12.4 Bsp. $(\mathbb{R},+)$, diskrete Vt.

a) $\mu = \varepsilon_x$, $\nu = \varepsilon_y$: $\varepsilon_x * \varepsilon_y = \varepsilon_{x+y}$ [Analog $\mathbb{R}$

$[\![\xi \equiv x,\ \eta \equiv y \Rightarrow \xi+\eta \equiv x+y]\!]$

b) Binomialverteilungen. $0 < p, q < 1$, $q := 1-p$.

(1) $\beta(1,p) * \beta(1,p) = \beta(2,p)$

$[\![(q\varepsilon_0 + p\varepsilon_1) * (q\varepsilon_0 + p\varepsilon_1) = q^2\varepsilon_0 + (pq+qp)\varepsilon_1 +$

(2) $\beta(n,p) * \beta(m,p) = \beta(n+m,p)$

(3) $\beta(n,p) = \underbrace{\beta(1,p) * \cdots * \beta(1,p)}_{n} =: \beta(1,p)^n$
(Faltungspoten

[Durch Induktion aus (1)]

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c) $\pi_\alpha * \pi_\beta = \pi_{\alpha+\beta}$ $(\alpha, \beta > 0$.

$$\left(\sum_{k\geq 0} e^{-\alpha} \frac{\alpha^k}{k!} \varepsilon_k\right) * \left(\sum_{j\geq 0} e^{-\beta} \frac{\beta^j}{j!} \varepsilon_j\right) = \Big(\text{Faltung a} \atop \mathbb{Z}_+$$

$$= \sum_{n\geq 0} e^{-(\alpha+\beta)} \left(\sum_{k=0}^{n} \frac{\alpha^{n-k}\beta^k}{(n-k)!\cdot k!}\right) \varepsilon_n = \sum_n e^{-(\alpha+\beta)} \frac{(\alpha+\beta)}{n!}$$

$$[\![\sum_{k=0}^{n} \left(\frac{\alpha^{n-k}\beta^k \cdot n!}{(n-k)!\cdot k!}\right) \cdot \frac{1}{n!} = \frac{1}{n!}(\alpha+\beta)^n]\!]$$

Insbes: $\forall \alpha > 0$, $\forall n \in \mathbb{N}$: $\left(\pi_{\frac{\alpha}{n}}\right)^n = \pi_\alpha$.

$\forall \alpha > 0$, $\forall n$, $p_1, \dots p_n > 0$, $\sum_1^n p_i = \alpha$:

$$\pi_\alpha = \mathop{\Large *}_{1}^{n} \pi_{p_i}$$