data set, teachers had been asked to introduce the topic in a first lesson and to deepen this topic with student-centred exercises in a second lesson – both lessons had been videotaped. According to the research focus on learning support situations, this analysis focuses on the second lessons. The fixed subject meant that specific representation registers as they can be labelled by “growth factor representation”, “rule of proportion”, “per cent bars (graphical)”, or “text description” can play an important role for solving many of the tasks the students had to work on. It is of course not necessary that all of these registers are used and related to each other, but for helping the students to build up conceptual knowledge that they can use flexibly, using multiple representations can be expected to be an important element. An accordingly designed top-down coding scheme was used to code the 30 lessons – an analysis which was done by two raters independently. In all cases of disagreement, a common code could be reached in a subsequent consensus process based on the video data and the respective criteria.

The coding categories focused on:

(1) The representation registers used in the learning support situations.

(2) The quality of the interactions related to the use of representations.

(3) Context factors of the situations such as who is the initiator or their duration.

Codes about quality characteristics (as mentioned in (2) above) concentrated on e.g. whether the teacher connected to the students’ representation register, whether the students were encouraged
to change between registers, or whether help was provided in order to help students connect representations. In line with the research questions, we will in the following concentrate on these quality characteristics. In line with research question (b), the overview approach is complemented by a deepening analysis of case examples, which uses Mayring’s (2015) content analysis focusing on criteria from the theoretical background introduced above.

Results

The descriptive results show that in 271 of 454 coded learning support situations (59.7%) it was possible to identify the representation register(s) involved in the situations from the lesson video. The results reported here concentrate on these 271 situations. In order to explore what role the use of multiple representations played in the teachers’ interaction with students and whether the teachers actively promote the use of multiple representations in learning support situations (research question (a)), we first coded the number of representational registers in the interactions. Figure 1 shows the frequencies. In the majority of the situations, only one representational register was used by the teacher and the students, which implies that none of them made an attempt to introduce or to connect with a different representation register in these situations.

Figure 1. Frequencies of numbers of representational registers in learning support situations

In the relatively infrequent case (17%, i.e., 46 situations) in which more than one representational register was subject of the interaction, these registers were rarely connected to each other. Representational registers were coded to have remained disconnected in 38 of the 46 situations; only in eight situations, the teacher or the students connected the representational registers in the interaction (Table 1). In two of these eight situations, there was explicit help in connecting representational registers. Compared to the 271 learning support situations where representational registers were identified, explicit help in connecting registers was very rare (0.7%).

Table 1. Frequencies of codes for the learning support situations with 2 or more representational registers

<table>
<thead>
<tr>
<th>Code</th>
<th>Number of learning support situations</th>
<th>Relative frequency</th>
<th>Relative frequency for 271 situations with representational registers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Registers connected</td>
<td>8</td>
<td>17.4%</td>
<td>3.0%</td>
</tr>
<tr>
<td>Registers disconnected</td>
<td>38</td>
<td>82.6%</td>
<td>14.0%</td>
</tr>
</tbody>
</table>

For exploring teachers’ mathematical language in such a case in more depth, we would like to turn to the transcript in Figure 2. It shows the interaction of a teacher with a student who struggles
with the problem that 120€ has to be reduced by 20%, so that the remaining prize (which has to be calculated) is 80% of the initial prize. From the interaction it can be reconstructed that she has multiplied the 120€ by 80 so far (yielding the result 9600€). She then asks the teacher for help.

As visible from the transcript, the teacher first encourages the student to draw an additional bar representation for the problem. He might expect that this additional representation could help the student to understand better how to proceed correctly in the calculation register. He further encourages the student to connect the given values with the bar representation. The teacher then switches to the student’s calculation register and appears to encourage her to compare the two registers by asking her about the “80 what” and by drawing her attention to the 9600€. However, the situation appears to change quickly, as the student changes the 80 in the calculation into 1.80. From here, the interaction shifts to connecting the value of 80% (register of percentage values) to 1.80.
a correct growth factor (register of decimal expressions for growth factors). The teacher’s help to connect these registers consists in asking the student additionally to translate 1.80 and 1.0 back into the register of percentage values, a known strategy for supporting students’ fluency with conversions in two directions (Duval, 2006).

Due to the length limits of this paper we are unable to elaborate on all details of our analysis of this learning support situation – yet the example shown in Figure 2 indicates how the teacher tries to connect the student’s representations using other ways of representing the problem and the values contained in it. However, several issues of connecting representations and their registers might have needed further clarification or reflection in the interaction of student and teacher. The interaction takes around three minutes and compared to the multitude of issues, which emerge related to conversions of representations, it is visible that slowing down the speed of interaction and at the same time intensifying the teacher’s strategies of connecting representational registers in his language might have improved the quality of the learning support in this situation.

**Discussion and conclusions**

The results of this video study show a need of improving the quality of learning support on several levels. Some findings indicate that both opportunities for learning (such as encouraging students to use multiple representations) and for focused learning support (such as connecting representation registers and providing corresponding help) have been frequently missed in the lessons under investigation (see Figure 1, Table 1). In terms of the mathematical language used by the teachers, an enriched language use should connect students’ reasoning better with multiple ways of representing mathematical objects and help them to reflect about connections between representation registers.

Beyond these main findings, considering case example situations marked by dealing with multiple representations (e.g., Figure 2) can give further qualitative insight into potential development needs of interaction in the mathematics classroom. In particular, in the case of the interaction shown in Figure 2, intensifying reflection about connections between different registers and reducing speed in interaction might be useful strategies for the improvement of the mathematical language used in learning support situations involving several representation registers.

Also on the methodological level, the representation-aware focus on the mathematical language in classrooms should be considered in follow-up research, including research on an international level. For instance, the methodology developed in the framework of this study calls for further use in intercultural video-based comparison studies that can contribute to deepened and shared knowledge about quality criteria of interaction in the mathematics classroom.

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**References**


Cooperative learning in mathematics and computer science learning environments

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When talking about media in primary school, often, one thinks of the use of computers during everyday classes every once in a while. When realizing that we live in a more and more digitalized world, this seems rather odd. The sole use of media in school is not sufficient to prepare children for a world that expects so much more. The fundamentals of computer science are strongly related to mathematics content and can therefore be linked to competences that children already learn today. The paper will present the evaluation of learning environments, developed with computer science content, to have a closer look at interactional processes that occur when working on a topic that is on the one hand strongly connected to e.g. mathematics but on the other hand fundamentally new for elementary school children and especially relevant in today’s society. We will focus on co-operative processes and consensus and dissent.

Keywords: Computer science, interaction, mathematical competences, digitalization.

Introduction

Digitalization is a topic that almost every professional branch has to deal with. The problem is not that structures are changing and new or modified competences are required, it rather is a problem that children in today’s primary schools in Germany are rarely prepared to sustain and work in a world where computing skills will be a needed in most branches. As terms like “media competences” or “media education” are watered-down (Krauthausen, 2012) understanding the underlying structures (e.g., logic, algorithms, programming, cryptography, data) is equally important. Most research projects that already exist, try to teach coding in a child friendly way (Garmann, 2016). It is essential to find a viable room where computer science contents can be worked on and looking at similar and equal competences, mathematics would be the ideal candidate for this. With this connection in mind we based our project on developing learning environments in the intersection of mathematics and computer science. We will not solely focus on content learning of mathematics and computer science but will have a closer look at learning in interactive processes because of our underlying understanding of learning as increasingly autonomous participation in negotiation processes.

Our research will focus on interactional processes and collective argumentations (Krummheuer & Brandt, 2001) in these mathematics/computer science learning environments. Krummheuer bases his research on mathematics content and it will be interesting to see how this theory is applicable to content material that is in many regards new to primary school children. Our view on learning in this regard is from an interactionist point of view. We look at how children negotiate meaning and focus on the interaction between them. The research question we are interested in also include these interactional processes, structures and negotiations of meaning when working on specific topics. According to Miller (2006) individuals that work in a group aim to solve dissent in their discourses. Different individuals bring different approaches and interpretations into the discussion. During collective argumentations, differences act as socio-cognitive conflicts (Miller, 1986), which induce further collective argumentations. Miller highlights that an ideal collective learning process requires a socio-cognitive conflict. He claims that to achieve collective learning, all individuals have to be aware of the ongoing conflict and a scheme of strategies has to be present for individuals to solve the conflict. In our research, it is interesting to see, whether
these conflicts, or dissents, are always present, and if so how it is treated by the individuals, and whether it is not rather a cooperative structure or a consent that fosters learning on the basis of a consent. For the purpose of this paper, we will narrow our research question down to focus on what roles the concepts of dissent and consensus in regard to cooperative learning play and how they influence the negotiation of meaning. In particular it is interesting whether a dissent can also be the basis to work onwards from or whether a working consensus is needed.

**Our view on learning and research focus**

Miller (1986) has developed a sociologic theory of learning in order to distinguish it from psychological approaches of learning. This theory of collective learning processes of at least two individuals is founded on the thought that the learning of new basic theories in early development of a child is determined through interactive dialogic processes. Miller describes this as fundamental learning (Schütte, 2009). A core aspect of this approach is collective argumentations which Miller differentiates from “communicative acting” in the sense of Habermas (1985) that rests upon indisputability. Hereafter, the essential attribute of collective argumentations is the aspect of rationality. Fundamental mathematical learning occurs in collective negotiations about what is rational. Krummheuer and Brandt (2001) have created an interactionist theory of learning mathematics based on the theory of collective learning processes related to Bruner’s (1983) approach of what he calls format. Based on this theory, the learning of mathematics can be seen as an increasing autonomous participation in collective argumentations that are produced and nurtured collectively by the group itself. Learning in this sense is subject-specific learning in and through interaction based on a social-constructivist understanding of learning resp. following epistemological approaches of symbolic interactionism (Blumer, 1969). Participants learn through participation in subject-specific interactions. These interactions occur when working collectively towards a common purpose (Borsch, 2010).

Working in smaller groups provides the advantage, that single children are not able to extract themselves from group tasks or discussions but rather that they are encouraged to participate. The foundation of learning can either be a dissent that has to be resolved (Miller, 2015) or a consensus (Krummheuer, 1992). Krummheuer also states that dissents do usually not appear in primary school as there are often not many possible solutions to discuss. As the topic of computer science is fundamentally new to primary school children it would not be surprising to observe dissents between the group members, as different perceptions and ideas come together. It is exciting to see how children handle these dissents. Another question would be whether a consensus has to follow from a dissent (Krummheuer, 2011). Traditional lessons in school usually try to avoid dissents but studies show that the accepting and resolving controversies can lead to better achievements with both high-performance and underachieving children. In addition to that, controversies foster acceptance and support between group members (Johnson & Johnson, 1986). Krummheuer speaks of the working consensus, a common denominator that serves as a basis for further collaboration and which is essential to continue. Possibly, participation patterns (Brandt & Tatsis, 2009), or special roles like the focal navigator who leads the group on a content basis (Schneeberger, 2009) can be transferred from mathematics to computer science lessons.

**Connection of mathematics and computer science**

Looking at today’s primary school teacher education in Germany computer science as a subject is not an option to choose. Sometimes schools have a media competence program but the underlying content differs from school to school. During further training, some teachers can accumulate more knowledge on how to use media and teach computer science knowledge but this is a very limited path that is not available to everyone. Looking at other countries, computer science seems to be more deeply established in the primary school curriculum already. In Great
Britain, the new computing curriculum from 2014 clearly cuts the strings to training specific software and concentrates more onto programming and understanding information technology, as it is mostly comprised of computer science content rather than merely using technology.

Teaching computer science in primary school in Germany cannot be accomplished as a separate subject from day one, because of the already set timetable that has not much room for additional subject (except for extracurricular activities that do not reach every pupil). To prove its importance, it would be necessary to connect and integrate computer science into another subject. Although almost all subjects that are taught in primary school can be connected to computer science, mathematics seems to be the ideal candidate. Looking into the core curriculum one can observe that competences that are necessary to perform well in mathematics are needed in computer science tasks at well. Looking into the core curriculum one can easily identify connections between the competences in mathematics and computer science. The connections at the competence level that can be found suggest, that mathematics and computer science are built on a similar basis. Learning one could benefit the other. Regarding the mathematical competences of space and shape, we found that e.g. spatial orientation is a key competence when it comes to programming a robot. Movements have to be defined, obstacles have to be kept in mind and motion sequences have to be predicted. This is only one example to illustrate the overlapping of competences. It would be difficult to modify the existing mathematical lessons to include computer science content. Rather, specific learning environments have been developed to show that learning computer science topics nurtures the competences that are necessary for both mathematics and computer science. The learning environments should always foster discussions between the participants that will lead to fruitful outcomes.

**CS learning environments in primary school - Methods**

The underlying qualitative study is located in the interactionist approaches of interpretive classroom research in mathematics education (Schütte, 2009). The empirical foundation consists of videos of what can be seen as common group work processes in primary school with a new content. The content is new in regard to the situation in which it is embedded. Surely, the concept of algorithms and logic in general is also partly broached in mathematics lessons but the actual developing of the algorithms or logical reasoning itself inside of a new topic is something that primary school children, at least in Germany, do not do. To analyse the interactional units, we orientate ourselves by a reconstructive-interpretive methodology. To analyze the collective processes of negotiation, video-recordings of group work conversations were made, transcribed and subject to interactional analysis (among others, Krummheuer, 2011). We partnered with one primary school in Dresden that enabled us to work with 19 students from grade 4. The learning environments were designed to cover three sessions of 90 minutes for each topic. The learning environments were developed during one of the authors’ empirical seminars for future elementary school teachers. The students in the seminar developed the first draft of the learning environments on their own. After a presentation and extensive reviews each learning environment was finalized into a researcher journal to hand to the children. The main topics are: logic (general and propositional), algorithms, cryptography, programming/robotics and technology. Of the three sessions, two sessions of 90 minutes are planned for the actual content-related activities and one session of 90 minutes reserved for documentation and evaluation of learning processes e.g. with a storyboard. The storyboard tool can also serve as an access to learning as it not only reviews what has been learned, it also enables the children to actively reflect on the content that they are working with. The tool of reflection therefore becomes a learning tool as well. The storyboard is also the ideal tool for the programming/robotics learning environment. The main goal after developing and building the robot itself will be to program its specific actions. It was important
for the learning environments to focus on interaction and talking between the participants as well as cooperative or co-dependent learning.

Each task should be designed in a way that fosters interactional processes between the participants as we focus on learning through collective argumentation and participation (Krummheuer, 2011). The primary task of every learning environment always is to get a first impression of what the participants already know (or think they know) about the concept. Very open questions like ‘What does logic mean?’ or ‘When does a person have to think logically?’ provide a wide variety of possible answers without any pressure for right or wrong. The focus here is not on content learning alone but rather on the learning that occurs between individuals whilst discussing and arguing about the specific topic since children learn through increasing autonomous participation (Krummheuer, 1992). This idea of learning can be transferred to the learning of basic competences in computer science. To build upon this concept, all ways of communication and interaction between the children have to be supported as much as possible. To ensure an efficient way to videotape the children working on the learning environments, we designed the environments to be worked on in pairs or groups of three, sometimes with a closing task, that included a larger group discussion in groups of up to six children. The learning environments that this paper will focus on are algorithms and logic. These have been specifically chosen because these are two learning environments that do not use a computer to work on computer science topics and competences. The children work on their tasks using pen and paper (or some other analogue material). It is important to understand that working on computer science does not always require a computer per se. The children did not seem to have a problem with the situation as no one complained about the lack of digital media during these situations. When thinking about this it seems quite obvious as the structure of an algorithm has to be learned before implementing it in a computer program, and this does not necessarily require a computer.

**Observations and conclusions**

To make first assumptions on dissent and consensus two short examples will be presented from the algorithm and logic learning environment. After learning what an algorithm does and discussing several examples, the children are asked to identify one algorithm that they already know from their mathematics lessons and write it down instruction after instruction. The hope here is that through actively discussing the structure of an algorithm the children can gain a deeper understanding of how the algorithm works. Most children use the algorithms correctly, because they know them and have practiced them, but when it comes to describing the structure, most children struggle to do so. The focus moves immediately to long arithmetic techniques and the children decide to work with long forms of calculation. Here, “long arithmetic techniques” will be used for written calculations with every interim stage clearly visible. The short version would be the mental process without any writing or if the child only writes down the Problem and the solution without any interim stages. The task is as follows: Can you find an algorithm in mathematics? If so, describe it step by step, so that a person that has no knowledge of the algorithm and its function can perform it. Test your algorithm with an example. The children first discuss what specific example they should use and here the first disagreement occurs:

- Theo: Let’s say…500 minus 55
- Ralph: That is not the long way Theo, I can solve that easily in my head….445
- Interviewer: So what is the long way?
- Theo: The long way is when you have 345 divided by 7 and then you write down these numbers and underline them, then you write down the equal sign…

The children have to find an example to demonstrate the long way of addition. Interestingly Theo proposes an example, apparently not expecting that Ralph could see an example as unsuitable.
(which is not possible, as the long form of any calculation can be done with every example). Ralph on the other hand considers the example unsuitable because, at least for him, it seems not necessary to perform the long way of the calculation because he can solve it mentally. For him, the necessity to perform this written version of the calculation seems to be strongly connected to his inability to do it in any other, shorter way as he dismisses the easier example. It could be interpreted that for Ralph the task has to have a certain complexity. Therefore, he proposes a much more complicated example (he even switches to division and not subtraction, although the full long form of division has not been topic in the classroom yet). Obviously there is a dissent, or according to Miller a socio-cognitive conflict, between the two pupils that has to be proceeded somehow. One possibility could be that the pupils discuss on eye level the different examples and argue whether one example would be more suitable than the other. As the overall situation suggest that an imbalance of power exists between the two children this possibility seems to be the least likely. In this example the conflict between the two pupils results in the search for a more complicated example and sets the standard for the rest of the conversation. Theo retracts after this excerpt and Ralph takes the lead. Theo does not even try to defend his proposition but accepts the proposal of Ralph. Ralph raises the bar for the entire task as Theo does not clarify that his example would be perfectly fine for the task. Regarding the whole situation, one can interpret that Ralph feels superior to Theo and he tries to impress his opinion without evaluating Theo’s statements for correctness or suitability.

The second task is taken from the learning environment with the topic: logic. After being introduced to the topic and a first introductory task to think about what the term logic means, the pupils had to decide whether a statement is right or wrong (e.g. when the street is wet, it rained). They have to discuss the task and decide on an answer. The statement of this particular transcript is: If something is round it is not pointy. Right or wrong? The children then engage into a lively discussion, where they have to decide what qualifies as being round:

Lukas: If something is round it is not pointy\ [reads]
     That does not have to be true\.
Simon:  (It is not\) [laughs].
Lukas:  That does not have to be true maybe it is so to say a pointy circle\.
Interviewer: How does a pointy circle work?\.
Lukas:  So. wait (4) [draws into his folder (see graphic, Figure 1)]
Simon:  [looks into Lukas’ folder]
Lukas:  Like this and so on and on \ [points at the drawing in his folder]
Simon:  Yes, but if something is round [draws a circle into the air with his pen] so this here [points at Lukas’ drawing] that is not round\ that is pointy\.
Lukas:  Yes, it is roundish but if it is round then you are right\.

Figure 1. Drawing of Lukas

Lukas starts the discussion with his remark that it does not have to be true and elaborates that it could be something like a pointy circle. The “pointy circle” turns out to be a polygon, with angles larger than 90 degrees (see graphic in transcript). From his sketch, one could assume that he would
also agree to call something round, when it follows the shape of a circle (or circular segment) even though it still has vertices. The pupils then do not agree on whether this would qualify as a round object which results in a socio-cognitive conflict that pushes Lukas and Simon to argue collectively. The problem that occurs is to decide what the characteristics of a round resp. a pointy object are. In the end, they do not move on with a dissent as they try to reach a consensus because otherwise they would not find an answer to the question because they only have right or wrong as possible answers. To resolve this dissent, Lukas revises his opinion and proposes that his drawing would qualify as roundish. That is, not completely round and also not completely pointy. Both pupils communicate their view of the matter and in the end, find a solution that might not be accepted as a general truth but which can help them to answer the question, knowing that they could defend their answer if they had to. In comparison to the first situation, the children here seem to be on par. They find a solution to their problem through discussion, whereas in the first example one child behaves superior to the other and raises the difficulty of the task by relying on his own abilities and ignoring the insights of his group member.

Remembering the question of dissent and consensus it becomes clear that in task 2 (logic learning environment), the children start with a dissent that turns into a consensus as they both try to find a solution for their struggle with the term pointy. The children are not able to find a solution based on their disagreement and therefore create a new term for the object that they need to describe. Example one on the other hand has a power component that seems to put one student above the other on an intellectual level or at least one child puts himself above the other in diminishing the other child’s answer as not suitable for the task although it would work as well as his own. This seems to eliminate the need for a consensus as the child in control dictates the terms for further discussion. The dissent is not a topic that is actively discussed but it remains in the situation and the children move on without resolving it. It remains the question whether this is still to be seen as autonomous participation and whether collective argumentation occurred or whether the children only moved on, on the basis of their different levels inside the group. It could be possible that a dissent that is not negotiated but resolved through retraction of one child and control of the other has to be viewed differently. The reasons for which this occurs should be analyzed more carefully to see whether the structure that occurs on the surface also represents the situation itself.

As for the research question, it seems that especially new topics that also connect to existing knowledge produce situations where dissents arise. The discussion takes part on a level that could also be part of a mathematics lesson but the basis is a computer science/mathematics learning environment which might have an impact on the view of the children onto the topic itself (e.g. the task to describe the algorithm step by step is a computer science problem that is not generally worked on in mathematics in primary school).

Since the analysis of the situations is still in progress and new situations are being taped, we will be able to identify similarities and describe the different situations of the pupils and their individual learning potential that occur during the group tasks. This will also benefit the understanding of dissent and consensus as the basis of collective argumentation.

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How language and culture affect the learning of fractions: A case study in the Kingdom of Tonga

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In this paper, I look at how different cultural practices go hand in hand with different discourses and how the two of them together have an impact on the learning of certain formal mathematical ideas. The findings are based on fieldwork carried out in the Kingdom of Tonga in 2011, with the aim of answering the question: How do the Tongan language and Tongan cultural practices shape discourses on fractions? I examine the place of fractions in the Tongan community of discourse. Importantly, the findings provide strong evidence to support the classical idea of linguistic relativism in the form of an updated version of the Sapir-Whorf hypothesis.

Keywords: Linguistic relativism, fractions, the Kingdom of Tonga, communities of discourse.

Teaching fractions in the Kingdom of Tonga

As a teacher of mathematics and statistics at Atenisi University in the Kingdom of Tonga (a group of Islands in the South Pacific) in 1994, and again in 2010, I experienced great difficulty explaining some of the mathematical concepts involved in the domain of probability. After some investigation, I discovered that part of the problem begins with the earlier concept of fraction, the use of which is indispensable for the understanding of the canonical Western way of interpreting and measuring probability. The research developed from an observation that a group of students, who were proficient at other subjects in the mathematics curriculum, had specific difficulty understanding both fractions and probability. In this paper, I concentrate on my findings concerning fractions and only mention in passing findings relating to probability.

Preliminary observations indicated that the Tongan language does not provide Tongan students with the tools and the intuitive ideas that are so important in developing the ideas of uncertainty, probability, and fractions. Although secondary and tertiary education is supposed to be in English, my students regularly switched to Tongan when discussing what I was teaching. Tongan is the language that mediates and organises these students' lives and activities. These observations motivated the research question: How do the Tongan language and cultural practices shape discourses on fractions? More generally, the hypothesis to be tested is that the linguistic tools provided by the Tongan language differ significantly from European languages and as a result, some of the Western concepts concerning fractions do not exist in the community of native Tongan discourse. This said, as consequence of the arrival of the English language and Western forms of life, both the discourse concerning fractions and activities that require dividing whole objects into equal parts are in the process of changing in Tonga.

My research experience has led me to the conclusion that the development of discourses on fractions are directly related to some of the cultural practices concerning dividing objects into parts in specific communities. In this paper, I draw on my fieldwork on this topic in an attempt to understand and explain the discourse on fractions, which I observed in the Kingdom of Tonga. The underlying research is in process and the findings that I present are preliminary. As it will be shown, data and findings are at this stage suggestive enough to warrant further research.
Language, culture, and mathematical thinking

The idea that language shapes people's views of the world (i.e., their worldviews) has a long history going back at least as far as Roger Bacon in the thirteenth century and Wilhelm von Humboldt in the nineteenth century (Gumperz & Levinson, 1993). The idea was popularised in the mid twentieth century by Benjamin Whorf and, since then, it is commonly known as the Sapir-Whorf hypothesis (Whorf, 1959). Initially there was great interest in the hypothesis and then for many years it did not receive much attention. Recently there has been a renaissance of interest (Deutscher, 2010). It is suggested that part of the reason that Whorf's ideas did not receive more support is that he over stated his case and did not base his ideas on clear definitions. He claimed that our mother tongue restricts how we think and prevents us from being able to think certain thoughts. The dominant approach today (known as neo-Whorfism or linguistic relativism) is that when we learn our mother tongue, we acquire certain habits of thought that shape our experience in significant and often surprising ways (Deutscher, 2010).

In his research, concerning the Oksampin communities in Papua New Guinea, Saxe develops the idea that not only language. However, culture and history are particularly related to how mathematical ideas are understood. He proposes a methodological approach "rooted in the idea that both culture and cognition should be understood as processes that are reciprocally related, each participating in the constitution of the other" (Saxe, 2012, p. 16). My research in Tonga originated with the idea that language affects understanding but it quickly became clear that the cultural and historical background of the community had to be taken into account.

Learning fractions

An overview of research into how fractions are learnt can be found in the seminal work of Lamon (2007). In her review she states that there has been relatively little research in this field during the last twenty years and that most references go back to the 1980s or earlier. There are a number of articles by Kieren (1988) and Thompson and Saldanha (2003), which describe the steps involved, and the difficulties faced, in the development of the discourse on fractions.

For a number of reasons fractions are seen as a particularly complicated and difficult subject to teach. One of the main explanations for this is that there are numerous definitions of a fraction. Fractions can be a number (two thirds is a number between half and three quarters), an operator (give her two thirds of the cake), an adjective (two thirds majority) or a measure (two thirds of a litre). The canonical use of fractions in the western education system assumes some type of synthesis of all these definitions into one concept.

The various diverse meanings of fractions are defined by Kieren (1988), who builds an ideal network of personal rational number knowledge, which links what he refers to as the “intuitive” to the “formal” definitions of rational numbers. As summarised by Thompson and Saldanha (2003, p. 100), Kieren’s basic approach is, “to break the concept of rational number into sub constructs – part-whole, quotient, ratio, number, operator and measure - and then describe rational number as an integration of those sub-constructs.” On the other hand, Lamon (2007) suggests that one of the ways of understanding fractions is through the process of sharing equally (for example, sharing three biscuits between seven people). This is relevant to some of my empirical findings concerning practices of and attitudes to sharing in Tonga. Nonetheless, at this stage I have found no literature in the field on how language or cultural norms affect the learning of fractions.

A discursive approach

I adopt the discursive approach in which mathematics is defined as a form of communication or discourse (Sfard, 2008). The discursive approach postulates that people are members of various
overlapping “communities of discourse”. Discourse is a specific form of communication defined by the vocabulary, the visual mediators, the communicational routines and the endorsed narratives used. A community of discourse is defined as those individuals capable of participating in any given discourse and by the endorsed narratives that they use. An individual can be a member of a number of overlapping communities of discourse. This approach supplied a powerful framework through which to understand and explain my observations in Tonga.

Using this theoretical framework and for the specific topic of fractions, the aim of the research is to identify and analyse the community of discourse observed in Tonga as well as how it compares with typical Western communities of discourse. In this paper, I aim to analyse the discourse about fractions, sharing, and dividing whole objects, which I observed and experienced in Tonga.

**Some findings concerning the use (or lack of use) of fractions in Tonga**

The seeds of this research emerged when I was trying (unsuccessfully) to help a student answer the question "What is the probability of getting two heads when two coins are tossed?" The student was an able university student. She was proficient at algebra and passed an introduction to calculus course. I recorded the following one-sided conversation:

Lecturer: So what is half of a half?
Student: [Blank face]
Lecturer: *Ko e ha e haafe 'o e haafe?* [I translated with help the question into Tongan]
Student: [Blank face]

As a result of this conversation I started asking colleagues, friends and acquaintances the following: “What is half of a half?” [*Ko e ha e haafe 'o e haafe?] and found that most Tongan speakers (including many well educated Tongans who speak good English) do not give the expected (Eurocentric) answer to this question. One of the most unexpected observations happened when I was discussing teaching fractions with a senior maths teacher at one of the best high schools. He described how a lack of ability with fractions causes problems when teaching other more advanced fields of mathematics. When I mentioned the question, “What is half of a half?” he answered, “I have no idea”.

Further early observations included the fact that in the fruit and vegetable market nothing is weighed and everything is sold by the "pile". Rarely is a price displayed because it is assumed that the price of a pile is three pa’anga (three Tongan dollars). When the price goes up the size of the pile goes down and when the price goes down the piles are bigger. The vegetable market is organised in a way that avoids the use of fractions either in measuring quantities or money.

These and other observations were the motivation for a systematic attempt to understand the underlying discourse on fractions in Tonga and how both the language and the cultural values of the society affect fractions use and understanding. The research started with a collection of anecdotes and progressed to include questionnaires, semi-structured interviews, lesson observations, audio and video recordings of conversations with children and adults, as well as interviews with individual professionals.

Sixty participants were interviewed from a cross section of the Tongan population. As well as a qualitative analysis of the answers, with the aim of identifying the underlying narrative, the interviews provided a quantitative basis to compare different groups in Tongan society. The questions asked in Tonga were based on an implicit assumption regarding the expected answers in a Western setting. In order to test this assumption a comparison group of fifteen Israeli participants of various ages and backgrounds were interviewed. The questions were asked in Hebrew and some questions were suitably adapted. This proved to be a rather dull procedure as with very few exceptions the comparison group gave some of the expected (Eurocentric) answers...
in line with Western contexts of culture. However, it was still important to confirm or refute the anticipated assumptions concerning the expected Western responses.

**Vocabulary for fractions**

Christian missionaries arrived in Tonga during the first half of the 19th century and were responsible for setting up an educational system based on European and Christian values. As well as introducing reading and writing, they standardised the counting system and created a Tongan vocabulary for fractions. There is no evidence of words for fractions in Tonga before the arrival of the Europeans. The missionary teachers introduced words for fractions into the language during the 19th century but they had little use outside the classroom. The expression for some fractions is cumbersome; for example eleven thirteenths is "vahe taha tolu e taha taha" [divide one-three take one-one].

There is considerable confusion caused by the order of numerator and denominator. In primary school, arithmetic is taught in Tongan and the pupils learn to express the denominator before the numerator: 2/3 is vahe tolu e ua [divide three take two]. Then in secondary school, they learn in English and the order is reversed ("two thirds"). In practice, I observed that many teachers resort to the hybrid language: ua ova tolu [two over three]. The teachers themselves seemed confused by all this and resorted to a technical definition of fraction as "the numerator divided by the denominator".

“Number” is translated into Tongan as “mata’i fika” but from my field interviews, it emerged that this phrase is understood to mean integers and does not seem to include fractions in the interpretation of Tongan people. Tongans use it in the way that we use the phrase “whole number”. In an interview with an expert on Tongan language, I asked:

**Interviewer:** How would you translate "the number half?"

**Expert:** “koe mata’i fika haafe”. But we would not say this.

**Interviewer:** How would you translate "Half is greater than zero and less than one?"

**Expert:** "oku lahi ange e haafe he noa pea si’isi’i ange he taha”. This also does not sound right. But we would not say this, mata’i fika is a whole number, not a fraction. There is not a word in Tongan which includes both whole numbers and fractions.

If the translation of the phrase "Half is greater than zero and less than one" does not sound right to ourselves, then it is not surprising indeed that Tongan students have a specific difficulty when working with fractions. Thus, there seems to be several language issues going on in the teaching and learning of fractions.

**Results of structured interviews and other findings**

There is only room here to give a brief written summary of some findings. I select the findings that illustrate most clearly differences between the discourse observed in Tonga and the canonical Western discourse on fractions, sharing, and dividing objects thought of as whole. I summarise these findings in the form of issues and questions relating to: i) naming and identifying fractions, ii) fractions as parts of something, iii) fractions as rational numbers, and iv) values involved in dividing objects.

That said, it is key to clarify some of the attitudes observed in the context of culture about general reciprocity. In pre-missionary Tonga there were no words for fractions and there is no evidence of objects being divided equally. Distribution of goods was based on general reciprocity so there was no need for accurate calculations as to who owed what to whom. All of the products in general use came in whole units (yams, cassava, coconuts, fish, etc). When gifts were presented they were typically counted and presented in pairs and as far as we know were never cut into parts. Tonga has undergone enormous changes in the last two centuries but a conscious attempt has been made
to preserve some of the traditional values and ways of doing things. From an early age, children are taught that giving is more important than receiving and the idea of sharing equally (so prevalent in Western culture) is not important.

Questions relating to naming and identifying fractions

Participants were asked to identify some simple fractions using flash cards. A large percentage (37%) of participants gave unexpected answers or did not understand the question. When asked to choose a flash card showing three quarters of a pie, 48% of the participants (29 out of 60) did not choose the expected picture. Almost all the participants in the Israeli comparison group gave the expected answers to all the relevant questions. These observations indicated that both the names and the symbols of fractions (the names of which were introduced by the missionaries) are not embedded in the Tongan discourse.

Questions relating to fractions as parts of something

Whereas those interviewed were fairly proficient at estimating fractions of various objects such as a fraction of the length of a stick or of a kilogram of sugar they had great problems when asked to calculate fractions of a number or fractions of another fraction. Only 38% of those questioned (23 out of 60) answered that half of a half is a quarter. When asked what is half of a half of an apple only 47% answered that it is a quarter of an apple. The most frequent other response was “half”. When asked “How many minutes are there in three quarters of an hour?” only 14 respondents (23%) answered “45 minutes” and most of those took a considerable time to calculate the answer. For example, one respondent took several minutes to calculate the answer in a notebook and most others took between five and fifty seconds to answer. Again almost all the Israeli comparison group gave the expected answers to the above questions.

Questions relating to fractions as rational numbers

I asked a number of people to place a set of numbers (on cards) on a number line between zero and two. The aim of this question was to test the hypothesis that I was observing a specific problem when fractions are to be understood as numbers (as opposed to operators). The fractions included decimal fractions (0.5, 0.71, 1.69) and regular fractions (1/8, 1/2, 3/4, 11/4). Of the sixteen respondents questioned, four appeared to have no idea what we wanted. To avoid embarrassing them we did not pursue the question. The remaining twelve had great difficulty placing the numbers so, for example, only one out of twelve placed “1/8” in the expected position and only two out of twelve placed “3/4” correctly. With one exception the Israeli comparison group put every one of the fractions in the correct position (one Israeli participant did not know where to put “3/4”).

Questions related to values involved in dividing objects

I found strong evidence that cutting whole objects up is seen in a negative light. Respondents were asked to divide five biscuits equally between three plates. Just over 50% (31 out of 59) of the participants made a reasonable attempt to divide the biscuits equally. Other responses included placing one biscuit on each plate and placing two whole biscuits in the middle; placing two biscuits on one plate and one and a half on the other two; placing one biscuit on each plate and eating the other two; or placing one biscuit on each plate and explaining how he would break the other two biscuits "but I don't do it". The most interesting (and unexpected) observation was that, almost without exception, the respondents asked if they were allowed to break the biscuits before breaking them into parts. In comparison, Israeli respondents did not request permission before breaking the biscuits. This was the first hint that I was observing a reluctance to break whole objects into parts. In order to test this idea we devised the following question: “You have three watermelons and are going to visit two respected aunties. How would you divide the watermelons as a present? Explain” Most respondents found ways of not cutting up any of the watermelons,
typically by giving two to one aunt and one to the other. A few respondents additionally said, "One and a half each" but many of those same participants added, "This is not good in our culture". When asked to explain why they did not want to cut a watermelon in half a typical response was, "I would want to give them a melon which is complete because here in Tonga if you give something that is not complete it is selfish". The words used for selfish were Kai Po, which literally translates as eating at night and implies eating in secret by oneself.

The place of fractions in the Tongan community of discourse

My findings provided strong evidence to justify the claim that a coalition between the Tongan language and daily practice make the use of fractions almost redundant in everyday discourse in Tonga. I suggest that fractions and dividing whole objects have a negative connotation in the context of culture and that, in many situations, practical alternatives have been created to avoid their use. My research supports the conjecture of a reciprocal dependence between discourses on fractions and daily activities that involve fractions:

- Discourses will only develop if they serve the needs of day-to-day activities.
- The day-to-day activities will only develop in tandem with the relevant discourse.

I have found evidence that the pre-European norms of general reciprocity and of gift giving have changed and developed over the last two hundred years but have not disappeared. Giving away excess is still seen as a value and dividing whole objects into parts still has a negative connotation.

When used, fractions function as operators rather than as objects (numbers). Thus when respondents were asked to estimate a fraction of an object the request was usually understood, but when asked to put the same fraction on a number line between zero and two this was not understood. It is thus possible to understand why so many Tongans were unable to answer the question "what is half of a half?" In their culture, the "half of" is understood as an operator that produces a part of a physical object, and it is in this sense that the expressions "half of an apple" or "half of a cake" are used in everyday discourse. In contrast, half is not understood to be a number between zero and one and therefore it is not clear how it is to be halved. It follows that half of any concrete object remains a half, even if the object itself is half of another concrete object. This explains why so many of my respondents referred to “half of a half of an apple” as “half” – it was half of the half-apple.

The research indicates that there is a need for further study concerning the effect of language and culture on how fractions, in particular, and mathematics, in general, are understood in diverse communities and contexts of culture. Mathematics cannot be taught in isolation from the linguistic and cultural practices of the students and this should be taken into account in the development of school curriculum as well as classroom practice.

All in all, this research led me to realise that when teaching mathematics, and particularly fractions and probability, it is essential to consider the linguistic rules and cultural norms of the society in which teaching and learning are placed. I learnt from my Tongan students, colleagues, and friends that there is not one all embracing approach to sharing and the need to divide objects equally. It became clear that mathematical ideas can be understood in radically different ways by different people and in different cultural contexts regardless of the mathematical definitions provided by the formal language of mathematics. At the same time, in the Tongan culture, there is a desire to learn the mathematical skills necessary to succeed in the “outside world”. King George Tupou I, the founder of modern Tonga, described the necessary synthesis between respecting traditional values and adopting Western methods. In a speech to the legislative assembly in 1882 the king stressed the importance of education and added:
“If there be anything in foreign lands which will be useful to us, it is right for us to desire to get it; but it is also right if there is any Tongan custom which is useful, for us to preserve it” (Campbell, 1957, p. 69).

References


Mathematics learners’ behaviour in CLIL bilingual lessons within L2 external setting

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This classroom-based study discusses the language behaviour of 12-15-year old mathematics learners in introductory CLIL bilingual lessons within L2 external setting. Based on qualitative analyses of audio-records of three lessons in grades 7-9, the argument made is that learners tend to use L2 whenever they are able to. The extent of their L2 use, mostly involving short utterances and phrases rather than long sentences, is much dependent on their L2 proficiency, CLIL experience and the teacher’s immediate performance in the classroom. CLIL seems to have a positive impact on learners’ alertness and engagement in the lesson. Moreover, the more requirement of mathematical thinking, the less L2 use by both teacher and learners.

Keywords: CLIL, bilingual mathematics education, language use, alertness, engagement.

Introduction

Currently bilingual education catches attention of researchers in various aspects, involving all educational levels and school subjects, including mathematics. One of the most popular and also controversial approaches in bilingual education is CLIL, i.e., Content and Language Integrated Learning. New approaches bring about initial enthusiasm as well as worries and dubiousness. As for CLIL in mathematics education in Slovakia, it seems to be still in its infancy, though the several related empirical researches carried out in the past years (Guffová, 2014; Lengyelfalusyová, 2013; Naštická, 2016; Páleníková & Naštická, 2017). Mathematics teachers who are eager to start using CLIL report lack of hands-on teaching materials, which is a critical point, especially when they feel inexperienced in implementing any innovative approach. Having started practising CLIL, they encounter many challenging educational situations. Our effort is to analyse such situations and devise recommendations that might facilitate CLIL mathematics teachers’ practice. Below, we present insights from a qualitative study of three lessons taught in a village school where bilingual education is not a daily practice. We investigate the learners’ behaviour, regarding their use of two languages (Slovak as their first language, and English as CLIL language, henceforth L1 and L2 respectively), learners’ alertness and engagement in the on-going mathematical activities.

Theoretical background, research sample and methods

Data from the lesson and learners are analysed within a sociolinguistic view of bilingualism that considers bilinguals as members of social groups using their languages for various functions in their everyday lives (Grosjean, 1994; Valdès-Fallis, 1978; particularly for mathematics education, Moschkovich, 2002; Planas, 2014). The teacher-researcher planned the lessons and designed the applied worksheet following CLIL principles (Mehisto, Marsh & Frigols, 2008). From the many CLIL principles, emphasis was put on inducement of mathematical communication and on integration of language and content educational objectives, such as active vocabulary acquisition and practising question forms (e.g., How many vertexes/faces does a shape have?), while discussing elementary geometrical concepts, distinguishing between two- and three-dimensional shapes, as well as hypothesizing and providing mathematical arguments. Since English is perceived by the learners as a foreign language, and is neither a minority language in Slovakia,
nor a dominant language in any of the neighbouring countries, the analysed learners are cases of Slovak-English bilingual mathematics learners in external L2 sociolinguistic setting (Siegel, 2003; Barwell, 2005).

The three 45-minute lessons in grades 7, 8 and 9 were attended by classes involving four girls and ten boys aged 12-13, eight girls and six boys aged 13-14, and eight girls and five boys aged 14-15 respectively. The lessons were taught in a village school as extraordinary mathematics lessons on St Nicholas celebration, when Slovak teachers often prepare unusual lessons. This is an important note since the school is not bilingual and its students rarely attend CLIL events. The learners are not cases of any continual bilingual education, their estimated L2 proficiency being or approaching A2 level (by Council of Europe, 2001) as required by the national curriculum. The lessons were the learners’ very first experience with bilingual or CLIL mathematics education. This brings about consequential perspective to the analysis. The learners’ regular mathematics teacher, who had invited the researcher to the classes on the initiative of the school authorities who ask for more frequent CLIL events, was present in the lessons, assisting the researcher, especially in calling the learners by their names, maintaining as normal flow of the lessons as possible. The teacher had already had some experience with CLIL environments and had been consulted previously so that the activity was well suited to the learners’ mathematical and language abilities. Although the lesson was not in the learners’ current flow of mathematics learning, the content of the lesson was in accordance with national curriculum across all the three grades. This experience was chosen for study as a case of introductory moments of CLIL implementation in mixed classes with learners who had not previously been selected and regrouped according to their mathematical and/or language proficiency, as it is normally done in bilingual schools.

Figure 1. Shadows of a tea candle on the paper screen of the projecting box

In order to stimulate and develop spatial imagination and argumentation skills in elementary geometry, learners were asked to watch projections of various objects put in a hand-made projecting box and back-lighted so that the objects cast two-dimensional shadows on the paper ‘screen’ (see elements of the teaching experiment in Figure 1). Then, they were asked to name the planar figures by means of mathematical terms in both L1 and L2 and complete a fill-in-the-gaps exercise in the CLIL worksheet (see Figure 2). Based on the shadows, learners’ task was to deduce what objects were projected, i.e., to name the shapes of the objects by mathematical terms denoting solid figures and think of everyday objects which might be hidden behind the screen.

The lessons were audio-recorded and orchestrated following a whole-class discussion structure. Based on the researcher’s participant observation and on the qualitative analysis of the transcribed records, this paper discusses what the learners’ observable responses were to the CLIL activity and the bilingual nature of the mathematics lessons. We phrase the research question as follows: How did the bilingual nature of lessons and CLIL approach affect learners’ speech, alertness and engagement in the mathematical activities during the introductory stage of CLIL implementation?

Alertness is considered as the learners’ state of active attention enabling them to perceive and response to external stimuli. By the learners’ engagement in the mathematical activities, we mean their active participation in the whole-class mathematical discussion and/or conversation with their classmates related to the mathematical activities. Although the mutual relation between alertness and engagement is not investigated here, we assume that learners are not necessarily engaged when they are alert, yet, they could hardly become engaged in activities if they were not alert. Alertness is, thus, only partially observable in the audio-records, and more information
would be obtained by video-records. On the other hand, learners’ engagement in the activities is reflected in their participation in the discussion, which is detectable in the transcripts. Frequent change of interlocutors in a discussion means that negotiation of meanings takes place, which reflects their engagement in the activity. By contrast, if the teacher becomes the only interlocutor, the discussion changes to a monologue, scarcely allowing learners’ active engagement.

Figure 2. Task from the CLIL worksheet

Qualitative analyses and results

In the following analyses, the classes where the situations emerged are not distinguished as they arose similarly in all the three grades. Interlocutors’ utterances originally spoken in L1 (Slovak) are typed in Arial Narrow italics, while additional comments describing the interlocutors’ actions are typed in (round brackets, italics). We show three of the emerging themes that came out of the qualitative analyses of lesson data with focus on the use of L1 and L2 throughout the activity. The illustrative pieces of data are only some of the pieces linked to the same theme.

Learners wish to use their L2

Learners in the study tend to use L2 whenever they are able to. This is demonstrated in high frequency of their language-switch, surprisingly also in utterances as short as three words, as shown in this transcript:

Researcher: What are the names of the two purple shapes?
Student: Pyramid and circle.

Their L1 and L2 speech acts –mostly responses to the researcher’s questions– involve rather short utterances, denoting numbers, colours, planar and solid figures, and expressing confirmation or rejection of preceding proposals:

Researcher: The figure of the cone in the worksheet, what colour is it?
Student(s): Yellow.
Researcher: It’s yellow. So, the yellow one is called cone --- in English (writing CONE on the blackboard). It’s a cone. Have we completed the first line?
Student: Yes. Maybe (meaning to express “It seems so.”)

The reason for learners’ short utterances in L2 is clear – their L2 proficiency, but also the type of the questions the teacher asks; ‘why’ questions would surely elicit longer responses, regardless of the language. However, the reasons for learners’ L1 short speech acts are not that obvious. We assume that learners are somehow ‘caught unawares’ by the bilingual nature of the lesson. They wish to be able to express their ideas in L2, to show off and flaunt their abilities, which slows down their (audible) reactions. This is but an obscure assumption and might be relevant only to
learners who have just started learning bilingually. Learners need certain amount of time to understand that in CLIL lessons usage and switching of both languages are accepted.

**Learners’ mathematical activity is not hindered by the bilingual setting**

Despite learners being slowed down by the bilingual setting, especially by their personal unnecessary focus on L2 use, their mathematical activity is not hindered. As shown in several transcripts here, learners were alert, obviously focused on and engaged in the lesson activities. Learners’ alertness and engagement in the activities were reflected in their frequent involvement in the whole-class discussion. All interlocutors regularly took turns. Also, learners discussed various mathematical questions in pairs, i.e., not addressing the teacher-researcher directly.

![Figure 3. The ‘squarish’ shadow of a playing dice](image)

The following transcript captures a short talk starting with the researcher’s question about the ‘squarish’ shadow cast by a playing dice (see Figure 3) and leading to a deep mathematical talk between two learners, which, unfortunately, was not clearly audible in the record, and was only noticed by the regular teacher taking field notes.

Researcher: How many sides are there in a square?
Student 1: One.
Student 2: Four. *They are four (addressing his classmate, Student 1)*
Student 1: *But now they are not four (inaudible record of his explanation to Student 2 follows)*

Student 1 was actually right. The shadow was not a square, and the playing dice was not a cube, not having the proper ‘spiky’ vertices. Student 1 was aware of this, and that is why he considered the whole perimeter of the shape to be the only one side of it, being un-interrupted by any vertices which would divide the perimeter in line segments. As noted by the regular teacher, the learners were highly alert and they perceived very critically the fact that the shadow was not a square. The whole-class discussion continued, again with intervention of the researcher:

Researcher: How many sides are there in a rectangle?
Student 2: Three.
Researcher: What is a rectangle?
Students: Four.
Researcher: *Rectangle, right?*
Students: Mm-hmm.
Researcher: So how many sides?
Students: Four.
Researcher: Four. And also in a square there are four sides. What’s the difference? *What is the difference between square and rectangle?*
Student 1: *That there --- that --- the rectangle is somehow longer than ------ than the --- cube.*
Student 2: *That------it is longer --- wider ---*
Researcher: *In other words?*
Student 1: *That it is bigger ---*
Researcher: *In a square the sides are ---*
Student 1: *Longer---*
Students: Equal.
The researcher’s and learners’ speech took turns regularly and frequently, indicating learners’ immediate alertness and engagement in the activity. As discussed below, the more the mathematical thinking was required, the more both the teacher-researcher and the learners used their L1 (compare the extent of L2 use in the first half of the transcript and in its second half).

We assume that the learners’ alertness and engagement resulted from several factors. First, the researcher’s presence was unusual, which made the learners behave in a slightly different manner than usual. Second, the use of two languages in mathematics lesson, the idea of learning mathematics bilingually was novel for them, and, as it seems, positively challenging. Third, the atmosphere of the lesson and the CLIL activity naturally demanded learners’ alertness and, thus, kept them engaged in the mathematical discussions.

**Teacher’s and learners’ L2 use decreases with the increase of mathematical activity**

A closer look in the situations captured in the records suggests that the more requirement and activation of mathematical thinking, the less L2 use by teacher and learners. The following transcript shows one of the discussions in which all interlocutors used only L1, despite other (mathematically less demanding) discussions having occurred in L2 in great extent and frequency.

**Researcher:** The shadow you can see is two-dimensional.

**Students:** Yes.

**Researcher:** It is planar.

**Student 2:** In fact, it must be a cylinder (the other students giggling)

**Researcher:** It can be a cylinder. If it were a cylinder and now I stood it on its circular base and back-lighted it, what would you see?

**Student 2:** A pillar. A column.

**Researcher:** (smiling) What would you see? If the object was a cylinder and I stood it on its circular base?

**Student 1:** A rectangle.

**Researcher:** You would see a rectangle, not a pillar. A pillar is not a mathematical term for any shape, it’s a real-life object. But the shape (paused) a pillar is in the shape of a cylinder, that is right, with that I do agree. Anyway, there is something inside the box and I have already turned it.

**Student 1:** Oh, then it is a sphere (the student asserted, seeing a circular shadow again)

**Student 2:** A cylinder, definitely (the rest of the class gave a loud giggle)

**Student 1:** It’s a sphere, P*** (calling Student 2 by his first name) How could a cylinder (paused) I mean, a cylinder does (paused, unable to communicate his thoughts in any language)

Not only learners, but also the teacher-researcher used exclusively L1 in the previous discussion. This seems to be a natural consequence of the learners’ level of L2 proficiency as well as the researcher’s effort not to put emphasis on languages at the expense of decreased mathematical activity. In other words, the researcher did not dare to use L2 in that situation, being aware that learners’ active engagement in the discussion was at stake.

**Final discussion**

Bilingualism in mathematics thinking and education has been subjected to both qualitative and quantitative research for decades. So far, many authors have reported that bilingual settings, usually requiring language-switching, might slow down one’s mathematical, especially arithmetic processes (McClain & Huang, 1982; Marsh & Maki, 1976; Saalbach et al., 2013). On the other
hand, these authors concede that laboratory experiments have only limited implications for classroom settings as well as for bilingualism in relation to other forms of mathematical processes (e.g., problem solving). It seems that although knowledge is not represented in a ‘language-independent’ way, certain amount of training in required language might lessen the ‘bilingual costs’ in relation to response times and accuracy of solutions. The findings of our study, not contradicting any of the above-mentioned studies, have direct implications for teaching and learning in bilingual mathematics classrooms. Given our preliminary results, we believe that unnecessary focus on language in CLIL mathematics lessons is a hindering factor in relation to learners’ mathematical activity. Our preliminary findings are, however, limited to classes where CLIL as a bilingual education approach is novel for the learners who, in addition, were not selected by their mathematical and/or language proficiency.

This study is part of an on-going dissertation research. The data and results will be subjected to further analyses and comparisons with data obtained in standard CLIL classrooms, i.e., where learners have already been exposed to CLIL approaches for longer periods and are not mixed in terms of specific language and mathematical skills. On the other hand, our empirical experience so far includes the interpretation of pieces of information provided by mathematics teachers who have been practicing CLIL in standard CLIL classrooms for several years, and whose students were once in such a position that CLIL mathematical lessons were novel for them –not in mixed groups, though–. At this stage and in anticipation of further evidence, we assume that the influence of students’ personal unnecessary focus on L2 use decreases over time. This fact subsequently clears the way for the emergence of merits of CLIL teaching in bilingual mathematics education.

Acknowledgements

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References


Mathematical agency and its connection to students’ multilingual resources

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By exercising agency, multilingual students are said to be able to direct the classroom discourse in ways that are conductive to their learning needs. It has been proposed that this is connected to the use of multilingual resources. In this study, a bilingual Turkish-German teaching intervention is investigated in regard to the question, whether exercising agency is connected to a specific use of Turkish or German or mixed. It employs positioning theory, assuming that exercising agency requires students to take positions from where they can articulate their problems/their learning needs. 176 instances of agency were identified in sessions two and four in 4 groups of the intervention (~720 minutes of video). Comparing the use of language in these instances with the distribution of languages in the intervention, there is no indication that exercising agency is specifically connected to the use of Turkish or mixed. Implications of this result are discussed.

Keywords: Student learning, mathematical agency, multilingual resources, positioning.

Introduction

The following conversation happens in a multilingual teaching intervention on fractions, where the students Rükiye, Atiye and Mediha try to determine 2/9 of 36 with the help of a fraction bar.

<table>
<thead>
<tr>
<th></th>
<th>Rükiye:</th>
<th>Off, ich versteh das nicht.</th>
<th>Off, I don’t understand that.</th>
</tr>
</thead>
<tbody>
<tr>
<td>73</td>
<td>Atiye:</td>
<td><strong>Burda nasil yazmis?</strong> Da kommt nicht 36 hin.</td>
<td>[Looks at her worksheet] <strong>How is it written here?</strong> 36 it doesn’t belong there.</td>
</tr>
<tr>
<td>74</td>
<td>Rükiye:</td>
<td>Was dann?</td>
<td>What else, then? [cancels out 36 in this column]</td>
</tr>
<tr>
<td>75</td>
<td>Mediha:</td>
<td>Dann tue eins weg. Nein!</td>
<td>Then take away one. No!</td>
</tr>
<tr>
<td>76</td>
<td>Atiye:</td>
<td>Nein! Du nimmst zwei Felder weg.</td>
<td>No, you take away two parts [from 9 parts in the fraction bar].</td>
</tr>
<tr>
<td>77</td>
<td>Mediha:</td>
<td>Dann sind das. <strong>Ozaman</strong> vier.</td>
<td>Then this is 36. Then four.</td>
</tr>
</tbody>
</table>

In this episode, there is no teacher to help in the learning situation, so that the students evaluate their work themselves and help each other. For that, the students change the direction of the discourse towards their learning needs. They change it towards a meta-level conversation about filling out a worksheet, and through this, explain the strategy of how to solve the task. It has been proposed that multilingual students can use their multilingual resources to overcome resistances by exercising agency (Langer-Osuna, Moschkovich, Norén, Powell & Vázquez, 2016). This episode is an example of such agency.

In this paper, the study reported is guided by the following research question: **Is there a connection between the use of multilingual resources and exercising agency?** Here, agency is understood as overcoming difficulties in understanding during collaborative work.

Multilingual students’ agency in the mathematics classroom

Agency captures humans’ capacity to act upon their world – “to reiterate and remake their world”– and not only to give significance to it (Holland, Lachicotte, Skinner & Cain, 1998, p. 42). It has

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been acknowledged as important for language learning, as language and multilinguality are means for acting upon the social world and for making meaning in it (Vitanova, Miller, Gao & Deters, 2015). In school learning, the construct of agency emphasizes that students are not objects in an unchangeable teaching-learning situation which is imposed on them. Instead, students can (co-)direct classroom conversations and enable themselves to participate, which can foster learning (Boaler, 2003).

Mathematical agency is here conceptualized as a discursive phenomenon, located at the intersection of everyday and mathematical discourses. At this intersection, students can engage in “dances of agency” to overcome problems of understanding (Pickering, 1995): students interweave their own ideas (conceptual agency) with outcomes of “standard routines and procedures” (disciplinary agency), thereby bridging the everyday discourse with mathematical discourses to better understand something (Boaler, 2003). Hence, the “dance” enables students to actively work on their difficulties of understanding and not surrender to them. Hence, mathematical agency is here conceptualized as the capacity of students to direct classroom conversations towards their learning needs, e.g., overcoming difficulties of understanding.

Multilingual students’ resources in mathematics learning

Multilinguality can be a resource for mathematics learning. Most prominently, Planas (2014) illustrated three ways in which multilingual resources can be conductive to generating learning opportunities in mathematics. She identifies the negotiation of mathematical words among peers, the invention of words, and translations to overcome difficulties with words as instances of students being able to activate their multilingual resources. All of these three examples are also instances of students exercising agency. In these examples, the students overcome difficulties in understanding by acting in meaningful ways with mathematical language in the given situation.

With this perspective, agency sensitizes for how multilingual students might draw upon their home language “to support communication in the language of instruction” (Langer-Osuna et al., 2016, p. 164). Langer-Osuna et al. (2016, p. 166-171) identify three vignettes for multilinguals’ agency, which illustrate how students position themselves in order to avoid difficulties with language or articulate their difficulties. Similarly, Norén (2015) shows how students change the direction of a conversation towards the negotiation of unclear word meanings.

These studies show how the construct of agency helps to overcome the traditional “monolingual bias” where multilingual students are assumed to be recipients of the dominant language, and allow for conceptualizing language learning as an active process. Agency materializes in the students’ attempts to direct the ongoing conversation towards their learning needs, either mathematical or language-related. Exercising agency requires students to take positions in conversations from where they can articulate their problems and from where they can engage others in working on these problems.

Positioning theory and agency

As argued above, exercising agency in the mathematics classroom is connected to taking certain positions in the ongoing conversation. Positioning theory allows to grasp such positions (Wagner & Herbel-Eisenmann, 2009). Attempts to direct the ongoing conversation towards language- or mathematics-related learning needs materialize in how the multilingual students deliberately position themselves in the classroom. For example, students can deliberately position themselves as not understanding a certain word, or as in need of help, and this way might direct the conversation towards clarifying the language/mathematics at hand (cf. van Langenhove & Harré, 1999, p. 24). In deliberate self-positionings, students take initiative for their positioning, so that it is strongly connected to exercising agency. In contrast, in forced self-positionings, the initiative for a position lies with someone else, for example the teacher or other students. The teacher
strongly influences how students exercise agency to overcome difficulties, as he or she can, as a representative of the institution school, force students to position themselves (van Langenhove & Harré 1999, p. 26). As illustrated above, studies on agency strongly suggest that the teacher has to give room for agency (e.g., Norén, 2015).

**Hypothesis**

In this study, multilingual students participate in a bilingual Turkish-German teaching intervention. Thus, the activation of multilingual resources will be connected to the use of Turkish. It can be hypothesized that: When multilingual students try to overcome resistances in understanding language or mathematics in collaborative settings –when they exercise agency–, they will use their Turkish language, resulting in a higher use of Turkish or of Turkish-German mixed in situations of agency.

**Background and methodological considerations**

In this study, a bilingual Turkish-German teaching intervention for fostering 7th graders conceptual understanding of fractions is investigated. The intervention consisted of 5 lessons á 90 minutes. 41 multilingual Turkish-German speaking students participated in 11 groups with 3-5 students each. Typically, small groups of 2-3 students were video-taped in the 11 groups. In their regular classrooms, these students are predominantly educated monolingually in German.

The intervention was implemented by trained teacher students. The bilingual intervention followed the relating registers approach, which poses that languages and registers need to be continually interlinked (Prediger, Clarkson & Bose, 2016). Following this approach, Turkish and German were not treated as separate languages, but as a unified resource. The teaching intervention implemented several principles for activating multilingual resources, among them the implementation of tasks which connect to the students’ everyday experiences and of activities of reflecting on differences in how languages conceptualize fractions.

**Distribution of Turkish and German in the teaching intervention**

To give background for this study, the use of Turkish in the teaching intervention in general is relevant. The students’ use of Turkish and German and their participation were investigated in a previous study. The sample of that study consisted of N=35 students who participated in the third teaching intervention session. The 16 x 90 min. of video material from this third session was analyzed with the software TRANSANA in regard to each participants’ turn-based contributions (S1-S5 and T). All utterances were measured for their length (in seconds), so that each participant’s speaking time could be determined (as the sum of the lengths of the utterances). Furthermore, each utterance was analyzed in regard to the language used. The results of this analysis are outlined in Table 1. As can be seen, the students in the intervention could be encouraged to speak Turkish or mixed languages, when the teacher invests in the use of Turkish (with 28% of language production time in Turkish and 39% in mixed utterances) (Schüler-Meyer, Prediger, Kuzu, Wessel & Redder, 2017).

<table>
<thead>
<tr>
<th></th>
<th>Share of German utterances</th>
<th>Share of Turkish utterances</th>
<th>Share of mixed utterances</th>
<th>Total time of language production</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher language productions</td>
<td>32%</td>
<td>28%</td>
<td>39%</td>
<td>99% (1% unidentified)</td>
</tr>
<tr>
<td>Student language productions</td>
<td>66%</td>
<td>16%</td>
<td>15%</td>
<td>97% (3% unidentified)</td>
</tr>
</tbody>
</table>

Table 1. Distribution of German and Turkish (averages) (Schüler-Meyer et al., 2017)
Selection of data and method

The data in this study are taken from the second and fourth session of the teaching intervention. These data are chosen because, first, it accounts for learning processes in the beginning of the teaching intervention, were students might not be familiar with using Turkish (Session 2), and the end of the intervention, where students likely have become familiar with using Turkish (Session 4). Second, these data accounts for different activities, where Session 2 is more exploratory in nature and incorporates everyday contexts, while Session 4 is about the guided reinvention of the procedure for determining $x/y$ of $a$, where $a$ is bigger 1. Session 2 is dominated by small group work (2-3 students), while Session 4 consisted mainly of large group work (all students in the group). To account for different teaching stiles, groups from three of the four teacher students are chosen. In sum, four groups are investigated; they were chosen for their rich discussions and interactions. One of these groups was videotaped with two cameras, so that for Session 2, there is data of five small groups working on the tasks. In sum, around 720 minutes of video were transcribed and categorized.

The data are analyzed with quantitative content analysis (Mayring, 2010) with categories of deliberate self-positioning. Only utterances were coded which occur when students collaboratively work on their difficulties with the mathematics or mathematical language. Collaborative means that at least two students interact, without the teacher’s guidance. The analysis was conducted in three steps:

1. Situations where students exercise agency are identified by linguistic markers that indicate self-positioning (I, me, myself, my), as these markers allow for a relatively good approximation of students positioning themselves as individuals (“lexical bundles”, in Herbel-Eisenmann, Wagner & Cortes, 2008). Agency might also be exercised collectively, but are not investigated in this study.

2. From these identified situations, only those are investigated further in which the students try to collaboratively overcome difficulties of understanding. These are categorized in regard to the language that the agentic student uses to exercise agency (Turkish, German, or mixed) and in regard to the nature of agency. The latter categories were generated from the material.

3. Relations between language use and nature of agency are quantified with the Software MAXQda.

Results

In the here analyzed four groups and over the course of Session 2 and 4 of the bilingual teaching intervention (720 minutes) there are 174 self-positionings by which students attempt to direct the discourse towards their learning needs (Table 2, right column). Overall, this illustrates that students exercise agency relatively infrequent.

<table>
<thead>
<tr>
<th>Session</th>
<th>Self-positioning for upholding participation</th>
<th>Self-positioning to signal learning difficulties / successes</th>
<th>Self-positioning of frustration / resignation</th>
<th>Self-positioning to engage in negotiation of ideas</th>
<th>Sums</th>
</tr>
</thead>
<tbody>
<tr>
<td>Session 2</td>
<td>3</td>
<td>42</td>
<td>10</td>
<td>12</td>
<td>67</td>
</tr>
<tr>
<td>Session 4</td>
<td>9</td>
<td>38</td>
<td>10</td>
<td>50</td>
<td>107</td>
</tr>
<tr>
<td>Sums</td>
<td>12</td>
<td>80</td>
<td>20</td>
<td>65</td>
<td>174</td>
</tr>
</tbody>
</table>
107 self-positionings occur in Session 4, and 67 in Session 2 (Right column in Table 2). As both sessions equal in the length of the analyzed videos, there is a slight imbalance in the number of instances where students exercise agency between Sessions 2 and 4. If analyzed per group, this imbalance can be found in three of the four analyzed groups (Table 3). It is unlikely that this imbalance is a result of the different variants of group work, where Session 2 was intended to be based on a lot of small group work, and Session 4 on large group work. Hypothetically, large-group work might have more opportunities for students to interact with each other, and thus, there could be more self-positionings. However, as groups H and I work consistently in small groups in both Session 2 and 4, there should have been an equal distribution of the number of self-positionings in these groups, but this is not the case (despite the smaller number of tasks, Table 3). Hence, the variant of group work likely does not explain the imbalance. Instead, this imbalance might be a result of the different tasks: The tasks in Session 2 are rooted in everyday contexts, while the task in Session 4 requires students to reorganize their previous knowledge about fractions, for which there are no everyday contexts. Hence, the tasks in Session 4 require students to reactivate previous contexts to engage in a dance of agency. This might lead to more difficulties to understand. As a result, the students might more often self-position themselves in order to direct the conversation towards their learning needs.

Table 3. Number of times agency exercised in Sessions 2 and 4 per group

<table>
<thead>
<tr>
<th></th>
<th>Group H*</th>
<th>Group I**</th>
<th>Group B</th>
<th>Group D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Session 2</td>
<td>7 (Hale, Sevin)</td>
<td>3 (Sevda, Oguz)</td>
<td>21 (Emrah, Deniz)</td>
<td>25 (Ilknur, Akasya)</td>
</tr>
<tr>
<td></td>
<td>29 (Hale, Sevin)</td>
<td>16 (Sevda, Oguz)</td>
<td>17 (Emrah, Deniz, Yusuf, Ceylan)</td>
<td>45 (Ilknur, Akasya, Halim, Hakan)</td>
</tr>
<tr>
<td>Sums</td>
<td>36</td>
<td>19</td>
<td>38</td>
<td>81</td>
</tr>
</tbody>
</table>

* Tasks 5, 6, 7, 9 not analyzed; ** Tasks 6, 7, 9 not analyzed

Table 3 shows that some students seem to exercise agency more often than others. For example, the students Akasya and Ilknur (Group D) exercise agency roughly twice the time than the students in the other groups, and even Halim and Hakan in the same group. This imbalance is likely not a result of the teaching style of the teacher student, as Groups B and D were led by the same teacher student. Accordingly, exercising agency might be connected to the personalities and individual features of the students. The overall sums for the different categories of self-positionings (Table 2) suggest that the imbalance in agency can be traced back to self positionings by which students engage in the negotiation of ideas. This might be explained by the affordances of the task, which requires students to interweave ideas about fractions, as developed in the previous sessions, with standard mathematical routines of calculating a·x/y with a>1, in other words, where students need to engage in a dance of agency.

Table 4. Students’ use of languages while exercising agency

<table>
<thead>
<tr>
<th>Language of utterance in which agency is initiated (sums)</th>
<th>Turkish</th>
<th>German</th>
<th>Mixed</th>
<th>Sums</th>
</tr>
</thead>
<tbody>
<tr>
<td>23</td>
<td>120</td>
<td>31</td>
<td>174</td>
<td></td>
</tr>
</tbody>
</table>

| Percentages                                               | 13,22%  | 68,97% | 17,82% | 100% |

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It was hypothesized that students more often use their Turkish language to exercise agency than German. Table 4 illustrates that this is not the case. When compared with the general language use as illustrated in Table 1, it can be seen that the students use of Turkish, German and mixed language while exercising agency equals the distribution of language use in Session 3. Thus, the initial hypothesis can be falsified.

**Discussion**

In this study, there is no indication that multilingual students’ exercising of agency is specifically connected to the use of Turkish or mixed German-Turkish language. This is an unexpected result, as it has been suggested that multilinguality is especially relevant in situations where students try to understand something (Norén, 2015; Langer-Osuna et al., 2016). There are several reasons why this might be the case.

In the present study with its specific conditions of a bilingual teaching intervention there is no indication that overcoming difficulties with understanding is especially connected to the activation of Turkish. Norén (2015) explicitly suggest that a potential for agency is connected to reform- and language-oriented classrooms where there is room for “creative changes within the mathematical discourse” (p. 181) and where power structures of dominant languages can be broken up as a result. As the here presented teaching intervention was relatively teacher centered and strict in the number of tasks that have to be worked on, there might not have been room for such creative changes. Thus, tightly clocked tasks which are typical for teaching interventions might compete with time for conversations about the meanings of language that stems from individually articulated needs for understanding certain language. As a result, the teacher might not give much room for exercising agency. This calls for studies that investigate the conditions which facilitate students to exercise agency.

The teaching intervention in this study is language oriented, but it does not break up the traditional role of the teacher as facilitator of learning. Accordingly, the students might rely on practices by which they ask for help or assistance that stem from the regular mathematics classroom, e.g., delegating difficulties of understanding to the teacher or of dropping out of the classroom conversation. This would explain why the distribution of languages while exercising agency is the same as the general distribution of languages, as agency is exercised like in the regular classroom, only now in multiple languages. There is a need for a comparative analysis of monolingual and multilingual learning processes to investigate language-related differences in students’ agency.

From a theoretical standpoint, the here presented construct of agency attempts a synthesis of proven constructs of agency, as for example put forth by Norén (2015), Boaler (2003) or Langer-Osuna et al. (2016). In these studies, on the one hand the conceptual function of agency is emphasized (Boaler, 2003 and others), while on the other hand its function for overcoming power structures resulting from dominant languages is emphasized (Norén, 2015; Langer-Osuna et al., 2016). Here, a combination of both is put forth, where agency is exercised for overcoming difficulties of understanding. These different notions call for a better operationalized model of agency that integrates these notions.

**Acknowledgments**

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References


The individualization of rational numbers
discursive routines

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The research presented deals with the process of learning rational numbers. Assuming that at the end of a successful learning process the formal routines taught at the school become useful for practical activities, we have documented the activities of children from different grade levels regarding school assignments and daily practical tasks. We document the participants' activities from their entrance to the 1st grade, where they have no school experience with fractions, all the way through the sixth grade, where they are expected to arrive at the fully satisfactory mastery of rational numbers. In analyzing the data, we examined how the formal routines for performing certain fractions-related school assignments changed over time and how (if at all) they converge with routines for daily practical tasks. In this paper, we present findings from two school tasks: (1) locating a fraction on the number line, and (2) naming a point on the number line.

Keywords: Fraction, rational numbers, discourse, individualization.

Introduction and theoretical background

In the research presented in this paper we investigate the development of students' thinking about rational numbers. The research is part of my doctoral dissertation under the supervision of Anna Sfard. In spite of the abundance of former research on the topic, this investigation may be expected to make a novel contribution because of two features that set it apart from former ones: (1) the study follows the process of development rather than just snapshots of children's performance with rational numbers; (2) the study is grounded in a conceptual framework markedly different from those that have been guiding the majority of other investigations.

In the last quarter of the twentieth century, learning rational numbers has been one of the topics most vigorously studied by mathematics education researchers. Numerous studies recognized the complexity of the concept and agreed that rational numbers should be characterised as a set of related but distinct constructs rather than as a homogenous single one (Behr, Lesh, Post & Silver, 1983; Kieren, 1976; Rappaport, 1962). Although this model has been the point of departure for many projects, some researchers argued that the division into interpretations of the rational number is insufficient for describing children's construction of the concept (Olive & Lobato, 2008). Psychologists took the research in a different direction and focused on the mental operations involved in constructing knowledge of non-integer quantities (Confrey & Scharan, 1995). Nevertheless, only few of the studies offered a continuous picture of the learning of rational numbers. In the Second Handbook on Mathematic Teaching and Learning, Lamon (2007) writes: “Multiplicative ideas, in particular fractions, ratio and proportion, are difficult and develop over a long period of time. Brief teaching experiments have had disappointing results. There seems to be no substitute for longitudinal research” (p. 651). She also writes that such research should consider students' intuitive and experimental knowledge as well as their formal school knowledge.

In this longitudinal study I interviewed 24 children, from a wide range of ages (6-13), for two years, looking at how their execution of an activity changed over time – before, during and after the formal learning of fractions.

By adopting a theoretical perspective that views individual learning as a collective endeavor (Vygotsky, 1978), I take yet another approach to the topic. According to the basic tenet of the
proposed conceptualization, known as commognitive (Sfard, 2008), mathematics is a form of communication – a discourse. The discourse on rational numbers is a mathematical discourse dealing with the mathematical object rational number. A mathematical object is introduced in order to account for the equivalence of many signifiers, which are the terms (names) and symbols used for communication. The term “three quarters” and the symbols 0.75, 3/4 and 6/8 are declared to be signifiers of the mathematical object called rational number. The leading example of mathematical signifiers featured in this article are fractions, whereas the objects signified by fractions are rational numbers. The discourse of rational numbers is identifiable by four characteristic features: its special words and their use, its visual mediators and their use, the narratives that are endorsed by the discourse community, and discursive routines, which are patterns of actions a person tends to perform in response to a task situation, a situation in which she feels obliged to act. Given a task-situation, the decision about what it is that needs to be done is made by the performer based on precedents - past task-situations she considers as similar enough to the present one to justify repeating at least some of the things that were done then. The participant is not always aware of what and why she chooses for repetition, or that she is even repeating anything, in the first place. The repetition-requiring elements constitute the task the performer feels obliged to perform. We will now say that routine performed in a given task-situation by a given person is the task the performer saw herself performing together with the procedure she executed to perform the task.

The origins of the historical discourse of rational numbers go back to early human attempts to expand some everyday practical activities, such as those that require comparisons of continuous quantities – length, area, etc. The development of that discourse through human history involved assigning a name (signifier) to germinal routines, routines of practical activities that are likely to evoke the use of fractional words. While most of these germinal routines can also be successfully dealt with without any mathematical discourse, the use of fractions allows refinement of communication: it makes it more compact, more accurate or more widely applicable. For example the activity of sharing fairly a roll of fabric among four women can be communicated as: "each woman get one quarter of the roll". It also involves consolidations of different germinal routines into a single one. Such consolidation was due to the use of a signifier which is applicable to several hitherto unrelated types of activities. For example, the deed of sharing a roll of fabric among four women and the deed of dividing land among four heirs can both be described as “finding one fourth of the whole”.

Today, it is through the process of learning that children gradually become participants of this historically established discourse. The way it happens is bound to deviate from the historical trajectory because the formal discourse has already been established and the child is not required to name signifiers on her own initiative or to formulate discursive patterns. Hence, learning rational numbers is the process of individualization of the formal discourse of rational numbers. It is the process at the end of which the learner fluently participates in the discourse, according to her needs. Individualization begins with the learner's exposure to new words and symbols embedded in such everyday expressions as “half an hour” or “quarter to six”, and proceeds with the formal learning of rational numbers in school. Since rational number is not a physical object, it cannot be displayed in class. Instead, the child is introduced to the discourse of rational numbers as a formal language: she is presented with the signifiers within its typical discursive context, and the formal routines. According to the commognitive approach there is no other way to begin the process of individualizing a new routine than by adopting it as a ritual. The adoption of a routine begins with an imitation of an expert’s moves (e.g., father, mother, and teacher) which is motivated by the child’s social needs, meaning participation is ritual. Being unaware of the practical application of the outcome, the child would not recognize the outcome of such routine as the exclusive aim of the performance. Later, the child's ritual routine undergoes de-
ritualization; there is a change of focus in the performance of the routine, from the procedure being performed to the desired result. Eventually it becomes exploration, that is, a routine whose focus is on the outcome and whose success is evaluated by answering the question of whether a new endorsed narrative has been produced (Sfard & Lavie, 2005).

This paper focuses on the individualization of two formal routines for manipulating rational numbers performed by Ada and Noa, two fourth graders who are new participants in the formal discourse of rational numbers. Through presenting the girls with the same two school-like tasks repeatedly, five times over two years, we describe the de-ritualization of these formal routines. Ada and Noa were randomly chosen. The way the girls acted appeared rather standard; what they did was similar to what was done by most other participants.

**Research questions and method**

The purpose of the study is to describe the process of individualization of formal discourse routines of rational numbers taught in school. In the part of the study presented we will describe one case of the individualization of two school routines: “locate a fraction on the number line (as a point)” and “name a point on the number line”. Hence we focus here on the questions:

1. How do these routines change throughout the school learning process?
2. Are they applicable in reproducing a point on the axis according to verbal instructions?

In order to answer these questions, I repeatedly interviewed 12 pairs of children from grades one to six (two pairs from each grade level) over two years. In Israeli schools, fractions are introduced in the third trimester of the third grade. The curriculum of grades 4, 5 and 6 was designed to gradually introduce fractions so that by the end of 6th grade, students would have satisfactory mastery of the fraction and its different interpretations. This is an ongoing study, in which interviews are conducted by introducing a predesigned assignments. The children are then engaged in the activity in order to accomplish the task. As a preliminary step to this project, we composed a battery of 28 assignments of two kinds: (i) school tasks; (ii) assignments which are meant to spur a performance of a germinal routine that occasions the use of rational numbers (such as finding parts and fair sharing of both discrete and continues quantities). This second group of assignments are meant to check the applicability of the formal routines. As we noted earlier in this paper, our aim is to characterize the way in which Ada and Noa individualized the formal routines of locating a fraction and naming a point on the number line (routines of lasting and naming, for short). We do so by looking at five consecutive interviews in which the girls repeatedly performed the respective school assignments and one practical activity. We labeled the interview as presented in Table 1.

<table>
<thead>
<tr>
<th>Labeled</th>
<th>ITV1</th>
<th>ITV2</th>
<th>ITV3</th>
<th>ITV4</th>
<th>ITV5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade</td>
<td>4th</td>
<td>5th</td>
<td>5th</td>
<td>6th g</td>
<td>6th</td>
</tr>
</tbody>
</table>

Ritual and exploration were defined by considering the task of the routine. We are dealing with a ritual if the routine is oriented exclusively at the process, that is, the task at hand – the set of elements of the precedent performances the performer regards as requiring repetition – is related to the process in its entirety. In other words, these repetition-requiring elements are the specific operations implemented in the precedent situation, not just their outcome. We are dealing with an exploration if only the outcome of precedent performances counts as important.
Table 2. Features of routine performances in a task-situation

<table>
<thead>
<tr>
<th>aspect of routine $R$</th>
<th>extremal types</th>
<th>examples of analysis-guiding questions about the available performances</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Agentivity</strong></td>
<td>Ritual</td>
<td>The performer, in a given type of task-situation, is always executing the procedure as it was in the precedent. She is not making independent decisions.</td>
</tr>
<tr>
<td></td>
<td>Exploration</td>
<td>The procedure implemented by the performer required making independent decisions; in some performances, the performer was task-setter.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>From one performance to another, was there any increase in the number of independent decisions made by the performer?</td>
</tr>
<tr>
<td><strong>Boundedness</strong></td>
<td>Ritual</td>
<td>Different steps of the performance do not depend on one another, even if they should.</td>
</tr>
<tr>
<td></td>
<td>Exploration</td>
<td>Outcome of one step in the performance feeds into the next one.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Does each step in the performance that should depend on the outcome of the former does, indeed, utilize that outcome?</td>
</tr>
<tr>
<td><strong>Objectification</strong></td>
<td>Ritual</td>
<td>The talk is about processes; The focal signifiers, if used, are either stand-alone or appear as adjectives or adverbs.</td>
</tr>
<tr>
<td></td>
<td>Exploration</td>
<td>The focal signifiers are used as a noun, a name of an independently existing object.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Was there an increase in the performer’s use of the focal signifiers as nouns?</td>
</tr>
<tr>
<td><strong>Flexibility</strong></td>
<td>Ritual</td>
<td>The performer, over time, uses the same single procedure in similar task-situations.</td>
</tr>
<tr>
<td></td>
<td>Exploration</td>
<td>In a given type of task-situation, the performer uses a range of procedures, with their choice depending on the parameters of the situation (such as the numbers that were given).</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Was there an increase in the number of different procedures P’s performs in reaction to what can count as the same task-situation?</td>
</tr>
<tr>
<td><strong>Substantiation</strong></td>
<td>Ritual</td>
<td>The performer is either unable to give any substantiation of the correctness of her performance or simply repeats the performance claiming its correctness.</td>
</tr>
<tr>
<td></td>
<td>Exploration</td>
<td>The performer argues for the correctness of her previous performance by employing a different procedure and showing that she gets the same outcome.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Is P now less dependent on other people’s judgement in gauging the appropriateness of the execution?</td>
</tr>
<tr>
<td><strong>Applicability</strong></td>
<td>Ritual</td>
<td>The routine is performed only if task-situation is reproduced in almost all its details.</td>
</tr>
<tr>
<td></td>
<td>Exploration</td>
<td>No restrictions</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Was there an increase in the use of the procedure in new contexts?</td>
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</tbody>
</table>

The analyst’s question is how she can diagnose a degree of ritualization of a routine or a change in it by considering the records of specific performances. According to the commognitive approach, there are features of performances that the analyst can actually see and can take as indicative of a degree of ritualization (or de-ritualization). The features of Table 2 indicate how much the performance is directed toward a specific procedure and how much toward the specific outcome (Lavie, Steiner & Sfard, 2018). Among those features are: i) vi the procedure is functional; iii) Agentivity: the degree of decision making while operating; iv) Substantiation: the performance arguments for the correction of the outcome; v) Objectification: the transition to referring to the rational number as an independent object in the mediating discourse; vi)
Applicability: the performer finds the routine useful in situations where there is no significant similarity between the two task situations.

Here are some principles that the analyst must keep in mind while performing such diagnosis:

1. Most of the features appearing in Table 2 cannot be diagnosed directly on the basis of any specific performance. Instead, one needs to look at a whole series of performances for what counts for an expert as the same task-situation.
2. The properties of performances are not independent from one another. Sometimes, when you decide on one of them, the other almost automatically turns thru as well.
3. Diagnosing the features is an interpretive activity, in which any claim is tentative and subject to change; the change in interpretation may come at any time as a result from broadening the context and considering additional performances.

Because of space limitations, we will present in this article illustration only for some of the properties listed in Table 2. We reiterate that although the examples we chose are typical, in that they represented phenomena we saw in the rest of Noa and Ada’s data and also in those of other participants, we will not present here finding but rather an illustration of the analytic method.

Illustrations of the analytic method

We will present representative examples showing how different features of routines summarized in Table 2 were identified in our data. The examples presented here are taken from the five consecutive interviews in which the girls repeatedly performed the following two fourth grade textbook assignments:

Could you draw the number-line with the numbers 0, 1, 2, 3, 4 on it? Is it possible that \( \frac{5}{12} \) is on the number-line that you have drawn? Can you show where?

**Figure 1. Text with Assignment 1**

Plato the Wolf runs from his house (marked with 0) to the bowl with food (marked with 1). What part of the way did he already make, approximately?

**Figure 2. Text with assignment 2**

Flexibility

The rise in the flexibility of a routine means that there is now more than one way to perform the task. Stating that a routine performed in a task-situation became more flexible means that there is now more than one optional procedure to perform the task. The routine become more flexible through the adaption of the procedure to typical features of the current task. Looking at Table 3, which present the procedures performed by Ada and Noa in the activity over Assignment 1, it might seems at first glance that the same procedure of dividing into twelfth and then reaching to the fifth twelfth is executed over and over again. However, despite the similarity between the procedures implemented by the girls, we argue that there is a process of streamlining - the adaptation of the procedure of the formal routine that they have learned to the specific assignment with which they are coping. Ada shifted from dividing the interval \([0,1]\) into twelfth to another method of division that was easier for her to implement. Then she made yet another alternation when she divided only the interval \([0,0.5]\).
Table 3. The girls’ procedures in implementing Assignment 1

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Noa</strong></td>
<td>(1) Divide all the intervals into halves</td>
<td>(1) Divide $[0,1]$ into half</td>
<td>(1) Divide $[0,1]$ into two halves</td>
<td>(1) Divide $[0,1]$ into half</td>
</tr>
<tr>
<td></td>
<td>(2) Divide $[0,0.5]$ and $[0.5,1]$ into sixths</td>
<td>(2) Divide $[0,0.5]$ and $[0.5,1]$ into halves</td>
<td>(2) Divide $[0,0.5]$ and $[0.5,1]$ into halves</td>
<td>(2) Divide $[0,0.5]$ and $[0.5,1]$ into halves</td>
</tr>
<tr>
<td></td>
<td>(3) Count five segments from zero</td>
<td>(3) Divide all new segments into three</td>
<td>(3) Divide all new segments into three equal</td>
<td>(3) Divide all new segments into three</td>
</tr>
<tr>
<td></td>
<td></td>
<td>equal parts</td>
<td>segments into three equal parts</td>
<td>segments into three equal parts</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(4) Count five segments from zero</td>
<td>(4) Count five segments from zero</td>
<td>(4) Count one segment from half</td>
</tr>
<tr>
<td><strong>Ada</strong></td>
<td>(1) Divide $[0,1]$ into twelfth</td>
<td>(1) Divide $[0,1]$ into half</td>
<td>(1) Divide $[0,1]$ into half</td>
<td>(1) Divide $[0,1]$ into half</td>
</tr>
<tr>
<td></td>
<td>(2) Count five segments from zero</td>
<td>(2) Divide $[0,0.5]$ into sixth</td>
<td>(2) Divide $[0,0.5]$ into sixth</td>
<td>(2) Divide $[0,0.5]$ into sixth</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3) Count five segments from zero</td>
<td>(3) Count five segments from zero</td>
<td>(3) Count one segment from half</td>
</tr>
</tbody>
</table>

Another sign of flexibility can be found in the excerpt taken from ITV3 where the equivalence of different procedures is explicitly recognized by Ada.

Ada: Ok, I did it between one to zero because I know it is not yet a whole and then I divided it into half, and then I sevise both halves to sixth, I did not complete the division of that one’ and here is the five.

Noa: I am dividing it to half to half and then divide into three.

Ada: But it is the method, like, it is less important, the how we divide it.

**Objectification**

For the discourse of rational numbers to be considered sufficiently developed, it is not enough that the discussant is acquainted with the different signifiers and realizations of rational numbers. It is also important that she objectifies rational numbers, that is, becomes able to see all of the relevant signifiers as signifying full-fledged objects in their own right. Hence, the increasing tendency of using the signifiers as nouns is yet another indication for the progress of de-ritualization. Throughout the repeated performances of Assignment 2 we could see changes in the linguistic use of the signifier fourth in the girls mediating discourse. The transformation from referring to the location of the wolf as: "It is in the middle of the second quarter" in ITV1, to utterance like this: "It is right in the middle between forth and two forth" in ITV3 indicates that the girls have objectified the signifier fourth. In the repeated performance of assignment 1 Ada have objectified the signifier five over twelve. Along ITV1,2 and 3, Ada did not related to the fraction five over twelve as a number, but rather as a set of twelfths with five entities. As can be seen in the following excerpt taken from ITV2, Five is the number of twelfths that we need to count from zero:

Ada: I divided it into twelfth and then I checked where the five was, because it is five over twelve.

The objectification of five over twelve in the discourse of Ada is expressed in the following excerpt taken from TLV4 as Ada referred to it as a name of the location that is needed to be found.

Ada: I divided the zero to one into twelfth and then I just found the five over twelve.

**Substantiability**

When focused only on the procedure, the performer would substantiate her actions by simply describing the procedure she executed. Hence, a step forward in the process of de-ritualization
would be substantiation of the outcome by showing that an alternative procedure would yield the same outcome. We found that over time the routines of the two task-situations became more substantive. One of the signs of substantiation can be found in the change in Noa's explanation of her choice to divide the interval unto eight equal parts in performing Assignment 2 (Table 2).

<table>
<thead>
<tr>
<th>Table 4. Signs of substantiation in Noa's explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>TV1</td>
</tr>
<tr>
<td>TV3</td>
</tr>
<tr>
<td>TV4</td>
</tr>
</tbody>
</table>

In ITV1 Noa is having hard time substantiate her actions. In ITV3 she is more confident and describes the calculation that led her to eighth, but she does not use words that are typical to the discourse of rational numbers as we would expect. In ITV4 she is describing the calculation again but this time she is using new words that are typical to the discourse of rational numbers that she didn’t use before, like half quarter and denominator. In addition Noa is using another procedure that of division that her partner Ada was using in TLV4 to in order to substation her action.

**Applicability of the two formal routines**

We speak about routine’s applicability while considering the range of task-situations for which its performances so far are likely to constitute precedents. In order to learn of the applicability of the two formal routines we presented the girls with the following activity that is supposed to evoke the use of the two formal routines. Each girl is given a card. Ada got the card in Figure 3 and had to instruct Noa over the phone to draw the point on her card (Figure 4) in the same location.

Using her demonstrated ability to name the point with a fractional number (23/4) would have helped Ada in this assignment, and would have been useful for Noa too, since she could translate a fraction into a number line location, as she had already shown during the execution of assignment 1. And yet, none of these routines were applied. We take this fact as an evidence of their ritualized character. Noa and Ada were asked to perform the telephone assignment over and over again in every session, but although their procedure underwent several refinements as is shown above, it did not involve point-naming and point-reading. It was only 18 months after ITR1 took place that the girls evoked that routine in that context, as shown in excerpt from ITR5:

Ada: Ok, divide between 2 and 3 into quarters (Looks at the page and marks with a finger the division of the interval between 2 and 3)  
Noa: Ok... three quarters? (Looks at her page)  
Ada: Yes (Looks at her page and smiles)  
Both: (Laugh)  
Noa: (Divides the interval between 2 and 3 into four more-or-less equal parts)  
Ada: Perfect. So two and three quarters, approximately (Looks at her page)

Here, it was the first time Ada was preforming the formal routine for naming the point to be copied with a fractional number. Later, Noa used the point-reading formal routine for reproducing
the desired location on the line. We can summarize by saying that in ITR5 we could finally recognize that the formal routines were applicable in a non-school-like assignment.

Concluding remarks

In the above, we have described the analytic method by which we operate in this study. As time-consuming and laborious a task it is to conduct this kind of analysis we feel that it is worthwhile to undertake this effort. Underlying the decision to study the notion of routine is our belief that conceptualizing learning as a process of routinization may generate new insights.

Conceptualizing the learning of rational number as de-ritualization of the formal school routines, which is reflected primarily in the shift of focus from the implementation of a certain procedure to achieving a desired outcome, allows us to look at familiar phenomena that are considered challenging in the world of education in a new and insightful way. We know from previous research that the process of de-ritualization is gradual and slow and only too often will not be completed in school. Such analytic method may provide answers to outstanding questions in the research of learning rational numbers. Questions like the ones listed in the Second Handbook on Mathematics Teaching and Learning (Lamon, 2007) may not get holistic and comprehensive responses. They are questions like: How does one measure rational number sense? How does a child come to understand a rational number as a single quantity as oppose to regarding it as a pair of numbers? What are the benchmarks by which to judge that children's knowledge is moving in a desirable direction? And how can we assess depth of understanding of the rational numbers?

References


Posters
More about teacher’s revoicing in the mathematics classroom

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Aim and context

These lines derive from my doctoral dissertation (Boukafri, 2017) regarding the study of revoicing in the mathematics classroom. I use the notion of revoicing accordingly with O’Connor and Michaels (1996) as recasting and reformulating contributions of participants in classroom contexts of talk. Despite the increased attention given to this language move in mathematics education research over the last years (e.g., Moschkovich, 2015), there is few work on analytical methods for the detailed study of revoicing as developed in the language of mathematics of the teacher in classroom discourse. My aim here is not to examine adequate theoretical frameworks – I generally draw on sociocultural theories of mathematics education that place the language of the teacher at the interplay of distinct voices in the social context of the classroom (Planas, 2012). Instead, I present some insights regarding the analytical deconstruction of the language of the teacher in the search of meaning for instances of revoicing. I examine revoicing at the successive structural levels of: 1) spoken turns, 2) episodes made of turns, and 3) lessons in order to explore language use and meaning production during mathematics teaching and learning. The specific tools developed for the organization and analysis of these levels of discourse jointly help to explain the complexity and variety of forms and functions of revoicing.

Tools for the analysis of revoicing

My lesson data come from video recording of whole-class discussions of two mathematics classrooms in two secondary schools of Barcelona. As a result, the term ‘discourse’ in my work refers to the institutional interactional settings embedded in the naturally occurring contexts of the mathematics classroom. Multimodal transcripts – talk into text and gesture into imagery – were elaborated for the initial identification of linguistic forms of revoicing and the reduction of data into pieces of language with empirical illustrations of this type of move. This process required the deductive refinement of linguistic forms into codes consistent with literature in the domain (e.g., O’Connor & Michaels, 1996; Forman & Ansell, 2001) but also the inductive production of newer codes (Planas, 2004). While linguistic codification was primarily undertaken through revision of talk into text in the transcripts, functional codification was more sophisticated in that the intention was to explore the impact of the teacher’s revoicing on particular turns and interactional episodes of a lesson for a number of lessons.

Figure 1. Examples of episode connections underlying revoicing

Figure 1 represents the schematic output emerging from the application of one of the tools to three lessons in the study. This tool connects episodes (e_i) in a functional sense with arrows that
represent directions of influence in the interaction between teacher and learners. It particularly establishes an inferential connection between the manifestations of revoicing in the language of the teacher and the manifestations of certain mathematical contents in the language of the mathematics classroom—accomplished by either the teacher or the students in the interaction. These connections are inferential with each other in that we can use them to make inferences as we go through the explanation of the use and role of revoicing in the language of the classroom related to the development of the mathematical task. On the other hand, in all these connections, revoicing is viewed as generating or implied in the traceable link between episodes. Such a link is not directly traceable at the beginning of the analysis, before the application of tools for the successive study of the following features: 1) the language of mathematics in the task of the lesson; 2) the linguistic forms and potential functions of revoicing in turns; 3) the potential relationships between turns with revoicing and episodes of mathematical content; and 4) the potential relationships across episodes with shared turns.

Concluding remarks

More than a mere illustration of particular data and results, the creation, application and validation of specific tools for the analysis of revoicing can help to enrich future research on the use and role of this and other language moves in mathematics teaching and learning. The position that the triad turn-episode-lesson is fundamental for the broader study of revoicing can be taken in further analyses of classroom-based language use. A sort of continuity shows the impact of revoicing not only during single turns or episodes but also throughout the lesson when the mathematical topic under discussion seems to have become out of focus. An expanded look at revoicing from the perspective of its manifestation in a turn of the language of the teacher to its manifestation in the overall language of the classroom can better explain relationships between mathematics learning and language use. That said, the major challenge is to approach this expanded look by means of a more comprehensive interpretation of the functions of language.

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Patterns of interaction and construction of shared mathematical meaning in classroom

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Background and introduction
My research with lesson data in a secondary mathematics classroom of Barcelona (Chico, 2014) drew on a sociocultural understanding of learning through talk-in-interaction to examine language use in situations that accomplish a mathematical discourse. In the tradition of symbolic interactionism, interaction is seen as a recursive process where individual actions are influenced and influence the interpretation of and response to the broader system of actions taking place in a given social context and time (Goffman, 1981; Krummheuer, 2007). Following Sfard (2008), student mathematical learning emerges through the practice of specific forms of discourse. Accordingly, I view the collective construction of the language of mathematics in classroom as an articulated process of individual actions of participants that have the opportunity to recognise, use and construct meanings in particular types of mathematical discourse. From this perspective, it is fundamental the study of social interaction in the midst of specific processes of collaboration, negotiation and construction of shared meaning in the mathematics classroom (Planas, 2014). This is why I examine class conversations among students and between students and the teacher and attempt to identify patterns of interaction as well as their impact on the production of a language of mathematics that can be interpreted as evidence of mathematical learning.

Lesson data and methods
My lesson data comes from the registration of class discussions in a sequence of five lessons with five problems about mathematical generalization. For each problem, the teacher introduces the task, and provides time for pair work followed by classroom group discussion. All problems consist of three successive moments of conceptual learning around: 1) Near generalization (work on particular cases that allow the use of drawing strategies); 2) Far generalization (work on particular cases that are not easily perceptual and allow the use of recursive strategies); 3) Algebraic generalization (work on an algebraic expression that represents the general case). In this way, there is increasing mathematical complexity in terms of the actions required to identify common generalizations that arise from the study of particular cases which are either numerical or geometrical. The application of constant comparative and inductive methods (Planas, 2004) served to reduce primary lesson data by producing related types of peer interaction and mathematical content. I looked for advances in the communication of mathematical content and for the types of interaction involved. In a more advanced stage of the research, and continuing with the application of similar methods of comparison, I came to some basic patterns of interaction made up of two consecutive types of peer interaction, both involved in the production of the language of mathematics during the discussion of a specific moment of generalization.

Some of the patterns of interaction produced
Based on the analysis developed, I can claim that some of the basic patterns of interaction constructed correspond to isolable situations of group and pair work with impact on the production of the language of mathematics of the learners. The mathematical product of these basic patterns functions as mediator of and contributor to the mathematical language in use during the resolution of the task. Overall, mathematically relevant moments of group interaction can be
de-constructed through basic patterns of interaction and a number of regular compositions among them. Despite the fact that the constitutive parts of a basic pattern tend to be contiguous, this is not always the case and exceptions may need further investigation. There are situations where, between the first and the second components of a pattern (e.g., ‘Initiating’ and ‘Sharing’), there is another basic pattern of interaction (e.g., ‘Initiating’, ‘Querying’ and ‘Sharing’) inserted or situations where several basic patterns are noticeable (see them and the detail of the research in Chico, 2014). Figure 1 illustrates three of the most frequent basic patterns found (first line in blue) in relation to the mathematical content underlying (second line in orange) that came out of the analysis. On the other hand, the composition of these basic patterns also seemed to play a role in the development of mathematical learning, particularly numerical and algebraic thinking.

Figure 1. Empirically-based examples of basic patterns of interaction

More generally, an interesting outcome of this research is the fact that shared meaning constructed in group discussion is produced as a non-consecutive process, with several turns in-between the development of specific mathematical reasoning and direct pairs of question-answer often being quite far in time during one same lesson. Hence, the production of the language of mathematics seems to be complex in terms not only of collaboration and of negotiation of meaning, but also due to discontinuities and interferences in the interaction and communication during group discussion. This sort of discontinuities are worthy of further study, for they may indicate the need to revise mathematical interaction and communication in more dialectical terms.

Acknowledgement


References


Reading mathematical texts in high-school: Expanding the discursive repertoire in the mathematics classroom

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Introduction

The mathematical classroom discourse is a collective creation where a teacher and her students are practicing mathematical activities while using mathematical language. It is the teacher who primarily uses various resources to present mathematical discourse to her class and foster her students as they become active participants in the evolving mathematical classroom discourse. The research I am about to introduce examines the use of mathematical texts as a resource, seldom used, for creating a unique mathematical activity (Borasi & Siegel, 2000). The term "mathematical text" is used here in a concrete way to signify original mathematical enrichment articles addressing mathematics teachers or mathematically oriented students. Good examples for such texts can be found in journals like The Mathematics Teacher and in various mathematical enrichment books (Posamentier, 2003 pp. 67-69; Usiskin, 1968). These articles are not meant to teach the students new chapters of an intended curriculum, but rather letting them recruit all the mathematics they are by now familiar with for "cracking" this newly introduced mathematical piece. The research aims at delineating the various levels of discourse that emerges when small groups of students guided by their mathematics teacher are engaged in a long-term course of reading mathematical texts.

Description of the research

In order to dwell into the processes of learning to read mathematical texts, a short course was designed to address small groups of 3-4 twelfth-grade math-majors. The texts to be read in the course were selected according to the following three complementary criteria:

1. They refer to mathematics that the students attending the course are generally familiar with, but on the other hand there should be gaps and obstacles in the texts that the students will need to learn how to overcome.
2. It should be possible to read and to accomplish some additionally planned activities within one or two lessons of 90 minutes.
3. The texts should be considered by the community of mathematics practitioners and educators as presenting an aesthetic dimension of mathematics.

The course was designed according to two leading principles. The first principle refers to the sequencing of texts according to increasing mathematical difficulty, and the second refers to the decreasing level of support (fading) provided by the teacher in the classroom. The last two lessons of the course were planned to let the students experience an independent preparation of a text of their choice, followed by a presentation of that text to an outside supportive audience. The course was enacted twice, addressing two different populations of students. All lessons were audio-recorded and carefully transcribed. The transcriptions were analyzed according to the commognitive framework (Sfard, 2008). Following this framework, the discourse was characterized according to its level (object-level or meta-level), its key words and endorsed narratives detected in it (are they representative of a specific community of discourse?) and its routines (are they ritual or explorative?).
Main findings

The discourse evolving while a group of students experience a long-term guided encounter with mathematical texts has three interlocutors, namely, the student, the teacher and the text which represents a specific kind of mathematical discourse. As the course advances, the discourse of the learning community slowly shifts towards becoming mainly a dual discourse between the students and the text. Eventually, the students learned to communicate with each other, to question the text and each other and to agree upon a coherent interpretation of the text in an expert-like manner. The students’ ability to engage in a meaningful communication with a mathematical text is mainly due to their adoption of some norms or meta-rules that govern that discourse. One of these norms states the legitimacy of sensing and expressing clearly the notion of not-understanding (or understanding) something in the text. Scrutinizing the development of the discourse reveals the process of how the students develop an independent sensitivity to what is clear and what is not. They increasingly express their sense of understanding and use it to share their interpretation of the text with their peers, and by doing that, expose it to criticism. In a similar manner, the students increasingly express their sense of not understanding and through this expression they sometimes implicitly ask for help from their fellow students or in a more active manner, use the text in hand to resolve their queries. These results are in accordance with previous research regarding reading in general (Pressley & Afflerbach, 1995). In addition, they may contribute to the community’s efforts to understand and maybe unravel the difficulties students encounter when they are reading mathematical text-books and proofs (Inglis & Alcock, 2012).

Implications

The study suggests that reading mathematical texts holds the potential to expand and enrich the way in which mathematics is taught in high school. The conclusions stated above, as well as the practical experience, accumulated through the design and enactment of the learning environment, may serve as an empirical and research-based foundation to assist policymakers and teachers in integrating the reading of mathematical texts within the advanced level high school mathematics curriculum.

References


Pre-service teachers learn to analyse classroom situations: How is their development related to the use of technical language?

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Introduction

Learning to analyse classroom situations can be considered as an important objective in mathematics teacher education. In this context, the use of technical language is assumed to be an important requirement for pre-service teachers to describe and discuss teaching practice. We conducted a one-semester university course focusing on pre-service teachers’ learning to analyse the use of multiple representations in mathematics classroom situations and investigated how their development in analysing was related to the technical language they made use of. At the beginning of the course, key elements of theory related to the use of multiple representations in the mathematics classroom were introduced (e.g., according to the approach in Duval, 2006). Based on these elements of theory, criteria for the analysis of classroom learning situations (e.g., presented as video clips) under the focus of multiple representations were derived. Correspondingly, the pre-service teachers learned to make use of relevant terms in the context of learning with multiple representations, such as register and conversion (Duval, 2006).

Theoretical background and research question

Due to the essential role representations play for the teaching and learning of mathematics, analysing the use of multiple representations has been described as a key aspect of mathematics teachers’ professional competence (e.g., Friesen & Kuntze, 2016). Mathematics teachers must be able, for example, to identify situations where students need support in using representations or in connecting different representations of the same mathematical object to each other (e.g., Mitchell, Charalambous & Hill, 2014). In this context, it is assumed that what teachers identify and interpret when observing classroom situations is not only channelled by their knowledge, but also by what they can name (Mesiti et al., 2017). Teachers’ analysis of classroom situations and its development in teacher education might consequently be related to the use of a technical language for describing practice and naming its components (Grossman et al., 2009). In our research, we were interested in pre-service teachers’ development in identifying and interpreting events relevant for students’ learning with representations. This resulted in the following research question: How is pre-service teachers’ competence of analysing related to the use of technical language regarding multiple representations before and after a corresponding university course?

Methods, results and discussion

We collected 112 written analysing results from seven pre-service teachers who each evaluated eight classroom situations regarding the use of representations before and after the university course. Each classroom situation was followed by an open-ended question: How appropriate is the teacher’s response in order to help the students? Please evaluate the use of representations and give reasons for your answer. Although no technical terms as introduced in the course were
found in the answers of the pre-test, 19 results indicated that events relevant for students’ learning with multiple representations have been identified and correctly interpreted. However, many of these results were characterised by a rather vague and intuitive argumentation. In the post-test, 35 results indicated a successful analysis regarding the use of multiple representations. Amongst them were nine answers with technical language related to representations as introduced in the university course.

The findings show that the use of technical language as introduced in the university course was not a necessary precondition for identifying and interpreting classroom events relevant for students’ learning with multiple representations. It indicates that it was possible for the pre-service teachers to give correct analysing results without the explicit use of relevant technical terms. On the other hand, however, all but one of the answers containing relevant technical language also indicated successful analysing of the corresponding classroom situation. Moreover, the argumentation in these analysing results was characterised by being more pointed and consistent. It might therefore be concluded that the use of technical language could help the pre-service teachers to interpret events they identified regarding the use of multiple representations and to improve their analysing results. The use of technical language for articulating analysing results should consequently be paid particular attention when pre-service teachers learn to analyse classroom situations. Specific support in applying relevant technical terms to classroom events, however, might be necessary. The reported findings encourage further research into the relation between the use of technical language and pre-service teachers’ learning to analyse classroom situations. A corresponding study with a bigger sample of pre-service teachers is currently being conducted at Ludwigsburg University of Education.

References


Flipped classroom and language in mathematics teaching

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Introduction and theoretical background

We present an action research project of a young secondary mathematics teacher introducing the “flipped classroom” approach into her teaching. The project was implemented and evaluated within the context of a larger initiative where teachers were supported by experienced teacher educators. During the project, the issue of language arose. The poster, written by the teacher and supported by the initiative leader, documents the teachers’ struggle in critically reflecting her project and finding ways to improve her teaching, in particular focusing on the issue of language.

Lage, Platt and Treglia (2000) defined the key-term “inverting” of flipped classroom as follows: “Inverting the classroom means that events that have traditionally taken place inside the classroom now take place outside the classroom and vice versa” (p. 32). Therefore, the flipped classroom is a newer teaching approach, where students have to watch videos with teacher instructions as homework most of the time, and the time of the lesson is mostly used for active, group-based or autonomous problem-solving activities.

The Project: Flipped classroom and its implications for language

Participants in this project were 180 students (grades 9-11) at an urban college of business administration in Austria. The flipped classroom model was implemented in six mathematics classes throughout the whole schoolyear 2016/17. The used 5-10 minute interactive videos were produced by the teacher (first author). The goals of the project were to give students the chance to control their own learning, to focus more on understanding than on recall and to reduce teacher lectures to open in-class time for implementing different teaching approaches.

During the school-year, the advisor of the project recommended to also focus on communication and the use of the flipped classroom model (flipping) in mathematics. There are only few research studies that look at these specific topics. The study of Murphy, Chang and Suaray (2016) showed that flipping a linear algebra course improved the students’ skills in communicating mathematical ideas regarding the subject topic.

Investigating one’s own teaching

The main reason for the teacher’s investigations was to get more knowledge about her (flipped classroom) mathematics teaching in order to interpret and improve it (in line with the principles of action research, see e.g., Altrichter et al., 2008). An anonymous online questionnaire with 12 statements was used at the end of the first semester to explore students’ attitudes towards the use of the flipped classroom in their learning of mathematics. The issue of language arose during the project and the idea about the poster presentation with reflections on the process much later. Thus, the methodology did not include special questions on language at that stage. However, there were relevant data related to that, not only because language is necessarily present in any teaching process, but also because numerous decisions about the role and use of language had to be taken.
Videos as a source of learning. An advantage of using videos in a flipped classroom is that everybody can learn at their own pace, because participants can pause or rewind a video at their convenience. If they need more processing time, they can watch difficult parts of the video several times. For example, students with migration background mentioned in their feedback to the project that it was helpful for them to look up some unknown German words. The mathematical language that teachers use in the videos is another interesting aspect of the flipped classroom. Howson (1980) thinks that if mathematics is not discussed in the language of the learners, they do not absorb it as part of their culture. While producing the videos for the described project, the teacher tried on the one hand to adjust the language to students’ competencies and on the other hand to be mathematically precise.

Reflections and future

This project has shown that using the flipped classroom model for teaching mathematics can leave students more space to communicate with their schoolmates and the teacher than in a more traditional classroom setting. For instance, there is time to discuss different ways of solving a mathematical problem or to sort out their problems regarding specific video content. In the future, it would be also interesting to try out and investigate mathematics learning in contexts of whole class discussion, as Boukafri, Ferrer and Planas (2015) did in their design experiment. From a methodological point of view, the questionnaire delivered some insights whether the flipped classroom model changed the classroom culture. However, mostly students’ comments helped to better understand the reasons for their answers. In the future, the teacher intends to sharpen the focus of her investigation and to try out instruments that allow deeper analysis. Even new media (e.g., mobile phones used for interviews) could be used. This would mean that both the teaching approach and the evaluation instrument reflect the culture of the digital age.

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