16th GAMM-Seminar on Microstructures

TU Dortmund
January, 20-21, 2017

This workshop is supported by:
Department of Mathematics of the TU Dortmund
The **16th GAMM-Seminar on Microstructures** is hosted by the TU Dortmund from January 20 to January 21, 2017.

**Location**
All lectures take place in room 1.001 at the first floor of the Seminarraumgebäude 1, Friedrich-Wöhler-Weg 6, 44227 Dortmund.

**Keynote Speakers**
Grégoire Allaire (Ecole Polytechnique)
Gilles Francfort (Université Paris-Nord)
Marc Geers (TU Eindhoven)
Bernd Schmidt (Augsburg University)
Augusto Visintin (University of Trento)

**Local Organizers**
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Website: www.mathematik.uni-dortmund.de/lsi/Gamm-2017

**Conference Office**
Saskia Stockhaus
In front of room 1.001 in the Seminarraumgebäude 1 or office M 635 in the mathematics building

Phone: (+49 231) 755-3053

**Important information**
Start of the conference Friday, 9:00
End of the conference Saturday, 15:40
GAMM activity group meeting Friday, 17:30 - 18:30, in room 1.004 of the Seminarraumgebäude 1
Informal dinner Thursday, 19:30, at the restaurant Hövels Hausbrauerei, Hoher Wall 5-7, 44137 Dortmund
Conference dinner Friday, 19:00, at the restaurant View, Leonie-Reygers-Terrasse, 44137 Dortmund
Lunch (Friday) Canteen of the TU Dortmund, Campus Nord
Lunch (Saturday) kostBar, Emil-Figge-Straße 40

We recommend the TU Dortmund-App for your smart phone (iOS and Android 4.x) for navigation on the campus.
Directions

By public transport
From Dortmund main station:

1. Take the S-Bahn (train) S1 (usually from platform 7)
2. Exit the train at “Dortmund Universität” (3rd stop)
3. Follow the route on the map below

Tickets:
You need a ticket “Erwachsene Preisstufe A”

For your way back to the city center, you can take the train S1 to “Dortmund Hbf” (Dortmund main station).
Informal dinner

On Thursday evening, January 19, at 19:30 there is an informal meeting at the restaurant Hövels Hausbrauerei, Hoher Wall 5-7, 44137 Dortmund (see the map below).

Conference dinner

The conference dinner takes place at the restaurant View in Dortmund’s famous landmark “the U”, Leonie-Reygers-Terrasse, 44137 Dortmund. From Dortmund main station, it is a 5 minutes walk to the restaurant (see the map below).
Program

Friday, January 20, 2017, 9:00-18:30

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<td>A relaxation approach to the modeling of the stochastic behavior of elastic materials</td>
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<td>An evolution equation based approach to topology optimization</td>
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<td>Anja Schlömerkemper</td>
<td>About an evolutionary model for magnetoelasticity in Eulerian description</td>
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Participants

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<td>Dr. Takayuki Yamada</td>
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Abstracts

Keynote speakers

Optimization of dispersive coefficients in the homogenization of the wave equation in periodic structures

Grégoire Allaire
Ecole Polytechnique, Palaiseau, France

We study dispersive effects of wave propagation in periodic media, which can be modelled by adding a fourth-order term in the homogenized equation. The corresponding fourth-order dispersive tensor is called Burnett tensor and we numerically optimize its values in order to minimize or maximize dispersion. More precisely, we consider the case of a two-phase composite medium with an 8-fold symmetry assumption of the periodicity cell in two space dimensions. We obtain upper and lower bound for the dispersive properties, along with optimal microgeometries. This is a joint work with T. Yamada (Kyoto).

A criticism of finite elasto-plasticity

Gilles Francfort
Université Paris-Nord, Villetaneuse, France

In this talk, I will propose a slightly different approach to finite elasto-plasticity which may (or may not) fare better than the traditional $F = F_e F_p$ decomposition. I will then proceed to show that, in any case, all of this makes little sense in the absence of regularization (hardening, gradient plasticity), except possibly when restricting the slip directions (that is for crystal plasticity).
Upscaling microfluctuation fields towards emergent phenomena in the mechanics of microstructures

Marc Geers, Ashwin Sridhar, Bart Vossen, Varvara Kouznetsova, Olaf van der Sluis
Eindhoven University of Technology, Eindhoven, The Netherlands

The bottom-up design of advanced materials with unprecedented mechanical properties is a grand challenge, requiring reliable multiscale methods. The mechanical performance of engineering materials at the large scale often reveals particular emergent properties that originate from the complexity at the underlying fine scale level. Relating and predicting the macroscopic emergent behaviour is a non-trivial task, for which classical homogenization methods may fall short. This presentation addresses this multi-scale challenge, whereby the microfluctuation fields at the fine scale are key in understanding and solving the problem. Two problems are addressed to illustrate the problem and potential solutions.

The first problem concerns the multiscale modelling of acoustic metamaterials, which are tailored to attenuate sound wave propagation for certain frequencies. The unique features of these metamaterials originate from the complex interaction of transient phenomena at the microscopic and macroscopic scales, with local resonance occurring within (one of the) micro-constituents resulting in effective band gaps at the macro-scale. To capture the complex multi-scale interaction, an enriched transient computational homogenization scheme is required, which homogenizes the evolution in space and in time of materials with a non-steady state microstructure [1]. The conventional separation of scales hypothesis is relaxed, using an enriched description of the micro-macro kinematics that allows for large spatial microfluctuations, originating from local resonance phenomena. The proposed multi-scale solution method enables innovative designs of finite-size microstructures for locally resonant acoustic metamaterials. Material nonlinearities can be incorporated in a natural manner, which is a new development in the field of mechanical metamaterials. In the particular case of a metamaterial with linear elastic phases, the homogenization method allows to eliminate the fine scale, yielding an extended (micromorphic) continuum [2].

The second problem concerns the multi-scale behaviour of delaminating cohesive interfaces of polymer-metal laminates, as used for stretchable electronics applications. Fibrillation mechanisms at the fine scale give rise to pronounced discretized microfluctuations that have a major contribution to the macroscopic work-of-separation. Standard homogenization frameworks fall short [2] and it is shown that the role of the discrete fluctuating fields originating from the fine scale are again essential.

References

On a quantitative piecewise rigidity result and Griffith-Euler-Bernoulli functionals for thin brittle beams

Bernd Schmidt
University of Augsburg, Augsburg, Germany

We study a planar thin brittle beam subject to elastic deformations and cracks described in terms of a nonlinear Griffith energy functional acting on $SBV$ deformations of the beam. In particular we consider the case in which elastic bulk contributions due to finite bending of the beam are comparable to the surface energy which is necessary to completely break the beam into several large pieces. In the limit of vanishing aspect ratio we rigorously derive an effective Griffith-Euler-Bernoulli functional which acts on piecewise $W^{2,2}$ regular curves representing the midline of the beam. The elastic part of this functional is the classical Euler-Bernoulli functional for thin beams in the bending dominated regime in terms of the curve's curvature. In addition there also emerges a fracture term proportional to the number of discontinuities of the curve and its first derivative.

Analogical Models and Homogenization Disagree

Augusto Visintin
University of Trento, Trento, Italy

We shall show that the outcome of the theory of homogenization is at variance with a classical engineering technique for constructing rheological models of mechanical materials. **Analogical models.** The mechanical constitutive behavior of several materials is often represented via discrete models, that are built up by arranging few elements. These are representative of the basic behavior of the material, viz., elasticity, viscosity and plasticity in continuum mechanics. In the case of univariate models, one may also use the pictorial image of parallel and serial arrangements. Similar constructions are also used elsewhere, e.g., in electro-magnetism.

As apparently these models are not derived from any representation of the mesoscopic structure, here we address the following question:

> may these models be retrieved by homogenizing an underlying lower-scale structure?

We shall answer in the negative: the main reason stays in an (unjustified) mean-field hypothesis, that is at the basis of these constructions. **Homogenization via Scale Transformations.** An alternative approach is illustrated, that rests on the following steps:

(i) formulation of a (space-distributed) model for an inhomogeneous material, with constitutive data that depend periodically on $x/\eta$;

(ii) derivation of a two-scale model as $\eta \to 0$;

(iii) upscaling, i.e. derivation of a coarse-scale model by averaging the mesoscopic fields over a reference set $\mathcal{Y}$;

(iv) downscaling, i.e., retrieval of a two-scale solution from a coarse-scale solution.

This shows that the single-scale and two-scale models are equivalent. This entitles us to regard the coarse-scale model as the genuine homogenized model. **Variational Approach.** This program is here developed for an initial- and boundary-value problem for a class of elasto-visco-plastic materials, see [4].
This process is given a variational formulation, which allows one to restate the homogenization result in terms of De Giorgi's $\Gamma$-convergence.

This is just an example of a more general set-up: analogous results have been derived for scale transformation for elasto-visco-plastic material of Prandt-Reuss-type, for electro-magnetic materials, for phase-transitions, see references.

References


Contributed talks

Computational modeling of hierarchical composites

Swantje Bargmann$^1$, Edgar Husser$^2$, Celal Soyarslan$^1$

$^1$ Chair of Solid Mechanics, University of Wuppertal, Germany

$^2$ Institute of Continuum Mechanics and Material Mechanics, Hamburg University of Technology, Germany

The impact of small scale geometric confinement on deformation mechanisms is subject of intense research in materials sciences nowadays. Nanoporous metals have a micro-structure with an extremely high volume-specific surface content. Due to a very high local strength and a relatively regular interconnection of the nanoconstituents as well as a low mass density, nanoporous metals are very good candidates for strong and light-weight structural materials.

The gold-polymer composite considered in this contribution is hierarchical. To date, no established understanding of how to design a microstructured composite exists. As a consequence, there is a strong need for experimental as well as numerical studies at all scales.

References

nanoporous gold submitted


Analysis of the embedded cell method for the numerical homogenization of metal-ceramic composite materials

Wolf-Patrick Düll1, Bastian Hilder1, Guido Schneider1
1 Universität Stuttgart, Stuttgart, Germany

In this talk, we analyze the embedding cell method, an algorithm which has been developed for the numerical homogenization of metal-ceramic composite materials. We show the convergence of the iteration scheme of this algorithm and the coincidence of the material properties predicted by the limit with the effective material properties provided by the analytical homogenization theory in three situations, namely for a one dimensional linear elasticity model, a simple one dimensional plasticity model and a two dimensional model of linear hyperelastic isotropic materials with constant shear modulus and slightly varying first Lamé parameter.

References

An evolution equation based approach to topology optimization

Philipp Junker1, Dustin R. Jantos2, Klaus Hackl2
1 Continuum Mechanics, Ruhr-University Bochum, Germany
2 Mechanics of Materials, Ruhr-University Bochum, Germany

The objective of topology optimization is to find a mechanical structure with maximized stiffness by minimizing the work of external forces for given boundary conditions[1]. In contrast to the common approaches, we solve this problem by introducing a variational problem for the field of the mass density defined by an internal variable [2]. Employing Hamilton’s principle for dissipative processes, we are able to find an evolution equation for the density field in a comparable manner to, e.g., plasticity. To this end, we introduce a viscous-like dissipation potential for the internal variable and add a Lagrange multiplier to control the total mass within the model space. A regularization with a discontinuous Galerkin approach for the internal variable enables us to suppress the well-known checkerboarding phenomena while evaluating the evolution equation within each finite element separately [3]. Thus, the numerical solution can be provided in a single finite element environment – no additional optimization tool is necessary. Since the density field is no additional field unknown but a Gauss-point quantity, the calculation effort is strongly reduced. We end our presentation by presenting
solutions of optimized structures for different boundary problems to show the potential of our model.

References

A Phase-Field Approach to Micro-Magneto-Mechanics at Finite Deformations
M.-A. Keip, A. Sridhar, C. Miehe
Institute of Applied Mechanics (CE), Chair of Material Theory, University of Stuttgart, Pfaffenwaldring 7, 70569 Stuttgart, Germany, E-mail: marc-andre.keip@mechbau.uni-stuttgart.de.

The talk addresses a variational formulation for finite-deformation micro-magneto-mechanics [1]. In line with our previous work [2], we develop a variationally consistent phase-field approach to micro-magneto-elasticity that incorporates the magnetization as order parameter. Key motivation is the generalization of micro-magnetic phase-field models to the general case of finite deformations. This will allow to solve micro-magnetically informed large-strain problems, which are found in a number of technological applications. A central example is given by magnetorheological elastomers (MRE), which are characterized by a microstructure composed of a soft elastomeric matrix and quasi-rigid, micron-sized ferromagnetic inclusions [3,4]. The effective behavior of MREs is driven by complex phenomena that span over multiple length scales: At micro-scale we observe magnetic domain wall motion within the embedded ferromagnetic particles. Particle-to-particle interactions as well as particle rotations then give rise to deformations within the soft matrix [5]. At macro-scale we eventually observe effective magneto-mechanical deformations that result from both micro- and macro-structural interactions [6,7].

In a series of numerical examples we will demonstrate the versatility of the model formulation. A particular focus will be on the modeling of ferromagnetic microstructure evolution in magnetorheological elastomers. Here we investigate the macroscopically driven micro-magnetic domain-wall motion as well as the resulting interactions between individual spherical and ellipsoidal particles which lead to an overall compression or expansion of the MRE.

References
Magnetic Domains in Thin Ferromagnetic Films with Strong Perpendicular Anisotropy

H. Knüpfer
Institute of Applied Mathematics, Heidelberg, Germany

We consider a regime of large and ultra-thin ferromagnetic films with strong anisotropy and easy axis pointing out of the film plane. Starting from the full three-dimensional micromagnetic model, we identify (to leading order) the critical scaling where the phase transition from single domain states to multi-domain states such as bubble or maze patterns occurs. Furthermore, we derive the asymptotic behavior of the energy in the single domain regime within the framework of Gamma-convergence. This is joint work with C. Muratov und F. Nolte.

Asymptotic rigidity and homogenization of layered materials with stiff components

Fabian Christowiak¹, Carolin Kreisbeck²
¹ Fakultät für Mathematik, Universität Regensburg, Regensburg, Germany
² Mathematisch Instituut, Universiteit Utrecht, Utrecht, The Netherlands

In the context of finite-strain elastoplasticity, we investigate the effective behavior of variational models for bilayered composite materials featuring large elastic constants in one component. Our particular interest lies in understanding whether the presence of the stiff layers forces a rigid macroscopic material response. The answer to this question is expected to depend on the scaling relation between stiffness and layer thickness. In this talk, we characterize the limit deformations of sequences of uniformly bounded energy as the thickness of the layers tends to zero, and identify two different scaling regimes. If the elastic constants diverge sufficiently fast, the observed macroscopic deformations coincide with those in the special case of completely rigid layers. As it is shown in [1], the latter correspond exactly to global rotations of shear deformations in layer direction, provided they are locally volume preserving. One major step in the proof is to quantify the layers’ stiffness with the help of the geometric rigidity estimate established in [2]. To show optimality of this regime, an explicit construction of a family of deformations based on bending of the individual stiffer layers is given, which yields macroscopically softer material behavior. These findings serve as a basis for proving a rigorous homogenization result via $\Gamma$-convergence.

References
Application of a magnetostrictive 3D Preisach model for the simulation of magneto-electric composites

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In this contribution we focus on the magneto-electric (ME) coupling of multiferroic materials. They combine two ferroic characteristics and exhibit an interaction between magnetic and electric fields. This magneto-electric coupling can find applications in sensor technology or in electric field-controlled magnetic data storage devices [1]. Since most ME single-phase materials show an interaction between electric polarization and magnetization far below room temperature and therefore outside of a technical relevant temperature range, the manufacturing of two-phase composites, consisting of a ferroelectric matrix with magnetostrictive inclusions, becomes important. They generate the ME coupling at room temperature as a result of the interaction of their constituents. Hence, the ME coupling of composite materials significantly depends on the material behavior of both phases. In order to determine the effective properties with respect to both aspects, a multiscale finite element (FE2) homogenization approach is performed, which combines via a scale bridging the macro- and microscopic level. Therefore, the microscopic morphology is considered by using a representative volume element [2,3]. Furthermore, the nonlinear properties of both phases are approximated with appropriate material models. On the one hand, the switching behavior of spontaneous polarizations of barium titanate unit cells are taken into account [4], which reproduces after a homogenization step the typical dielectric and butterfly hysteresis loops of the ferroelectric matrix material. On the other hand, the nonlinear remanent magnetizations of the magnetostrictive inclusions are described with a Preisach operator [5]. Both models affect the strain-induced coupling, such that we obtain a nonlinear behavior of the ME coefficient.

References
Formulas for relaxed disarrangement densities

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Structured deformations provide a multiscale geometry that captures the contributions at the macrolevel of both smooth geometrical changes and non-smooth geometrical changes (disarrangements) at submacroscopic levels. Recently, Owen and Paroni evaluated explicitly some relaxed energy densities arising in Choksi and Fonseca’s energetics of structured deformations. In this talk we will show that a different approach to the energetics of structured deformations, that due to Baía, Matias, and Santos, confirms the roles of the relaxed densities established by Owen and Paroni. In doing so, we give an alternative, shorter proof of Owen and Paroni’s results, and we establish additional explicit formulas for other measures of disarrangements. This is joint work with A. C. Barroso, J. Matias, and D. R. Owen.

A relaxation approach to the modeling of the stochastic behavior of elastic materials

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The material behavior even in the simple linear elastic range is not deterministic but rather fluctuates randomly around some expectation values. The knowledge about this characteristic is obviously trivial from an experimentalist’s point of view. However, it is not considered in the vast majority of material models in which “only” the deterministic behavior is taken into account.

One very promising approach to the inclusion of stochastic effects to the modeling of materials is provided by the so-called Chaos Polynomial Expansion. It had been used, e.g., for the derivation of the so-called stochastic finite element method. This method yields results which are exactly of the desired kind but unfortunately at drastically increased numerical costs.

This contribution aims at proposing a new ansatz which is also based on a stochastic series expansion at the Gauss point level. Appropriate energy relaxation allows for a synthesized (deterministic) stress measure while simultaneously offering stochastic properties as e.g. the standard derivation. The total procedure only needs negligibly more computation effort as compared to a simple elastic calculation.

Stochastic homogenization of rate-dependent systems of monotone type

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In this work we are concerned with the stochastic homogenization of the initial boundary value problems of monotone type. The models of monotone type under consideration describe the deformation behaviour of inelastic materials with a microstructure which can be characterised
by random measures. Based on the Fitzpatrick function concept we reduce the study of the asymptotic behaviour of monotone operators associated with our models to the problem of the stochastic homogenization of convex functionals within an ergodic and stationary setting. The concept of Fitzpatrick’s function helps us to introduce and show the existence of the weak solutions for rate-dependent systems. The derivations of the homogenization results presented in this work are based on the stochastic two-scale convergence in Sobolev spaces. We present also the convergence results, which are related to the classical $\Gamma$-convergence results.

A Large-Scale Regularity Theory for Random Elliptic Operators on the Half-Space

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We are interested in large-scale regularity properties of linear elliptic operators with random coefficients. It is well-known that for general heterogeneous coefficient fields $a$, functions that are $a$-harmonic in the whole-space may fail to be Lipschitz or (in the systems case) even locally bounded. As was already noted in the 80’s by Avellaneda and Lin in the context of periodic coefficients, the situation improves when homogenization occurs. The philosophy here is to transfer the regularity properties of the constant coefficient homogenized operator to the heterogeneous coefficient operator. More recently, a large body of literature devoted to extending the strategy of Avellaneda and Lin to the setting of random coefficient fields has developed.

The main emphasis of this talk will be on boundary regularity: We consider operators on the half-space with coefficients that are the restriction of a random coefficient field on the whole-space. In the case of homogeneous Dirichlet (or Neumann) boundary data, under certain assumptions on the random coefficient field $a$, we obtain a first-order Liouville principle, which states that the space of subquadratic $a$-harmonic functions vanishing on the boundary has the same dimension as in the Euclidean case. This Liouville principle arises as a corollary of a large-scale $C^{1,\alpha}$-decay of an appropriate version of tilt-excess, the proof of which relies critically on the availability of a sublinear half-space-adapted homogenization corrector. The main contribution of our work is the construction of such a sublinear half-space-adapted corrector given a whole-space corrector satisfying a slightly quantified sublinearity condition (the existence of which may be ensured with a quantitative ergodicity assumption). To the best of our knowledge, while within the context of periodic homogenization boundary correctors have already been introduced by Avellaneda and Lin in the Dirichlet case and Kenig, Lin, and Shen in the Neumann case, this is the first use of boundary correctors in the stochastic setting.

References

Quantitative homogenization in nonlinear elasticity

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We consider a nonlinear elastic composite with a periodic micro-structure described by the non-convex integral functional

$$I_\varepsilon(u) = \int_\Omega W\left(\frac{x}{\varepsilon}, \nabla u(x)\right) - f(x) \cdot u(x) \, dx,$$

where $u : \Omega \to \mathbb{R}^d$ is the deformation, $f : \Omega \to \mathbb{R}^d$ is an external force, $\varepsilon > 0$ denotes the size of the micro-structure, and $W(y,F)$ is a stored energy function which is periodic in $y$. As it is well-known, under suitable growth conditions, $I_\varepsilon$ $\Gamma$-converges to a functional with a homogenized energy density $W_{\text{hom}}(F)$, which is given by an infinite-cell formula. Under appropriate assumptions on $W$ (namely, $p \geq d$-growth from below, frame indifference, minimality at identity, non-degeneracy and smoothness in a neighborhood close to the set of rotations) and on the microstructure, we show that in a neighborhood of rotations the homogenized stored energy function $W_{\text{hom}}$ is of class $C^2$ and characterized by a single-cell homogenization formula. Moreover, for small data, we establish an estimate on the homogenization error, which measures the distance between (almost) minimizers $u_\varepsilon$ of $I_\varepsilon$ and the minimizer of the homogenized problem. More precisely, we prove that the $L^2$-error as well as the $H^1$-error of the associated two-scale expansion decays with the rate $\sqrt{\varepsilon}$.

About an evolutionary model for magnetoelasticity in Eulerian description

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A fundamental issue in the modeling of magnetoelastic materials is that elasticity is phrased in Lagrangian coordinates whereas magnetism is phrased in Eulerian coordinates. We discuss an evolutionary model that is completely phrased in Eulerian coordinates and takes microstructures of the magnetization into account. The model presented is a system of partial differential equations that contains (1) a Navier-Stokes equation with magnetic and elastic terms in the stress tensor obtained by a variational approach, (2) a regularized transport equation for the deformation gradient and (3) the Landau-Lifshitz-Gilbert equation for the dynamics of the magnetization. We prove existence of weak solutions in two dimensions and study their uniqueness.
An FFT-based fast gradient method for elastic and inelastic unit cell homogenization problems

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Building upon the previously established [1] equivalence of the basic scheme of Moulinec-Suquet’s FFT-based computational homogenization technique [2] with a gradient descent method, this work concerns the impact of the fast gradient method of Nesterov [3] in the context of computational homogenization. Nesterov’s method with speed restarting [4] leads to a significant speed up compared to the basic scheme for linear problems with moderate contrast, and compares favorably to the (Newton-)conjugate gradient (CG) method for problems in digital rock physics and (small strain) elastoplasticity. We present an efficient implementation requiring twice the storage of the basic scheme, but only half of the storage of the CG method.

References

Fully periodic RVEs of matrix inclusion composites in practical multiscaling: not worth the effort

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The identification of effective material responses or macroscopic constitutive laws is on of the major goals in computational micromechanics. State of the art are FE2-methods or homogenization approaches that utilize representative volume elements (RVEs) which incorporate the microstructural information. In this regard, periodic model setups featuring a fully periodic RVE topology and discretization are known to perform the best. However, generating such RVEs and setting up a proper model might become a cumbersome task. Therefore, it is highly questionable if the tremendous effort of generating such complex RVEs is justified over other simplification current in an engineering analysis. In the present work we investigate the effect of utilizing simpler RVE model setups to determine effective material parameters and responses. Especially the influences of different RVE topologies, discretizations...
and boundary conditions are studied. The case of a fully periodic RVE with exact periodic boundary conditions will act as a reference solution for the evaluations. Special emphasis is paid to so called matrix-inclusion composites, widely found in industrial applications. General trends for linear and non-linear material behavior will be presented.

References

Rigidity of Shape Memory Alloys undergoing cubic-to-tetragonal transformations
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Microstructures in Shape Memory Alloys (SMAs) undergoing cubic-to-tetragonal transformations typically exhibit phase transitions parallel to six different hyperplanes relative to the crystal orientation. The simplest case is laminar structures involving two martensite variants called twins. The same phenomenon also occurs, however, when transitioning between a twinned structure and austenite (habit planes). The orientations of these hyperplanes can phenomenologically be explained by requiring the involved strains to be rank-one connected.

Building on work by Dolzmann and Müller [2], a more quantitative analysis of twins in the framework of linearized elasticity has been given by Capella and Otto [1]. Starting from a variational model including a term penalizing surface area between phases, they identified the energy regime where, asymptotically, only twins can occur. Their result is sharp in the sense that the regime reaches up to the energy of the well-known Kohn-Müller branching construction [3] of habit planes.

Starting from the result of Capella and Otto, we present our findings on the rigidity of SMAs in the regime where branching may occur. While planes may cross, we do recover the predicted orientations if we additionally assume that the strain either lies in $BV$ or is independent of some arbitrary direction. We also give some insight into the regularity of possible microstructures by using tools from microlocal analysis to track the “internal” orientation of the twins. Roughly speaking, the results suggest that in three space dimensions habit planes may cluster on a set of Haussdorff dimension $3 - \frac{2}{3}$.

References
Variational framework of phase field modeling of ductile fracture in isotropic and anisotropic porous solids at finite strains

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This work outlines a rigorous variational-based framework for the phase field modeling of fracture in isotropic and anisotropic porous solids undergoing large elastic-plastic strains. It extends the recent work [1] to a formulation of porous plasticity with particulate microstructures characterized by spherical pores or by ellipsoidal voids which beside changing their size are also able to change their shape and orientation due to a macroscopic deformation [2]. The phase field approach regularizes sharp crack surfaces within a pure continuum setting by a specific gradient damage modeling with geometric features rooted in fracture mechanics. A gradient plasticity model for isotropic and anisotropic porous plasticity with a simple growth law for the evolution of the void fraction is developed, and linked to a failure criterion in terms of the local elastic-plastic work density that drives the fracture phase field [3]. It is shown that this approach is able to model basic phenomena of ductile failure such as cup-cone failure surfaces in terms of only two material parameters on the side of damage mechanics: a critical work density that triggers the onset of damage and a shape parameter that governs the postcritical damage up to fracture.

The coupling of gradient plasticity to gradient damage is realized by a constitutive work density function that includes the stored elastic energy and the dissipated work due to plasticity and fracture. With this viewpoint at hand, the thermodynamic formulation is outlined for gradient-extended dissipative solids with generalized internal variables. The proposed canonical theory is shown to be governed by a rate-type minimization principle, which describes the coupled multifield evolution problem of plasticity-damage, where on the side of plasticity an additive split of the total Hencky strain is used. The variational nature of the formulated problem is exploited on the numerical side by inherent symmetry properties. Another aspect of this work is the regularization towards a micromorphic gradient plasticity-damage setting [4] by taking into account additional internal variable fields linked to the original ones by penalty terms. This enhances the robustness and facilitates the finite element implementation, in particular on the side of gradient plasticity.

References

Stochastic homogenization of a maximal monotone relation

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We study the stochastic homogenization of a quasilinear model of Ohmic electric conduction with Hall effect. We prove that, as the homogenization parameter vanishes, the solution of the random system converges to the solution of a deterministic system having the same structure. The proof relies on Birkhoff’s ergodic theorem, on the maximal monotonicity of the nonlinear operator, and on the scale integration method introduced by A. Visintin. (Joint work with L. Lussardi and S. Marini)

Conditions for phenomenological anisotropic damage models derived from micromechanics

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The talk investigates the modeling of irreversible anisotropic brittle and ductile damage based on second and fourth order damage tensors and its connection to micromechanical material models. In particular, the conditions for strictly increasing damage are discussed. Restrictions on the form of the free energy function and the damage evolution law are proposed. These are motivated by energetical, scale-bridging considerations for growing cracks and pores. Moreover, conclusions are drawn related to the damage-induced evolution of the yield surface in plasticity.

High Order Homogenization for Elastic Wave Problems Using the Asymptotic Expansion Method

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Dispersion is a phenomenon appearing in heterogeneous media, by which waves of different wavelengths propagate at different velocities [1]. In the scalar wave equation for periodic media, the dispersive effect is modeled as an additional forth-order term in the homogenized wave equation [2,3,4]. This effect must be taken into account in industrial applications because the size of periodic microstructures is finite in manufactured products. In this study, the dispersive effect is formulated in the elastic wave problem for periodic media by using the asymptotic expansion method, in the same manner as for scalar wave propagation problems.

References
Optimal design of fractured media with prescribed macroscopic strain

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An optimal design problem for two-component fractured media, for which a macroscopic strain is prescribed, is studied. Within the framework of structured deformations, we derive an integral representation for the relaxed energy functional. We start from an energy functional accounting for bulk and surface contributions coming from both constituents of the material; the relaxed energy densities, obtained via a blow-up method, are determined by a delicate interplay between the optimization of sharp interfaces and the diffusion of microcracks. This model has the far-reaching perspective to incorporate elements of plasticity in optimal design of composite media.