... π_j bereits konstruiert, $1 \le j \le k$ mit
$\pi_j \in P_m(j,j)$, $j \le k$, $\xi_j(\pi_\ell) = x_j$, $j \le \ell \le k$.

Sei $x_{k+1} =: \ell$.

$\boxed{\ell = 1}$: π_{k+1}: $\begin{cases} k+1 \longmapsto 1 = x_{k+1} \\ j \longmapsto \pi_k(j) + 1, \ 1 \le j \le k \end{cases}$

$(\pi_{k+1}(k+1) \le \pi_{k+1}(i) \ \forall i)$

$\boxed{\ell = k+1}$: π_{k+1}: $\begin{cases} j \longmapsto \pi_k(j), \ 1 \le j \le k \\ k+1 \longmapsto k+1 = x_{k+1} \end{cases}$

Allgemein!
$\ell \in \{2, \dots k\}$ $\quad \pi_{k+1}(j) := \begin{cases} \pi_k(j), : \pi_k(j) \le \ell - 1 \\ \pi_k(j) + 1 : \pi_k(j) \ge \ell \\ \ell = x_{k+1} : j = k+1 \end{cases}$

D.h. π_{k+1} : $\begin{cases} M_k \ni j \longmapsto f_{k+1}(\pi_k(j)) \\ k+1 \longmapsto \ell = x_{k+1} \end{cases}$!

Da f_{k+1} strikt $\nearrow$ gilt:

$j \le k$: $\pi_{k+1}(j) \le \pi_{k+1}(j') \Longleftrightarrow \pi_k(j) \le \pi_k(j')$

Daher $\xi_{k+1}(\pi_{k+1}) = x_{k+1}$, $\xi_k(\pi_{k+1}) = \xi_k(\pi_k)$

Nach Induktionsann.: $\xi_j(\pi_{k+1}) = \xi_j(\pi_k)$
$j \le k$.

.....
$\pi_N =: \pi$