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„Take Me to the Mathematical Circle! “

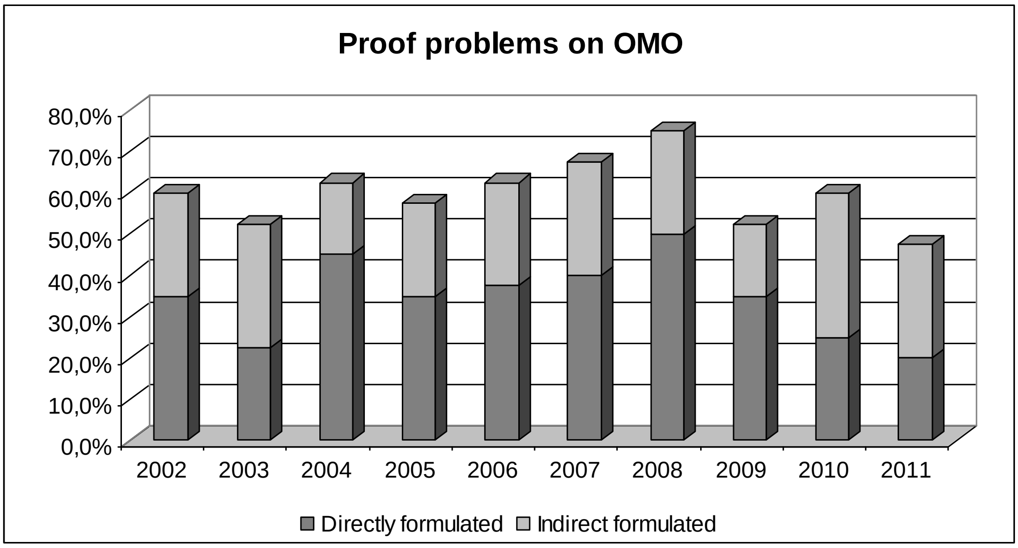
Introduction

Investigating challenging problems and preparing the students for various mathematical contests are the key goals of mathematical circles in Latvian schools. One of the reasons why mathematical circles for students of primary schools are organized rather seldom is that problem sets of Mathematics Olympiads are mainly created for students of 5^{th} to 12^{th} grades. Younger-grade students can participate in the Open Mathematical Olympiad. This opportunity points to the need of introducing some basic problem-solving principles to the children.

The activities of *Mathematical Circles* (**MC**) in Soviet times were the way of independent thinking. The problem sets of *Mathematics Olympiads* (**MO**) were widely used and researched to raise the mathematical competences of MC participants. A variation of this approach is extant in contemporary post-Soviet states. New, colorful ideas inspired by the significant work of Western education science and didactic scientists flow into the content of MCs for younger grade students. On the other hand, the phenomenon of MCs in Russia and post-Soviet republics is an established fact worldwide. We can now see how the MCs activities based on the aforementioned experience progress in the USA (web site of AMC).

Mathematical circles in Latvia

The MOs in Latvia have been taking place for more than 60 years. The content of problem sets has changed - the share of continuous mathematics has diminished, while the share of discrete mathematics has increased. This tendency is especially conspicuous in problems sets of the Open Mathematical Olympiad that is very popular in Latvia. About 40% of problems created for the young students do not need any special mathematical knowledge. This does not mean that there is no need for comprehension of some problem solutions strategies, reasoning and proofs. Looking at the past 10 years we can see that on average the problems of proof account for more than half of all problems for all grades. Proof problems are usually formulated directly. The indirectly formulated problems are existence or estimations problems (see graph 1). The problem set for the 5^{th} grade includes one or two such problems. When evaluating Olympiad works of younger-grade students, the most common mistakes in solutions of proof problems are identified. These are: misunderstanding the problem, guessing the answer, finding a particular example not the general solution, absence of investigation of given objects, and deficient argumentation. These results demonstrate the importance of the first steps of problem-solving: analysis and classification of objects given in the problem.



Graph 1. Proof problems on OMO.

The principles of lesson design for younger grade students

Problem solving (**PS**) takes the central place in the mathematical circle. This term includes deep content from the view point of teacher. When choosing a problem the teacher cogitates about the solution: its level of difficulty, creation of a solution plan, useful methods, additional sub-problems, comprehension of students, possible questions and explanations. The problem must be challenging and at the same time accessible.

Any MC lesson has been designed with consideration of two important guidelines: the ideas of psychology scientists and the basics of Olympiad mathematics.

According to Piaget studies, children aged 7 to 11 are in the operational stage of development and they can solve problems about the classification, arrangement and grouping, but they don’t understand abstract rules. Vigotsky’s research work in the field of epistemology shows that children’s learning processes have to be assisted by a skilled teacher and be organized progressively. Having studied how students learn, Bruner recommended the „spiral-like” learning where mathematical concepts must be revisited time after time, extending the content of the subject (Gage, Berliner).

The content of MCs in general must be like a good textbook: for example „There must be a lot of problems of different nature and different level of difficulty” and „the deductive elements must be introduced step-by-step” (Bonka, Andzhans).

PS processes cover different steps of intellectual activities that include detailed analysis of the given objects, mastering various PS strategies, generalization competence, and intuition. This complex process can be structured. For example, Professor Tomas Teepe developed a scheme of the PS process that combines the mind map with the great ideas of George Polya, Arthur Engel and Paul Zeitz (Teepe). In accordance with the findings of psychologists, primary school children have to start with the first level – understanding and analysis of the given objects. Teachers have to provide many additional practical activities in MCs so that children can touch and investigate objects, research the properties of given objects, sort them and experiment. Physical properties of objects improve the comprehension of elements' inter-correlation and create the basis for the formation of abstract thinking in children. New PS strategies have to be introduced step by step.

Any MC lesson has to be designed combining such different components as topic, problem set, PS methods, students' work forms; the teacher has to provide an open, joyful and motivating ambience in the classroom as well.

Practical experience

In Latvian MCs for young grade students there are varied miscellaneous work forms to consider the mathematical topics on number theory, arithmetic, algebra, combinatorial geometry, graph theory and others. Discussions, work in pairs or in groups, problem solution using supporting aids and materials, and games are used to teach students many solution strategies: try and check, research simple special case, make a table, draw, paint, search extreme element, use the Dirichlet's box principle and others. Special attention is paid to investigation and classification of the objects described in the given problem.

<u>Example 1.</u> A lesson about the system of linear equations was held in Adazhi high school MC. There were demonstrated PS without introducing symbolic variables. The solution methods used were making a table, drawing a picture, complete enumeration, and the trial and error method. The children actively participated in discussions and found their own solutions:

Problem 1. Small things are set into two magic boxes. In the white box there are 4 more things than in the red one. If we take two equal white boxes and a red box, the total number of small things is 23. One of the girls, Laila, solved the problem very fast. She stated that in every box there is more than one thing. Then she observed that white boxes contain an even number of things, therefore the number of things in the red box is odd. She tried 3 that did not fit. Number 5 was the correct answer. In her solution Laila used the number theory results considered in previous classes.

Example 2. In another lesson only one problem was investigated:

Problem 2. A large cube is made from 27 unit cubes. The large cube is dipping into a pot of red paint so the whole outer surface is covered. Then we rearrange the large cube in a different way and dip it in paint again. How many times do we have to dip the large cube into paint for all faces of all unit cubes to be red?

Here the lesson plan was carried out starting with simple introductory cases when the large cube consists of 1 or 8 unit cubes. The children announced some hypotheses. Very serious attention was paid to discovering particular properties of unit cubes after the first paint. The minimal and maximal number of dipping of any unit cube was detected. Then children worked out an algorithm, checked the hypothesis and tried to prove the minimality of the solution. First perplexity happened when encountering the problem – "What do we have to do with the large cube after the first dipping in paint?". Other difficulties came to the front when the algorithm of rearranging had to be described. Children practiced with unit cubes and colored stickers as to "paint" the large cube, decompose it and experiment with the arrangement of the large cube.

Conclusions

Any teacher of young students has to take into account the fact that every child is an individual with their own thinking model. The teacher's responsibility is to comprehend the solution method discovered by the student and not to impose the spatial pattern of a solution. The teacher must facilitate the students' creativity and show different ways of solving the problem. Activities in MCs of elementary school not only improve and expand students' knowledge, but also increase the professional qualification of teachers.

Literature

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