

Ant $k=1$ $x_1=1, M_1 = \{1\}, \pi_1: 1 \mapsto 1.$

$k=2$ $x_1=1 \Rightarrow \pi_2(2) \leq \pi_2(1) \Rightarrow \pi_2: \begin{cases} 1 \mapsto 2 \\ 2 \mapsto 1 = x_2 \end{cases}$

$x_1=2 \Rightarrow \pi_2(2) \geq \pi_2(1) \Rightarrow \pi_2: \begin{cases} 1 \mapsto 1 \\ 2 \mapsto 2 = x_2 \end{cases}$

$k=3$ $x_2=1$ $\pi_3(3) \leq \pi_3(i) \forall i \Rightarrow \pi_3: \begin{cases} 3 \mapsto 1 = x_3 \\ 1 \mapsto 3 \\ 2 \mapsto 2 \end{cases} \begin{matrix} \\ \pi_2+1 \\ \end{matrix}$

$x_2=2$

$x_2=1$ $\pi_3: \begin{cases} 3 \mapsto 1 = x_3 \\ 1 \mapsto 2 \\ 2 \mapsto 3 \end{cases} \begin{matrix} \\ \pi_2+1 \\ \end{matrix}$

$x_3=3$ $\pi_3(3) \geq \pi_3(i) \forall i$

$x_2=2$: $\pi_3: \begin{cases} 1 \mapsto 1 \\ 2 \mapsto 2 \\ 3 \mapsto 3 = x_3 \end{cases} \begin{matrix} \\ \pi_2 \\ \end{matrix}$

$x_2=1$: $\pi_3: \begin{cases} 1 \mapsto 2 \\ 2 \mapsto 1 \\ 3 \mapsto 3 = x_3 \end{cases} \begin{matrix} \\ \pi_2 \\ \end{matrix}$

$x_3=2$

$x_2=2$ $\pi_3: \begin{cases} 1 \mapsto 1 & (\pi_2) \\ 3 \mapsto 2 = x_3 \\ 2 \mapsto 3 & (\pi_2+1) \end{cases}$

$x_2=1$ $\pi_3: \begin{cases} 2 \mapsto 1 & (\pi_2) \\ 3 \mapsto 2 = x_3 \\ 1 \mapsto 3 & (\pi_2+1) \end{cases}$

Allgemein: $p_{k+1}^{(e)}: \begin{cases} j \mapsto j, j \leq k-1 \\ j \mapsto j+1, j \geq k \end{cases}$

$p_{k+1}^{(e)}$ strikt \uparrow (injektiv) $M_k \rightarrow M_{k+1}$