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Understanding the development of the proving process within a dynamic geometry environment

Abstract. In this paper we investigated the development of the proving process within a dynamic geometry environment in order to provide tertiary students with a strategy for proving. As a result, we classified different levels of proving and designed an interactive help system corresponding with these levels. This help system makes a contribution to bridge the cognitive and structural gaps between conjecture and proof. We also propose three basic conditions for understanding the development of the proving process.

Keywords. Proving process, level of proving, help system, abduction.

1. Introduction

In this research we consider proof as the final product of the proving process. Therefore, understanding the development of the proving process contributes in gaining insight into the invention of mathematical ideas and the difficulties in constructing proofs. That is also the reason why tertiary students should learn how to write, read, understand, and construct proofs. To support students in learning proofs, we provided them a methodological model with seven levels of proving and built the interactive help system based on this model (see Fig. 1). This help system contains open-ended questions and explorative tasks with two functions: to *direct thought* and to *convey information*. An open-ended question was used to help students look for geometric invariants and combine valid arguments into a formal proof. An explorative task was used to help students explore the problem on their own. During students' proving process, by answering open-ended questions as well as tackling explorative tasks, the idea of proofs may emerge gradually and arguments are produced as well.

2. Methodology

The proving process is a sequence of mental and physical actions, such as writing or thinking a line of a proof, drawing or visualizing a diagram, producing arguments, etc. Therefore, we classified seven levels of proving that represent the developmental phases in the proving process. These levels are described as follows: *level 0 (information)* provides students with clear information aimed at pointing out the principal parts of the problem, the unknown, the data, and the conclusion; *level 1 (construction)* guides students to model and construct the figures in a dynamic geometry environment; *level 2 (invariance)* guides students to search for geometric invariants that support in generating the ideas for proofs; *level 3 (conjecture)* supports students in formulating conjectures that often originate from experimental activities; *level 4 (argumentation)* guides students to produce arguments by explaining 'observed facts' and validating formulated conjectures; *level 5 (proof)* guides students to write proofs based on produced arguments; *level 6 (delving)* suggests students to delve into the problem such as generalization, specialization, analogy, etc.

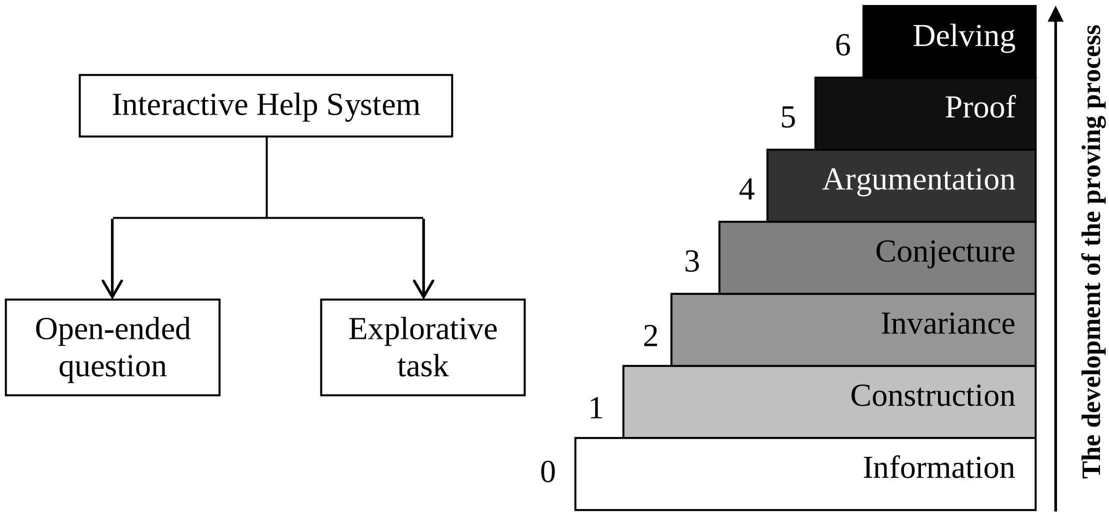

Fig. 1 A methodological model for understanding the proving process

The data of the empirical research was collected during the summer semester 2010/2011. The students were enrolled in a required elementary geometry classes for a teacher training course and divided into groups of three who sat together at one computer. We also installed Wink® software on each computer in order to capture and audio-record of all the working worksheets and group discussions. In this research, we also used Toulmin model in order to analyze the structure of argumentation during students' discussion in their group. According to this model, in any argumentation the first step is expressed by a *claim* (**C**) such as an assertion, an opinion or a conjecture. The second step consists of the production of *data* (**D**) supporting the claim. The *warrant* (**W**) can be expressed as a principle, a rule or a theorem for supporting for the data-claim relationships (see Toulmin, 1958). This model is not only useful to represent a deductive step but also a powerful tool to represent an abductive structure, which can be used to explicate the role of abduction in transition from conjecturing to proving modality (see e.g. Pedemonte & Reid, 2011). The following model describes the way students finding data (?) for validating their claim when they know one rule for supporting the claim:

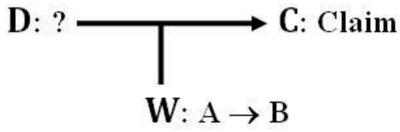

Fig. 2 Abduction in Toulmin model of argumentation

3. Understanding the development of the proving process

In this paper we provide three basic conditions for understanding the development of the proving process: (1) *Realizing geometric invariants for generating ideas for proofs*. This phase supports students in getting more data for proving and searching geometric invariants for generating the ideas of proofs. (2) *Constructing a cognitive unity in the transition from conjecture to proof*. This process produces arguments for validating conjectures and writing proofs. In other words, cognitive unity is a phenomenon where some arguments, which are produced for the plausibility of the conjecture and become ingredients for the construction of a proof (Boero et al., 1996). (3) *Organizing arguments in order to write a formal proof*. This is one of the most difficult phases in the proving process because students need to organize (select and combine) produced arguments as a chain of logical valid arguments for writing proofs.

We chose the discussion of one typical group, which was audio-recorded by using Wink® software, to analyze students' arguments during proving process through the following problem: *A river has straight parallel sides and cities A and B lie on opposite sides of the river. Where should we build a bridge in order to minimize the traveling distance between A and B (a bridge, of course, must be perpendicular to the sides of the river)?* The interactive help system provided students with some open-ended questions and explorative tasks like "What is relationship between two lines *AD* and *EB* when the length of the broken line *ADEB* is minimal?", "Compare the length of the broken line *ADEB* and the length of the broken line *AGHB*", and so on. Firstly students realized a key geometric invariant by using GeoGebra software: "the line *AD* is an image of the line *EB* under a translation in the vector $\overrightarrow{ED}$ direction when the length of the broken line *ADEB* is minimal". Then they determined two points *G* and *H* which are best places for building the bridge and some spontaneous arguments were also produced for validating this conjecture. The following dialogue was extracted from three-student group's discussion and Toulmin model was used to represent the structure of argumentation:

♣ Student 1: It is obvious that the length of the broken line *AGHB* is smaller the length of the broken line *ADEB*. How can we prove this inequality when we have the following data $ED = HG = BB'$, $HB = GB'$, $EB = DB'$?

$\mathbf{D_1}$ **= ?** ⟶ $\mathbf{C_1}$: $AG + GH + HB \leq AD + DE + EB$ (1)

$\mathbf{W_1}$: $ED = HG = BB'$, $HB = GB'$, $EB = DB'$

♣ Student 2: We may consider the inequality $AG + GB' + B'B \leq AD + DB' + B'B$ (2)

$\mathbf{D_2}$ **= ?** ⟶ $\mathbf{C_2}$: $AG + GB' + B'B \leq AD + DB' + B'B$

$\mathbf{W_2}$: BB' is common summand

♣ Student 3: Look at the inequality! We have BB' as a common summand and three points *A*, *G*, *B'* are collinear. Therefore, we need to prove that $AG + GB' = AB' \leq AD + DB'$.

$\mathbf{D_3}$ **= ?** ⟶ $\mathbf{C_3}$: $AB' \leq AD + DB'$ (3)

$\mathbf{W_3}$: Triangle inequality ($\Delta ADB'$)

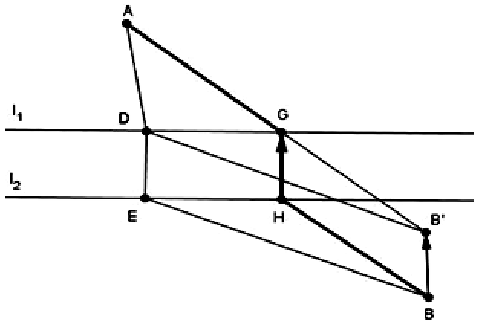

Fig. 3 One-Bridge problem

In order to write a formal proof, students followed a sequence of the inequalities (3) → (2) → (1). Therefore, by using Toulmin model, we interpret that students always reverse 'abductive structure' so that they can find the data for validating the claims, produce arguments, and write a formal proof.

4. Conclusions

This paper proposes a methodological model and three basic conditions for understanding the development of the proving process within a dynamic geometry environment. This model also provides tertiary students with appropriate strategies and tools as a means of exploration, discovery, and invention. The interactive help system can support students in realizing geometric invariants, producing arguments, and writing a formal proof. The findings of this research also provide mathematics teachers with a strategy for teaching proof and the proving process at the tertiary level.

References

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