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$$= \Phi(x) + \frac{x}{2n}\Phi'(x) + O\left(\frac{1}{n^2}\right) + \frac{1}{2}\beta^2 \Phi''(\alpha x) + o\left(\frac{1}{\sqrt{n^3}}\right)$$

$$= \Phi(x) + \frac{x}{2n}\Phi'(x) + \frac{1}{2n}\left(\Phi''(x) + \frac{x}{2n}\Phi'''(x) + o\left(\frac{1}{n^2}\right)\right)$$
$$+ o\left(\frac{1}{\sqrt{n^3}}\right)$$

$$= \Phi(x) + \frac{x}{2n}\Phi'(x) + \frac{1}{2n}\Phi''(x) + \underbrace{\Big[\overbrace{\frac{x}{4n^2}\Phi''(x)}^{\leq \frac{1}{\sqrt{n^3}}} + \overbrace{\frac{1}{2n}\cdot o\left(\frac{1}{n^2}\right)}^{\leq \frac{1}{\sqrt{n^3}}} + o\left(\frac{1}{\sqrt{n^3}}\right)\Big]}_{\leq 3\cdot\frac{1}{\sqrt{n^3}} = o\left(\frac{1}{\sqrt{n^3}}\right)}$$

$$= \Phi(x) + \frac{x}{2n}\Phi'(x) + \frac{1}{2n}\Phi''(x)$$
$$+ o\left(\frac{1}{\sqrt{n^3}}\right)$$

~~Es~~ gilt: $\Phi''(x) = -\frac{1}{\sqrt{2\pi}}\, x e^{-\frac{x^2}{2}}$

$$= -x \cdot \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} = -x \cdot \Phi'(x)$$

$$\Rightarrow E\left(\Phi(\alpha x - \beta X_{n+1})\right) = \Phi(x) + O\left(\frac{1}{n^{3/2}}\right)$$

Dann folgt aus $d(F_{n+1}, \Phi) \leq d(F_n, \Phi) + \sup_{x \in \mathbb{R}} \left| E\left(\Phi(\alpha x - \beta X_{n+1})\right) - \Phi(x)\right|$

$$d(F_{n+1}, \Phi) \leq d(F_n, \Phi) + O\left(\frac{1}{n^{3/2}}\right)$$

und weiter für geeignete von n, N unabhängige $U > 0$:

$$d(F_{n+1}, \Phi) \leq d(F_n, \Phi) + U n^{-3/2}$$