β) $n$ Beobachtungen, $n \geq 2$ { $x_1 \dots x_n$ nicht alle gleich.

$$L((a,\beta); x_1, \dots, x_n) = \prod_1^n L((a,\beta); x_i) =$$

$$= \frac{\beta^{n/2}}{(2\pi)^{n/2}} \cdot \exp\left(-\frac{\beta}{2}\sum_1^n (x_i - a)^2\right)$$

$$\log L((a,\beta); x_1 \dots x_n) = \sum \log L((a,\beta); x_i) =$$

$$= \frac{n}{2}\log\beta - \log(2\pi)^{n/2} - \frac{\beta}{2}\sum_1^n (x_i - a)^2 \stackrel{.}{=} g(a,\beta)$$

Kritische P.: 

$$\frac{\partial g(a,\beta)}{\partial a} = \frac{\beta}{2} \cdot 2 \sum_1^n (x_i - a) \stackrel{!}{=} 0 \quad ①$$

$$\frac{\partial g(a,\beta)}{\partial \beta} = \frac{n}{2\beta} - \frac{1}{2}\sum_1^n (x_i - a)^2 \stackrel{!}{=} 0 \quad ②$$

$$① \Rightarrow \hat{a} = \frac{1}{n}\sum_1^n x_i \Rightarrow \text{in } ② \quad \hat{\beta} = \frac{n}{\sum_1^n (x_i - \hat{a})^2}$$

$\neq 0$, da $n \geq 2$

Daher

$$\boxed{\begin{aligned} \hat{a} &= \frac{1}{n}\sum_1^n x_i =: \overline{X}_n \quad ① \\ \hat{\sigma}^2 &= \frac{1}{n}\sum_1^n (x_i - \overline{X}_n)^2 \quad ② \end{aligned}}$$

Globales Max: z.B. 2-te Ableitungen

① „Stichprobenmittel"

② ≈ „Stichprobenvarianz"

[Häufiger:
$$\frac{1}{n-1}\sum (x_i - \overline{X}_n)^2$$
(später: „erwartungstreu")]