The Number Pyramid with a Japanese Twist
‘Incompleteness’ as a Trigger to Wake Children’s Sense of Mathematical Beauty

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1. Introduction
(By Shinya) 8 min.

(1) Greetings


Unsere Forschungsgruppe beschäftigt sich zur Zeit mit dem Projekt, Lektionen für den Mathematikunterricht zu entwerfen und im Unterricht einzuführen, die den Schülern Spaß machen und gleichzeitig nützliche Fähigkeiten wie Berechnung, räumliches Bewusstsein, mathematisches Denken etc., weiter zu entwickeln. Wir sind an dem innovativen Mathematiklehrbuch „Das Zahlenbuch“ sehr interessiert, und wir benutzen es, um daraus die mathematischen Aufgaben für die Kinder zu entwickeln.

Es ist nicht einfach und es dauert wirklich eine lange Zeit für uns, die deutschen Texte im Lehrbuch zu lesen und zu verstehen. Aber wir bedienen uns eines sehr vertrauten Freunds: Ein Deutsch/Japanisches Wörterbuch!

(Our research group is currently undertaking a research project in which we try to design and implement lessons in which children would enjoy mathematics, and at the same time they would develop useful skills such as calculation, spatial awareness, mathematical thinking, etc.

We are very interested in the innovative German mathematics textbook, Das Zahlenbuch, and we are developing mathematical tasks for children and lessons by studying this textbook.

It actually takes a long time for us to read and understand the German texts in the textbook. But, we manage to read them with a German to Japanese dictionary. The dictionary is one of the most familiar friends for us.)

By the way, (My German is under construction now, So I will speak English from now)

,of those tasks in the textbooks, we feel, one of the most attractive learning environments for Japanese children is die Zahlenmauern, which we call the ‘number pyramid’. We particularly designed a lesson with number pyramids for Japanese primary school children and put the plan into practice to Year 4 children (aged from 9 to 10). The main problem of the lesson designed for Japanese children is ‘How many different combinations of the numbers are there which would make the top number 20?’
In this problem, there are some beautiful mathematical patterns, and we really hope that children would pick up on this beauty and appreciate this. To design this lesson, we considered how we could introduce this problem to children so that children would pay attention to the mathematical beauty in this problem. We are inspired by the words of a Japanese tea master which we will mention later.

(2) Aims of this workshop
In this workshop we would like to share our experiences with the number pyramids which we particularly designed for Japanese primary school children, and we would like to discuss how we develop a good lesson plan from the perspective of ‘Mathematics Education as a design science’ with you.

(3) Plan of this workshop
Our plan of this morning/afternoon consists of the following 4 parts. First, we would like to talk about Japanese primary schools and children briefly. And then, before we talk about our children’s learning activities, we would like YOU to experience the main problem which we gave our children. We hope you would experience and discover some mathematical patterns in this problem. My students will help you, so please do not hesitate to ask any questions. We will then introduce how our children undertook this task, and how they started paying attention to the mathematical patterns in this problem.

Finally, we will summarize this session and discuss how we develop a good lesson plan from the perspective of ‘mathematics education as a design science’. We are very much interested in your opinions about our lesson, so please do not hesitate to let us know your thoughts!

From now on, my colleague Shin will take over the workshop to talk about Japanese primary schools and children.

2. Overview of Japanese primary schools and children (By Shin, 8 min)
Good morning/afternoon. I am a teacher in Kumamoto Fuzoku primary school. I am very happy to report our experiences to you, our German colleagues. In Kumamoto, Japan, we have a research seminar every week, and we review and study Das Zahlenbuch. We are very enjoying reading this textbook and would like to know more about German mathematics education to improve mathematics teaching in Japan.
Now, what I would like to report to you today is that our teaching experiences with the number pyramids in my school, and I will be very happy if we can exchange productive opinions about our experiences.

First, I would like to make a brief introduction about my schools and children. This is my school, Kumamoto Fuzoku primary school. This school is 130 years old. ‘Fuzoku’ means ‘attached’, and our school is actually attached to Kumamoto University. Our school is basically no different from normal primary schools, but we have additional aims such as training student teachers from Kumamoto University, and undertaking classroom research. In fact I am undertaking such research with Shinya and his students.

At the moment, there are 23 teachers and 720 children from Year 1 to 6, aged from 6 to 12. Each class has 40 children. Basically teachers teach all subjects as normal primary school teachers do, but my particular interest is mathematics education.

Now, I am teaching Year 5 children, aged 11… these my children. We have 4 mathematics lessons (45 min.) in a week. I will show our mathematics lessons …

They all really like mathematics indeed. I have designed and implemented a lesson by using the number pyramids. They are not included in normal Japanese textbooks but, as you know, we consider they are very worth introducing to Japanese classrooms.

I posed the main problem mentioned above ‘How many different combinations of the numbers are there which would make the top number 20? This is a really challenging task for my children, but I wanted them to discover mathematical patterns and beauty through investigating this problem.

Now, I would like to explain more about our problem, but I don’t think that a good teacher teaches the secret of mathematical patterns at the beginning of a lesson. And in fact, a good way to learn mathematics is, I believe, actively playing with mathematics. So I would like YOU to become my students, and experience with us the lesson which we designed for my children in Japan.

After this, we will explain some pedagogical background of the lesson, and we will be happy to exchange our opinions. From now on, my colleague Taro will take over the workshop next. You will experience the main problem with us and discover some mathematical patterns here.

3. Experience of the main problem with us (By Taro, 30min)

Hello, my name is Taro. I am a Japanese, but at the moment I am working for Plymouth University in the UK. I was also Shinya’s student. I am also interested in the learning environments in Das Zahlenbuch.

First I would like YOU to consider the following introductory problems.
Could you complete the following 5 pyramids?

So, could you complete those pyramids? What have you noticed from the five pyramids?

Which of these pyramids stands out, as if it disturbs the consistent pattern between the pyramids?

(Focusing on the top 20 pyramid...)

If we change the numbers in this pyramid ... do you think now the pattern is more consistent??

Now, we have two different pyramids whose top number is 20.... It seems there are other combinations of the numbers which would make the top number 20.... So, shall we investigate this combination?

You can use only whole numbers, i.e. 1, 2, 3, ... and you cannot use 0 today. Also, we consider the symmetrical ones are still different combinations.

Symmetrical pyramids

So, let’s start! When you will have completed the task, please ask Shin, so he will record your answers on the blackboard (or whiteboard)

(Activities of Participants for about 10 min.)

It seems there are so many different combinations.... but could we find a pattern here?
As you hopefully have noticed, you can organize the combinations if you pay attention to the middle number of the bottom. The number of combinations increase by 1, 3, 5, 7, …, and there are 81 different combinations in total. This is a square number! Actually the sum of the consecutive odd numbers from 1 is a square number indeed. I will show you this in informal proof. We wanted our children to discover this pattern through undertaking the main problem. We think this pattern is quite beautiful,

Shin will report how our children undertook this problem next.

4. Children’s activities with the pyramids (By Shin, 8 min)

Now, we hope that we all had some ideas and could recognise the pattern of the problem, and now, we would like to report how our children undertook this problem, and developed their mathematical thinking during the lesson.

First, let us see what our children commented on the introductory problems.

This child is saying that these 5 pyramids look like a sandwich. she means the middle one is a piece of ham, and the other ones are pieces of bread

This child is saying that if the numbers were 12 and 8, the pattern would be completed.

This child is suggesting that the pattern be completed if he can put ‘9’ on the bottom row
Children started investigating the main problem individually, and then in small groups.

We shared ideas and answers by focusing on the middle numbers in the bottom row.

This child noticed that the number of combinations is increased by 2, when the middle number of the bottom row is decreased by 1. It was useful to ‘see’ the pattern when we organized the pyramids like this. Some children could find all combinations, 81 different pyramids.

Of course, not all children could reach the answer, but they really worked hard and collaboratively.

In summary, the introductory problems did work to make children pay attention to the number of different combinations of the top 20 pyramids.

5. Summary (By Shinya, 8 Min)

I hope you have enjoyed our problem, and find our teaching practice interesting. We know, there are beautiful patterns in mathematics, but children would not always notice the patterns, and in fact mathematics lessons, as you know, does not always follow a path which a teacher wants children to follow.
Thus an important task for us, designers of mathematics lessons, was how we could organise a lesson plan so that children would spontaneously start paying attention to the mathematical patterns in a problem.

After many discussions, we finally found an idea in philosophy of Japanese traditional art, *Tea*, in particular the following words by a Japanese tea master, Okakura.

*True beauty could be discovered only by one who mentally completed the incomplete*  
(K. Okakura, 1906, p. 40)

He means here, that a tea master intentionally organizes his tea room as somewhat ‘incomplete’ so that his gusts would complete this incompleteness in their mind and complete the beauty of the tea room and spirit of tea by themselves.

Let me explain how we understand this idea in the context of the mathematics lesson. Incompleteness here is the inconsistent pattern in the introductory problems which we have experienced. Please remember that only the top 20 pyramid disturbs the pattern in the five pyramids, and we emotionally wanted to ‘complete’ this ‘incompleteness’, which led us to pay attention to the ‘top 20 pyramids’, i.e. we intentionally proposed an incomplete situation.

And we expected our children to pay attention to the patterns of the top 20 pyramids by completing the incompleteness of the introductory problems.

As we have seen, our children did notice this incompleteness, and through actively completing this incompleteness, they started to pay attention to the pattern in the top 20 pyramids. We therefore believe the tea master’s philosophy did work in our teaching practice.

Wir sind besonders daran interessiert, wie wir das mathematische Denken der Kinder durch die täglichen Mathematiklektionen anregen können, und wir versuchen permanent, unseren Mathematikunterricht zu verbessern.

Wir würden uns sehr glücklich schätzen, wenn Sie unsere Unvollständigkeit mit Ihren Erfahrungen, Ihrem Fachwissen und Ihren Meinungen vervollständigen könnten!

Vielen Dank für Ihre aktive Mitarbeit!

(We are particularly interested in how we could provoke children’s mathematical thinking through everyday mathematics lessons, and we are trying to improve our mathematics teaching, since we are still ‘incomplete’ in mathematics teaching! So we are very happy if you notice our incompleteness and ‘complete’ us from your opinions! Thank you very much for your active cooperation.)
Discussion (By Taro)