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Preservice Teachers' Patterns of Metacognitive Behavior During Mathematics Problem Solving in a Dynamic Geometry Environment

1. Introduction

The educational community holds a general acceptance of the important role metacognition and technology play in problem solving. Even though a plethora of research reported on the role of metacognition in problem solving (e.g., Garofalo & Lester, 1985; Schoenfeld, 1981, 1985) and on the importance of technology as a tool for mathematics problem solving (e.g., Fey, Hollenbeck, & Wray, 2010; NCTM, 2005; J. W. Wilson, Fernandez, & Hadaway, 1993), no study addressed the impact of working in dynamic geometry environments, such as the Geometer's Sketchpad, on student mathematics problem solving. New technological tools are becoming available and continually transform mathematical classrooms (Fey et al., 2010; NCTM, 2005), however, little is known about students' mathematical achievement with dynamic technology tools, problem solving schemas and mental models when solving nonroutine geometry problems.

In this paper I examined the metacognitive processes of two preservice teachers when solving nonroutine geometry problems in a dynamic geometry environment, namely the Geometer's Sketchpad. The main purpose of the study was to uncover and investigate patterns of metacognitive processes two preservice teachers exhibited and to understand how and why observed metacognitive processes emerged when problem solving in dynamic geometry environment. Moreover, this study sought to understand student perceptions about the importance of the Geometer's Sketchpad when faced with nonroutine geometry problems.

2. Theoretical Framework

For the purpose of uncovering and investigating patterns of metacognitive processes two preservice teachers exhibited when problem solving, a problem solving model adapted from Pólya (1945/1973), and Schoenfeld (1981, 1985) was used. In order to better understand the nature and interplay of the cognitive and metacognitive processes within each of the episodes, the nature of participants' answers with respect to their metacognitive awareness, metacognitive evaluation and metacognitive regulation (J. Wilson & Clarke, 2004) was taken into account. The resulting model was characterized by the following episodes: reading the problem, understanding the problem, analyzing what needs to be done, exploring different possibilities, planning the best solution, implementing the plan, and verifying the answer is a solution, together with junctions between episodes (transition). Artigue's (2002) instrumental approach was used to uncover what circumstances, interactions and situations promoted metacognitive behaviors when problem solving using the Geometer's Sketchpad; together describing the effects of tool use on the participant's activity (instrumentation) and transformation of the tool to fit participant's activity (instrumentalisation).

3. Methodology

Case studies were conducted of two mathematics education preservice teachers, from the mathematics education program at a large southeastern university in the United States, who had previously completed a semester of college geometry and had prior experience working in Geometer's Sketchpad. Data sources for this study consisted of different verbal reports (think aloud protocol, concurrent verbalization methods, such as prompts and probing), individual interviews after each problem-solving session, students' written solutions, researcher's observation notes, video files of problem solving sessions and a final interview. Each participant solved individually one nonroutine geometry problem per problem solving session. Three types of problems were used for this study: construction, applied, and exploration problem; that allowed exhibiting different mathematical thinking processes, both cognitive and metacognitive, multiple solution paths, the use of different strategies and different uses of the Geometer's Sketchpad using a variety of available functions. All collected data was analyzed using constant comparative method for both the within- and cross-case analysis.

4. Findings of the Study

Problem solving of the two participants was described through identifying the metacognitive processes within each problem-solving episode, and associating them with the Geometer's Sketchpad use. During the reading, understanding, and analysis episodes, the participants engaged in monitoring behaviors such as sense making, drawing a diagram, and allocating potential resources and approaches that helped make productive decisions. During the exploring, planning, implementation, and verification episodes, the participants made decisions to access and consider knowledge and strategies, make and test conjectures, monitor the progress, and assess the productivity of activities and strategies and the correctness of an answer. With respect to metacognitive processes within each of the episodes, it was evident that awareness of one's knowledge triggered selective attention, evaluation of one's thinking helped better planning for effective solution approaches, and regulation of one's thinking helped monitor progress, select appropriate problem solving strategies, and regulate missteps. Geometer's Sketchpad played an important role in supporting these metacognitive processes; it appeared to be integrated into the problem solving processes and strategies (trial-and-error, bottom-up) used by the participants. Both participants shared belief that Geometer's Sketchpad was important and useful tool during problem solving centering around these qualities: problem solving activities and processes, visualization, speed, and accuracy. For instance, it helped explore, gather information, experiment, conjecture, better understand the problem, relearn mathematical concepts, aided attaining accurate visual input and "fitting" all the pieces together, and triggered possible solution possibilities. Hence, Geometer's Sketchpad proved to be an important resource when working on nonroutine problems supporting flexibility in thinking, transfer of mathematical knowledge to unfamiliar situations and extension of previous knowledge.

The findings furthermore showed that substantial mathematical knowledge, prior problem solving experience, reliance on the use of technology, use of metacognitive questions, and affective behaviors, such as perseverance and frustration were related to participants' success when problem solving. The findings of this study suggest that effective management of negative affective behaviors and the presence of positive affective behaviors, such as perseverance were important factors contributing to successful problem solving.

In summary, the effectiveness of solution approaches was dependent on the presence of managerial decisions. Cognitive problem-solving actions not accompanied by appropriate metacognitive monitoring actions appeared to lead to unproductive efforts. Redirection and reorganizing of thinking in productive directions occurred when metacognitive actions guided the thinking and when affective behaviors were controlled. Hence, productive problem solving in a technology environment depends on factors, such as well-connected mathematical knowledge, metacognitive and reflective processes, generative knowledge of a DGE, and regulation of affective behaviors.

5. Implications

The findings of this study may be applied to the development of teaching materials for methods and problem solving courses to help consolidate preservice teachers' problem solving abilities and skills and to facilitate an understanding of their students' metacognitive activity. On the other hand, taking into consideration the influence of an increasingly global and technological society on teaching practices, teachers need to become aware of the pedagogical and cognitive implications of technology and be able to take advantage of technology as a powerful and engaging teaching tool. The opportunity to experience genuine problem solving, reflect on their metacognitive behaviors that are consistent with the use of the Geometer's Sketchpad, discuss curricular, pedagogical, and learning issues with respect to that mission in variety of contexts, and identify the possible effects they have on mathematical problem solving, teaching and learning is powerful.

References

Artigue, M. (2002). Learning mathematics in CAS environment: The genesis of a reflection about instrumentation and the dialects between technical and conceptual work. *International Journal of Computers for Mathematical Learning, 7*(3), 245–274.

Fey, J. T., Hollenbeck, R. M., & Wray, J. A. (2010). Technology and the mathematics curriculum. In B. J. Reys, R. E. Reys, & R. Rubenstein (Eds.), *Mathematics curriculum: Issues, trends, and future directions* (pp. 41–49). Reston, VA: National Council of Teachers of Mathematics.

Garofalo, J., & Lester, F. K. (1985). Metacognition, cognitive monitoring, and mathematical performance. *Journal for Research in Mathematics Education, 16,* 163–176.

National Council of Teachers of Mathematics. (2005). *Technology-supported mathematics learning environments*. Reston, VA: Author.

Pólya, G. (1973). *How to solve it: A new aspect of mathematical method*. Princeton, NJ: Princeton University Press. (Original work published in 1945).

Schoenfeld, A. H. (1981, April). *Episodes and executive decisions in mathematical problem solving.* Paper presented at the Annual meeting of the American Educational Research Association, Los Angeles, CA.

Schoenfeld, A. H. (1985). *Mathematical problem solving*. Orlando, FL: Academic Press.

Wilson, J., & Clarke, D. (2004). Towards the modelling of mathematical metacognition. *Mathematics Education Research Journal, 16*(2), 25–48.

Wilson, J. W., Fernandez, M. L., & Hadaway, N. (1993). Mathematical problem solving. In P. S. Wilson (Ed.), *Research ideas for the classroom: High school mathematics* (pp. 57–77). New York: Macmillan.